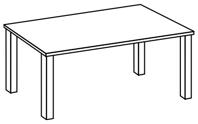

Problem of the Week

Problem C and Solution

Cover Up

Problem

Sara Mictile has an unlimited supply of square tiles. She has 1 cm by 1 cm tiles, 2 cm by 2 cm tiles, 3 cm by 3 cm tiles, and so on. Every tile has integer side lengths. A rectangular table top with an 84 cm by 112 cm surface is to be completely covered by identical square tiles, none of which can be cut. Sara knows that the table can be completely covered with 1 cm $\times$ 1 cm tiles, 9408 in total, since $84 \times 112 = 9408$ cm^2. However, Sara wants to use the minimum number of identical tiles to complete the job in order to reduce the overall material cost. Determine the minimum number of identical tiles required to completely cover the table top.

Solution

To use the smallest number of tiles, Sara must use the largest tile possible. The square tile must have sides less than or equal to 84 cm. If the tile has side length greater than 84 cm, it would have to be cut to fit the width of the table.

Since the tiles are square and must completely cover the top surface, the side length of the tile must be a number that is a factor of both 84 and 112. In fact, since Sara needs the largest side length, she is looking for the greatest common factor of 84 and 112.

The factors of 84 are

$$1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, \text{ and } 84.$$

The factors of 112 are

$$1, 2, 4, 7, 8, 14, 16, 28, 56, \text{ and } 112.$$

The largest number common to both lists is 28. Therefore, the greatest common factor of 84 and 112 is 28. The required tiles are 28 cm $\times$ 28 cm. Since $84 \div 28 = 3$, the surface is 3 tiles wide. Since $112 \div 28 = 4$, the surface is 4 tiles long. The minimum number of tiles required is $3 \times 4 = 12$ tiles.

The number of 28 cm $\times$ 28 cm tiles required to cover the top of the table is 12. This is the minimum number of tiles required.