Problem of the Week
Problem C and Solution
On the Button

Problem

Enzo has two piles of buttons. In each pile there are red and blue buttons. In one pile, the ratio of the number of red buttons to blue buttons is $1 : 2$. In the second pile the ratio of red buttons to blue buttons is $3 : 5$. If Enzo has a total of 20 red buttons, what are the possible number of blue buttons?

Solution

Solution 1

In the first two columns of the following table, the only possible combinations of red buttons and blue buttons in pile 2 are shown. In the third column, we determine the number of red buttons in pile 1 by subtracting the number of red buttons in pile 2 from 20, since the total number of red buttons is 20.. Then, we find the number of blue buttons in pile 1 by multiplying by the number of red buttons by 2. In the final column we determine the total number of blue buttons from both piles.

PILE 2		PILE 1		
Number of Red Buttons	Number of Blue Buttons	Number of Red Buttons	Number of Blue Buttons	Total of Blue Buttons
3	5	$20 - 3 = 17$	34	$5 + 35 = 39$
6	10	$20 - 6 = 14$	28	$10 + 28 = 38$
9	15	$20 - 9 = 11$	22	$15 + 22 = 37$
12	20	$20 - 12 = 8$	16	$20 + 16 = 36$
15	25	$20 - 15 = 5$	10	$25 + 10 = 35$
18	30	$20 - 18 = 2$	4	$30 + 4 = 34$

Therefore, there are six possible values for the total number of blue buttons. The six possible values are 34, 35, 36, 37, 38 and 39.

Solution 2 is on the next page.

Solution 2

In the first pile, the ratio of red buttons to blue buttons is $1 : 2$ so let the actual number of red buttons in this pile be a and the actual number of blue buttons in this pile be $2a$. We know that a is a positive integer.

In the second pile, the ratio of red buttons to blue buttons is $3 : 5$ so let the actual number of red buttons in this pile be $3b$ and the actual number of blue buttons in this pile be $5b$. We know that b is a positive integer.

In total, there are 20 red buttons so the number of red buttons in pile 1 plus the number of red buttons in pile 2 sums to 20. This can be represented by the equation $a + 3b = 20$. Since there are $2a$ blue buttons in pile 1 and $5b$ blue buttons in pile 2, the total number of blue buttons is $2a + 5b$.

We consider all the possible values for a and b that satisfy the equation $a + 3b = 20$. Using these values of a and b we can find the possible values of $2a + 5b$ and hence the possible values for the total number of blue buttons.

The results are shown in the table below.

$a + 3b$	a	b	$2a + 5b$
20	2	6	34
20	5	5	35
20	8	4	36
20	11	3	37
20	14	2	38
20	17	1	39

No other positive integer values for a and b exist that satisfy the equation $a + 3b = 20$.

Therefore, there are six possible values for the total number of blue buttons. The six possible values are 34, 35, 36, 37, 38 and 39.