Problem of the Week
Problem B and Solution
Sh-Rinking

Problem

Over the summer, Randy and Sarah have built the base for a rectangular outdoor skating rink which is 25 m by 10 m, surrounded by boards. Now it's winter, and time to make the rink!

a) How many litres of water will they need to fill the rink to a depth of 10 cm, assuming the ground is level?

b) To resurface the rink, their Dad gets out his 4-wheeler and blade and scrapes off 3 mm of ice. What is the volume of ice removed?

EXTENSION: In Science class, they learned that water expands by 9% when it freezes. How much less water would be needed so that the ice will still be 10 cm deep?

Solution

a) Converting 25 m and 10 m to centimetres, we obtain 2500 cm and 1000 cm, respectively.

The volume of water required, in cubic centimetres, is $10 \times 2\,500 \times 1\,000 = 25\,000\,000\,\text{cm}^3$.

Since 1 litre is equal to $1\,000\,\text{cm}^3$, this volume of water will be equivalent to $25\,000\,000 \div 1000 = 25\,000$ litres.

b) Converting 3 mm to centimetres, we obtain 0.3 cm. The volume of ice removed is $0.3 \times 2\,500 \times 1\,000 = 750\,000\,\text{cm}^3$ or 750 litres.

EXTENSION: Whatever amount of water we require must be such that this amount plus 9% of this amount sums to 25 000 litres. That is, 109% of the amount must be 25 000 litres. This means that 1.09 times the required amount of water should be 25 000 litres.

If we divide 25 000 by 1.09, we obtain approximately 22 936 litres.

It follows that approximately $25\,000 - 22\,936 = 2\,064$ fewer litres are needed.

NOTE: It is tempting to think as follows: We don't need 9% of the 25 000 litres, or 2 250 litres. But if we follow that to a conclusion, we'd use $25\,000 - 2\,250 = 22\,750$ litres of water, which would give an amount of ice equal to only $1.09 \times 22\,750 = 24\,797.5$ litres, not quite enough.

Many solvers would have used some sort of guess and check approach to obtain the answer. A possible guess and check approach is shown on the following page.

Finding the amount of water needed using a guess and check approach is tedious. We want an amount of water such that when we increase that amount by 9% we have 25 000 litres of water.

Amount of Water (in litres)	9% of Amount of Water (in litres)	Total Amount of Water (in litres)	Conclusion
20 000	1800	21 800	not enough
22 000	1980	23 980	not enough
23 000	2070	25 070	too much, but close
22 900	2061	24 961	close but under
22 950	2065.5	25 015.5	a bit too much
22 940	2064.6	25 004.6	so close
22 935	2064.15	24 999.15	close but under
22 936	2064.24	25 000.24	close but over

We will restrict our search to finding a whole number amount which when increased by 9% gives an amount closest to 25 000 litres. We just need to determine which amount is closest to 25 000 litres when it expands by 9%.

Looking at the last two rows of the table we see that 25 000.24 is closer to 25 000 than 24 999.15.

It follows that approximately $25\,000 - 22\,936 = 2\,064$ fewer litres are needed.