


Problem of the Week
Problem E and Solution
Math and Movement

Problem

A walker and a jogger left the town of Pi at the same time, heading toward the town of Epsilonville. The walker and jogger travel at constant speeds of 6 km/h and 10 km/h, respectively. At the same time, a biker left Epsilonville, heading toward the town of Pi. The biker travels at a constant speed of 14 km/h. If the biker passed the walker 4 minutes after passing the jogger, how far apart are the towns Pi and Epsilonville?

Solution

Let d be the distance, in km, between the towns Pi and Epsilonville.
Let t be the time, in hours, until the jogger and biker meet.
The biker meets the walker 4 minutes, or $\frac{1}{15}$ hours, after meeting the jogger. Therefore, $\left(t+\frac{1}{15}\right)$ is the time, in hours, until the biker meets the walker.

Using the formula distance = speed × time,
In t hours, the biker travels $14t$ km.
In t hours, the jogger travels $10t$ km.
Between the biker and jogger, they travel the total distance between Pi and Epsilonville.
Therefore, $d = 14t + 10t = 24t$ (1).

Again, using the formula distance = speed × time,
In $\left(t+\frac{1}{15}\right)$ hours, the biker travels $14\left(t+\frac{1}{15}\right)$ km.
In $\left(t+\frac{1}{15}\right)$ hours, the walker travels $6\left(t+\frac{1}{15}\right)$ km.
Between the biker and walker, they travel the total distance between Pi and Epsilonville.
Therefore, $d = 14\left(t+\frac{1}{15}\right) + 6\left(t+\frac{1}{15}\right) = 20\left(t+\frac{1}{15}\right)$ (2).

But $d = d$ in (1) and (2). Therefore,

$$\begin{aligned} 24t &= 20\left(t+\frac{1}{15}\right) \\ 24t &= 20t + \frac{4}{3} \\ 4t &= \frac{4}{3} \\ t &= \frac{1}{3} \text{ hours} \end{aligned}$$

Substituting $t = \frac{1}{3}$ into (1), we find $d = 24t = 24(\frac{1}{3}) = 8$ km.

Therefore, the towns of Pi and Epsilonville are 8 km apart.