Problem of the Week

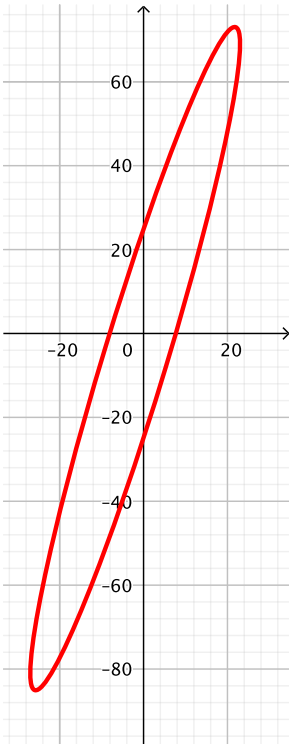
Problem E

Elliptical Workout

When we graph the equation $(x+1)^2 + (y-2)^2 = 100$, we get a circle centred at the point $(-1, 2)$ with a radius of 10.

When we graph $10x^2 - 6xy + 4x + y^2 = 621$, we get a shape known as an ellipse. This is the graph shown below.

List all the ordered pairs (x, y) of non-negative integers x and y that satisfy the equation $10x^2 - 6xy + 4x + y^2 = 621$.



NOTE: The following idea might be useful.

By completing the square,

$$x^2 + y^2 + 2x - 8y = 83$$

can be rewritten as

$$(x+1)^2 + (y-4)^2 = 100.$$

One solution to this equation is $(x, y) = (5, 10)$.