



## Problem of the Week

### Problem E and Solution

### Measure Twice, Cut Once

#### Problem

You are given 100 cm of rope and are asked to cut the rope exactly once so that:

1. You have two pieces.
2. The first piece is used to form a rectangle with one side 8 cm long.
3. The second piece is used to form a square.
4. The area of the square and the area of the rectangle are equal.

Where should the cut be made?

#### Solution

Let the length of the piece of rope used to form the square be  $4x$  cm. This would be the perimeter of the square. Then the side length of the square would be  $4x \div 4 = x$  cm. The area of the square is  $(x)(x) = (x^2)$  cm<sup>2</sup>. (1)

The length of the piece of rope used to form the rectangle is  $(100 - 4x)$  cm. This would be the perimeter of the rectangle. If the width of the rectangle is 8 cm, then there are  $100 - 4x - 8 - 8 = (84 - 4x)$  cm left to form the lengths of the two other sides of the rectangle. Therefore, the length of the rectangle is  $\frac{84-4x}{2} = (42 - 2x)$  cm. The area of the rectangle is  $(8)(42 - 2x) = (336 - 16x)$  cm<sup>2</sup>. (2)

But the area of the square equals the area of the rectangle, so by equating (1) and (2) we obtain:

$$\begin{aligned} x^2 &= 336 - 16x \\ x^2 + 16x - 336 &= 0 \\ (x - 12)(x + 28) &= 0 \\ x = 12 \quad \text{or} \quad x = -28 \end{aligned}$$

Since  $x$  is the length of the side of the square,  $x = -28$  is inadmissible. Therefore,  $x = 12$  cm. The area of the square is  $12 \times 12 = 144$  cm<sup>2</sup>. The length of the rectangle is  $42 - 2x = 42 - 24 = 18$  cm. The area of the rectangle is  $18 \times 8 = 144$  cm<sup>2</sup>. (These calculations were not required but are provided as a check of the correctness of the result.)

Then  $4x = 4(12) = 48$  cm. The cut should be made 48 cm from one end creating a 52 cm piece for the rectangle and a 48 cm piece for the square.

