



Problem of the Week

Problem E and Solution

Extra Support

Problem

A container for bird seed is built in the shape of a trapezoidal prism. That is, the container is a prism in which opposite parallel ends are congruent trapezoids. The opposite parallel sides of each trapezoid are 18 cm and 8 cm. The non-parallel sides of each trapezoid are 8 cm and 6 cm. The container is 20 cm long. The container has volume 1248 cm^3 . In order to make the container stronger, a cable is attached from the top left corner of the front trapezoid to the bottom right corner of the trapezoid on the back end. Determine the length of the cable, in centimetres, to the nearest tenth.

Solution

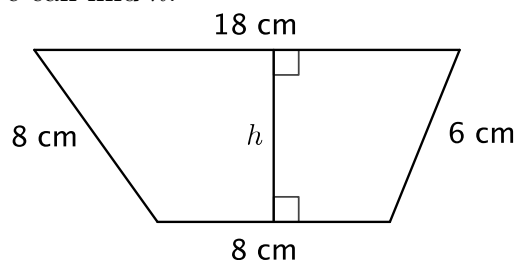
Let h represent the height of the trapezoid. We will determine the height using two different methods.

Method 1 Finding the Height Using the Given Volume

To find the volume of a prism, V , we multiply the area of one of the congruent bases by the perpendicular distance, d , between the two bases. Since the bases are trapezoids, we can calculate the area of the base using the formula $A = \frac{h(a+b)}{2}$, where h is the perpendicular distance between the two parallel sides a and b . We know

$V = 1248 \text{ cm}^3$, $d = 20 \text{ cm}$, $a = 8 \text{ cm}$, and $b = 18 \text{ cm}$. We can find h .

$$\begin{aligned} V &= \frac{h(a+b)}{2} \times d \\ 1248 &= \frac{h(8+18)}{2} \times 20 \\ 1248 &= 13h \times 20 \\ 1248 &= 260h \\ \therefore h &= 4.8 \text{ cm} \end{aligned}$$



Method 2 Finding the Height Without Using the Given Volume

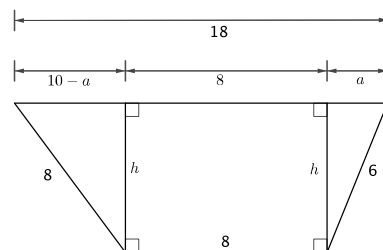
Break one of the trapezoids into two right triangles and a rectangle as shown in the diagram below. The longer side breaks into pieces $a \text{ cm}$, 8 cm , and $18 - 8 - a = 10 - a \text{ cm}$, respectively. Using the Pythagorean Theorem, we can find two different expressions for h .

$$h = \sqrt{8^2 - (10 - a)^2} \quad \text{and} \quad h = \sqrt{6^2 - a^2}$$

Since $h = h$, $\sqrt{8^2 - (10 - a)^2} = \sqrt{6^2 - a^2}$.

Squaring both sides and expanding, $64 - 100 + 20a - a^2 = 36 - a^2$.

Simplifying further, $20a = 72$ and $a = 3.6 \text{ cm}$.

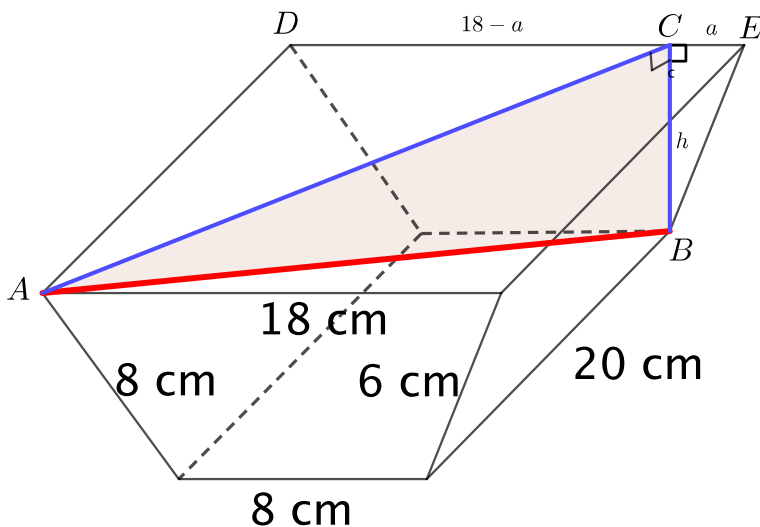


Substituting $a = 3.6$ into $h = \sqrt{36 - a^2}$, $h = \sqrt{36 - 3.6^2} = 4.8 \text{ cm}$.



Let A be the point on the top left of the front trapezoid and B represent the point on the bottom right of the back trapezoid. We want to find the length of AB . Let DE represent the length of the longer side of the back trapezoid with C on DE such that $DC \perp BC$. It follows that $BC = h = 4.8$ cm.

From method 2, we know that $CE = a = 3.6$ cm. Then $DC = DE - CE = 18 - a = 14.4$ cm.



The sides of the container are perpendicular to the ends so $AD \perp DC$ and $\triangle CDA$ is right-angled. Using the Pythagorean Theorem,

$$\begin{aligned} AC^2 &= DC^2 + AD^2 \\ &= 14.4^2 + 20^2 \\ &= 607.36 \end{aligned}$$

To find the required length, AB , we note that AC is a line segment drawn across the top of the container. BC is a line segment perpendicular to the top and bottom of the container. It follows that $\angle ACB = 90^\circ$ and $\triangle ACB$ is right-angled. We will use the Pythagorean Theorem to find the length AB .

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ &= 607.36 + 4.8^2 \\ &= 630.4 \\ AB &\doteq 25.1 \text{ cm, since } AB > 0 \end{aligned}$$

Therefore, the support wire is approximately 25.1 cm.

