



Problem of the Week

Problem C and Solution

Picture Perfect Planets

Problem

A distant solar system in the Andromeda Galaxy has a star with 4 orbiting planets, Yrucrem, Sunev, Htrae and Sram. Each of these planets happens to have a different orbital period. Yrucrem's is 2 years, Sunev's is 6 years, Htrae's is 9 years and Sram's is 15 years.

If all 4 planets were aligned in the year of 2018 (as shown above left), in what year will all 4 planets have the same alignment?

Solution

Since Yrucrem has an orbital period of 2 years, it will be back in its original position every 2 years. Therefore, Yrucrem will be in the same position in years 2, 4, 6, 8, 10, 12, . . .

Since Sunev has an orbital period of 6 years, it will be back in its original position every 6 years. Therefore, Sunev will be in the same position in years 6, 12, 18, 24 . . .

Therefore, both Yrucrem and Sunev are back in their original, aligned positions after years 6, 12, 18, . . . This happens because these are *common multiples* of the orbital periods of Yrucrem and Sunev.

If we want to determine when all 4 planets will next align, we need to find the *lowest common multiple* (LCM) of all 4 orbital periods. We shall solve this in two ways.

Solution 1

The first way to find the LCM is to list off multiples of each orbital period, until you find a common multiple in each list.

Yrucrem \rightarrow 2, 4, 6, 8, 10, 12, 14, . . ., 86, 88, **90**, 92, 94, . . .

Sunev \rightarrow 6, 12, 18, 24, 30, 36, 42, . . ., 72, 78, 84, **90**, 96, 102, . . .

Htrae \rightarrow 9, 18, 27, 36, 45, 54, 63, 72, 81, **90**, 99, 108, . . .

Sram \rightarrow 15, 30, 45, 60, 75, **90**, 105, 120, . . .

So the LCM for the 4 orbital periods is 90. Therefore, it will take 90 years until this alignment occurs again. This will be the year 2108.

Solution 2

The second way to determine the LCM is to rewrite each number as a prime or a product of prime numbers. (This is known as *prime factorization*.)

$2 = 2$, $6 = 2 \times 3$, $9 = 3 \times 3$, and $15 = 5 \times 3$

The LCM is calculated by determining the greatest number of each prime number in any of the factorizations (here we will have one 2, two 3s and one 5), and then multiplying these numbers together. This gives the $\text{LCM}(2, 6, 9, 15) = 2 \times 3 \times 3 \times 5 = 90$. Therefore, it will take 90 years until this alignment occurs again. This will be the year 2108.

NOTE: The second method is a more efficient way to find the LCM. Especially, when the numbers are quite large.

