

Problem of the Week

Problem D and Solution

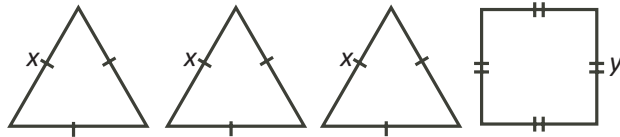
Roped In

Problem

A rope of length 200 cm is cut into four pieces. Three of the pieces are used to form identical equilateral triangles with integer side lengths. The fourth piece is used to form a square with integer side lengths. Determine all possible side lengths for each triangle and square.

Solution

Let x represent the integer side length of each equilateral triangle and let y represent the integer side length of the square.



The perimeter of each figure is the length of the piece of rope used to form it. For each triangle, the length of rope is $3x$ and for the square the length of rope is $4y$. The total rope used is $3(3x) + 4y = 9x + 4y$. But the length of the rope is 200 cm. Therefore,

$$\begin{aligned} 9x + 4y &= 200 \\ 9x &= 200 - 4y \\ x &= \frac{4(50 - y)}{9} \end{aligned}$$

Since both x and y are integers, $4(50 - y)$ must be a multiple of 9. But 4 is not divisible by 9, so $50 - y$ must be divisible by 9. There are five multiples of 9 between 0 and 50, namely 9, 18, 27, 36, and 45. So $50 - y = \{9, 18, 27, 36, 45\}$ and it follows that $y = \{41, 32, 23, 14, 5\}$. The corresponding values of x are computed in the chart below.

| y | $4y$ | $200 - 4y$ | $x = \frac{200 - 4y}{9}$ |
|-----|------|------------|--------------------------|
| 41 | 164 | 36 | 4 |
| 32 | 128 | 72 | 8 |
| 23 | 92 | 108 | 12 |
| 14 | 56 | 144 | 16 |
| 5 | 20 | 180 | 20 |

When the side length of the square is 41 cm, the side length of each triangle is 4 cm; when the side length of the square is 32 cm, the side length of each triangle is 8 cm; when the side length of the square is 23 cm, the side length of each triangle is 12 cm; when the side length of the square is 14 cm, the side length of each triangle is 16 cm; and when the side length of the square is 5 cm, the side length of each triangle is 20 cm.

