



Problem of the Week

Problem D and Solution

Concentrate



Problem

A large jug of lemonade contains a mixture of lemon juice concentrate and water. When 1 litre of water is added to the jug, the ratio, by volume, of juice concentrate to water is 1 : 2. When 1 litre of juice concentrate is added to the new mixture, the ratio becomes 2 : 3. Find the original ratio, by volume, of lemon juice concentrate to water in the jug.

Solution

Let j be the amount of juice, in litres, in the original mixture.

Let w be the amount of water, in litres, in the original mixture.

When 1 litre of water is added, the ratio of juice concentrate to water is 1 : 2.

This tells us that

$$\frac{j}{w+1} = \frac{1}{2} \quad (1)$$

Simplifying (1), we obtain $w+1 = 2j$ and $w = 2j - 1$ follows. (2)

When 1 litre of juice concentrate is added to the new mixture, the ratio becomes 2 : 3. This tells us that

$$\frac{j+1}{w+1} = \frac{2}{3} \quad (3)$$

From (2), since $w = 2j - 1$, we can substitute for w in (3), obtaining:

$$\begin{aligned} \frac{j+1}{2j-1+1} &= \frac{2}{3} \\ \frac{j+1}{2j} &= \frac{2}{3} \\ 2(2j) &= 3(j+1) \\ 4j &= 3j+3 \\ j &= 3 \end{aligned}$$

Substituting $j = 3$ in $w+1 = 2j$ we obtain $w+1 = 6$ and $w = 5$ follows.

So there are 3 litres of juice concentrate in the container and 5 litres of water.

The ratio of juice concentrate to water is 3 : 5.

