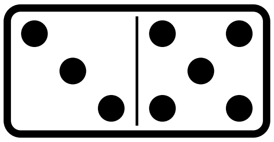

Problem of the Week

Problem C and Solution

Dominoes

Problem

A 10-set of dominoes contains all the tiles with the number of pips on any end ranging from 0 to 10, and no two dominoes can be the same. How many tiles are in a 10-set of dominoes?

Solution

Since rotating a domino tile does not change the domino, let's orient each non-double tile so that the smaller number is always on the left end of the tile.

For each possible number on the left side of the domino, we examine the possible numbers that can occur on the right side of the domino and compile this information in a table.

Number on Left Side	Possible Numbers on Right Side	Total Number of Dominoes
0	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	11
1	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	10
2	2, 3, 4, 5, 6, 7, 8, 9, 10	9
3	3, 4, 5, 6, 7, 8, 9, 10	8
4	4, 5, 6, 7, 8, 9, 10	7
5	5, 6, 7, 8, 9, 10	6
6	6, 7, 8, 9, 10	5
7	7, 8, 9, 10	4
8	8, 9, 10	3
9	9, 10	2
10	10	1

Therefore, the total number of dominoes in a 10-set is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66.$$

Did you know?

Do you know the quick way to calculate the sum $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11$?

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = \dfrac{(11)(11 + 1)}{2}$. Why is this true?

Let S represent the sum.

$$\begin{array}{ccccccccccccccccccccccc}
S & = & 1 & + & 2 & + & 3 & + & 4 & + & 5 & + & 6 & + & 7 & + & 8 & + & 9 & + & 10 & + & 11 \\
S & = & 11 & + & 10 & + & 9 & + & 8 & + & 7 & + & 6 & + & 5 & + & 4 & + & 3 & + & 2 & + & 1 \\
\hline
2S & = & 12 & + & 12 & + & 12 & + & 12 & + & 12 & + & 12 & + & 12 & + & 12 & + & 12 & + & 12 & + & 12
\end{array}$$

$2S = 11(12)$ and so $S = \dfrac{(11)(12)}{2} = 66$.

In general, it can be shown that if n is a positive integer, then the sum of the integers from 1 to n is $\dfrac{(n)(n + 1)}{2}$. In other words, $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$.