



$$A = B$$

Problem of the Week

Problem C and Solution

Exponential Primer

Problem

We are given two expressions:

Expression A	Expression B
72×5^x	225×2^y

Given that x and y are positive integers, find all ordered pairs (x, y) so that Expression A equals Expression B .

Solution

Solution 1

We write each expression as the product of prime factors.

Expression $A = (2^3)(3^2)(5^x)$ and Expression $B = (3^2)(5^2)(2^y)$.

Since x and y are each positive integers and the expressions are equal, then the corresponding exponents for each prime number must be equal. Therefore, $x = 2$ and $y = 3$ are the only integer solutions for x and y , and the only ordered pair is $(2, 3)$.

Solution 2

Setting the two expressions equal to each other,

$$72 \times 5^x = 225 \times 2^y.$$

Dividing both sides by 9,

$$8 \times 5^x = 25 \times 2^y.$$

Expressing each side of the equation as the product of prime factors,

$$2^3 \times 5^x = 5^2 \times 2^y.$$

Since x and y are each positive integers and the expressions are equal, then the corresponding exponents for each prime number must be equal. Therefore, $x = 2$ and $y = 3$ are the only integer solutions for x and y , and the only ordered pair is $(2, 3)$.

