

Problem of the Week

Problem C and Solution

Cut Up

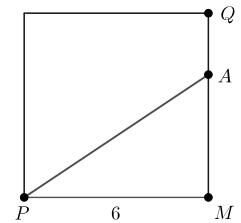
Problem

The 6 m by 6 m square shown above is to be divided into three equal areas using two cuts. One possible way for you to cut the square would be to make a horizontal slice through H and a second horizontal slice through K . This method of cutting the square works but is not very creative. To make things a little more interesting, you must still make two straight cuts but each cut must start at point P . Each of these two cuts will pass through a point on the outside of the square. Which other labelled points will the cuts pass through?

Solution

The area of the entire 6 m by 6 m square is $6 \times 6 = 36 \text{ m}^2$. Since the square is divided into three regions of equal area, the area of each region must be $\frac{36}{3} = 12 \text{ m}^2$.

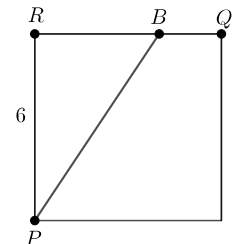
Consider the line through P that passes through some point on the vertical line segment QM . Let A be the point where this line intersects QM . Since $\angle PMQ = 90^\circ$, $\triangle PMA$ is a right triangle with base $PM = 6 \text{ m}$ and height MA .



Using the formula $\text{area} = \frac{\text{base} \times \text{height}}{2}$, we have $\text{area of } \triangle PMA = \frac{6 \times MA}{2} = 3 \times MA$.

We require that the area of $\triangle PMA = 12 \text{ m}^2$. Therefore, $3 \times MA = 12$ and so $MA = 4 \text{ m}$. Since H is the point on QM with $MH = 4 \text{ m}$, we must have $A = H$. Therefore, one line passes through the point H .

Consider the line through P that passes through some point on the horizontal line segment RQ . Let B be the point where this line intersects RQ . Since $\angle PRQ = 90^\circ$, $\triangle PRB$ is a right triangle with height $PR = 6 \text{ m}$ and base RB .



Using the formula $\text{area} = \frac{\text{base} \times \text{height}}{2}$, we have $\text{area of } \triangle PRB = \frac{RB \times 6}{2} = 3 \times RB$.

We require that the area of $\triangle PRB = 12 \text{ m}^2$. Therefore, $3 \times RB = 12$, and so $RB = 4 \text{ m}$. Since V is the point on RQ with $RV = 4 \text{ m}$, we must have $B = V$.

Therefore, the other line passes through the point V .

Therefore, one line passes through the point H and the other passes through the point V .

Extension:

Now try three cuts to make three equal areas. Make one cut from point P to a point inside the square located at a vertex common to four of the smaller squares. From this inner point make two different cuts to points on the outside of the square. Once the three cuts are performed, there are three sections of equal area. How are these cuts made? (There are many solutions.)

