Problem of the Week
Problem D and Solution
Cubicles

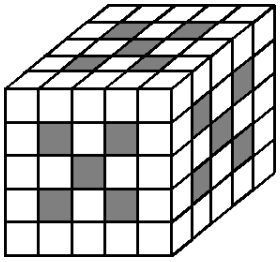
Problem

A 5 by 5 by 5 cube is formed using identical 1 by 1 by 1 cubes. A number of the smaller cubes are removed by punching out the fifteen designated columns from front to back, top to bottom, and side to side. The columns to be removed are shaded grey on the diagram. What percentage of the volume of the original cube remains following the removal of the fifteen columns?

Solution

Solution 1

In this solution, we analyze how many cubes are removed at each of the following stages: when removing the columns from front to back, when removing the columns from top to bottom, and finally when removing the columns from side to side.



When removing columns from the front to the back, 5 smaller cubes are removed from each layer. A total of 25 cubes are removed during this stage.

When removing cubes from top to bottom, the number of cubes removed from each layer is no longer the same. In order from top to bottom, the number of cubes removed at each layer is 5, 1, 4, 1, and 5. A total of 16 additional cubes are removed during this stage.

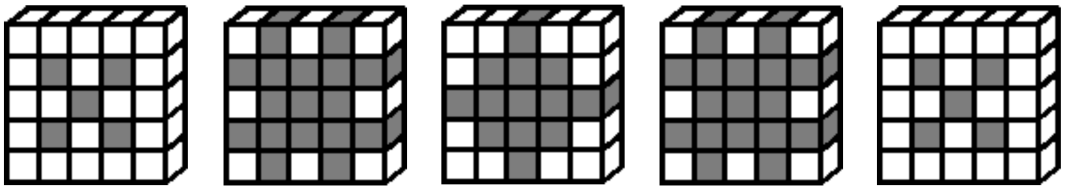
Finally, when removing cubes from side to side, the number of cubes removed at each layer is 5, 1, 4, 1, and 5, the same as the number removed in going from top to bottom. A total of 16 additional cubes are removed during this final stage.

The total number of cubes removed is $25 + 16 + 16 = 57$. The original 5 by 5 by 5 cube had $5 \times 5 \times 5 = 125$ of the smaller 1 by 1 by 1 cubes. The number of cubes remaining is $125 - 57 = 68$. The percentage of the original cube remaining after the removal of the fifteen columns is $68 \div 125 \times 100\% = 54.4\%$.

Solution 1 forces the solver to visualize the solution. The second solution will be more concrete.

Solution 2

After the columns have been removed, peel off each of the layers from front to back. Each layer is illustrated in the diagrams shown below.



In the first layer, 20 of the 1 by 1 by 1 cubes remain. In the second layer, 8 of the 1 by 1 by 1 cubes remain. In the third layer, 12 of the 1 by 1 by 1 cubes remain. In the fourth layer, 8 of the 1 by 1 by 1 cubes remain. And in the final layer, 20 of the 1 by 1 by 1 cubes remain.

A total of $20 + 8 + 12 + 8 + 20 = 68$ of the 1 by 1 by 1 cubes remain. There were 125 of the 1 by 1 by 1 cubes in the original 5 by 5 by 5 cube.

The percentage of the original cube remaining after the removal of the fifteen columns is $68 \div 125 \times 100\% = 54.4\%$.