

Problem of the Week

Problem D and Solution

How are We Related?

Problem

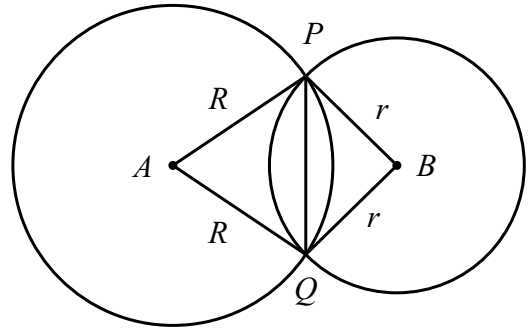
Two circles, one with centre A and one with centre B , intersect at points P and Q such that $\angle PAQ = 60^\circ$ and $\angle PBQ = 90^\circ$. How is the area of the circle with centre A related to the area of the circle with centre B ?

Solution

Let R be the radius of the circle with centre A and r be the radius of the circle with centre B .

Join P to Q .

We will determine the length of PQ in terms of R and then in terms of r in order to find a relationship between R and r .



Consider $\triangle APQ$. Since $AP = AQ = R$, $\triangle APQ$ is isosceles and so $\angle APQ = \angle AQP$. Since $\angle PAQ = 60^\circ$, $\angle APQ = \angle AQP = \frac{180^\circ - 60^\circ}{2} = 60^\circ$. Therefore, $\triangle APQ$ is equilateral and $PQ = AP = AQ = R$.

Consider $\triangle BPQ$. We are given that $\angle PBQ = 90^\circ$. Therefore, $\triangle BPQ$ is a right-angled triangle. The Pythagorean theorem tells us that $PQ^2 = BP^2 + BQ^2 = r^2 + r^2 = 2r^2$.

We have $PQ = R$ and $PQ^2 = 2r^2$. Therefore, $R^2 = 2r^2$.

The area of the circle with centre B and radius r is πr^2 .

The area of the circle with centre A and radius R is

$$\pi R^2 = \pi(2r^2) = 2(\pi r^2) = 2 \times (\text{the area of the circle with centre } B).$$

Therefore, the area of the circle with centre A is twice the area of the circle with centre B .

