



Problem of the Week

Problem C and Solution

Edible Math

Problem

Kuri Uz is given some red licorice that is wrapped in a coil. Upon flattening the licorice, Kuri discovers that the total length is 60 cm. She then cuts the licorice into two pieces such that the ratio of the lengths of the two pieces is 7 : 3. Each piece is then bent to form a square. What is the total area of the two squares?

Solution

Since the licorice is cut in the ratio 7 : 3, let the longer piece be $7x$ cm and the shorter piece be $3x$ cm. Then $7x + 3x = 60$ or $10x = 60$ and $x = 60 \div 10 = 6$. Therefore, the longer piece is $7x = 7 \times 6 = 42$ cm and the shorter piece is $3x = 3 \times 6 = 18$ cm.

Each of the two pieces is then bent to form a square. The perimeter of each square is the length of the licorice used to form it. The side length of the longer square is $42 \div 4 = 10.5$ cm and the side length of the shorter square is $18 \div 4 = 4.5$ cm.

To find the area of each square, we multiply length by width. In effect, to find the area of the square, we square the side length. The area of the larger square is $10.5 \times 10.5 = 10.5^2 = 110.25$ cm² and the area of the smaller square is $4.5 \times 4.5 = 4.5^2 = 20.25$ cm².

Therefore, if Kuri Uz cuts the licorice into pieces of length 42 cm and 18 cm, she can form two squares with area 110.25 cm² and 20.25 cm², respectively. The total area of the two squares is $110.25 + 20.25 = 130.5$ cm².

For Further Thought

The ratio of the area of the larger square to the area of the smaller square is $110.25 : 20.25 = 11025 : 2025 = 441 : 81 = 49 : 9 = 7^2 : 3^2$.

The ratio of the perimeters is 7 : 3 and the ratio of the areas is $7^2 : 3^2$. In general, if the ratio of the perimeters of two squares is $a : b$, is it true that the ratio of the areas of the two squares is $a^2 : b^2$?

