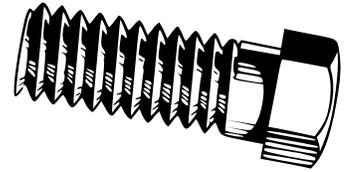


Problem of the Week

Problem E and Solution

Too Little, Too Much



Problem

A company produces steel bolts. Each bolt should have a mass of 7.0 g. However, it has been determined that each individual bolt may be heavier or lighter by as much as 2.14%. A box contains a number of these bolts and the total mass of the bolts in the box is exactly 1 kg. Determine the maximum and minimum number of bolts that could be in the box.

Solution

- **Maximum Number of Bolts**

The smallest possible mass for a single bolt is $7.0 - 0.0214 \times 7 = 7 \times 0.9786 = 6.8502$ g.

In order to find the maximum number of bolts, we need each bolt to be as light as possible. If all of the bolts were as light as possible, we would have approximately $\frac{1000 \text{ g}}{6.8502 \text{ g}} \approx 145.98$ bolts.

We cannot have a non-integer number of bolts. Therefore, there will be at most 145 bolts. If there were 146 bolts, then the total mass would be $146 \times 6.8502 = 1000.1292$ g > 1 kg.

To get the 145 bolts to have a mass of 1 kg, we could start with each bolt at the minimum mass and then make each slightly heavier. (145 bolts, each with a mass of 6.8502 g, would have a total mass of 993.279 g. This is under 1 kg by 6.721 g. Divide 6.721 g by 145 and add this small amount to each of the minimum mass bolts.)

- **Minimum Number of Bolts**

The largest possible mass for a single bolt is $7.0 + 0.0214 \times 7 = 7 \times 1.0214 = 7.1498$ g.

In order to find the minimum number of bolts, we need each bolt to be as heavy as possible. If all of the bolts were as heavy as possible, we would have approximately $\frac{1000 \text{ g}}{7.1498 \text{ g}} \approx 139.86$ bolts.

We cannot have a non-integer number of bolts. Therefore, there will be at least 140 bolts. If there were 139 bolts, then the total mass would be $139 \times 7.1498 = 993.8222$ g < 1 kg.

To get the 140 bolts to have a mass of 1 kg, we could start with each bolt at the maximum mass and then make each slightly lighter. (140 bolts, each with a mass of 7.1498 g, would have a total mass of 1000.972 g. This is over 1 kg by 0.972 g. Divide 0.972 g by 140 and remove this amount from each of the maximum mass bolts.)

The minimum number of bolts is 140 and the maximum number of bolts is 145.

