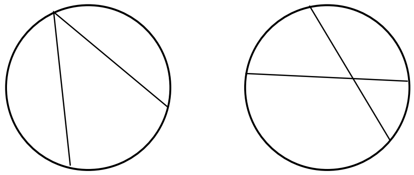


Problem of the Week
Problem B and Solution
This Strikes a Chord

Problem

A line segment that has its endpoints on a circle is called a *chord*.

a) What is the maximum number of pieces into which a circle can be divided by three chords?

b) Sketch all the ways a circle can be subdivided by three chords. How is the number of intersections related to the number of pieces?

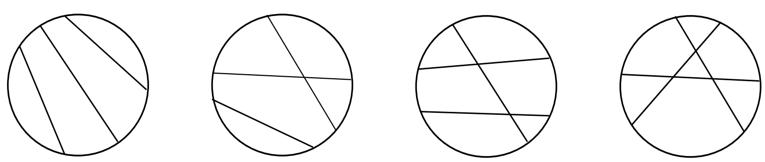
c) Find all the ways a circle can be subdivided by four chords. Is the number of intersections related to the number of pieces in the same way as in part b)?

d) If you used six chords, what would you predict to be the maximum number of pieces? Explain your reasoning.

Solution

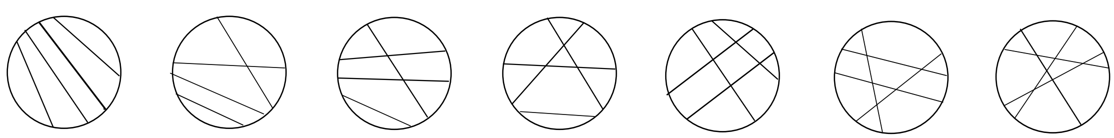
In the following, the letter **i** refers to the number of intersections, and the letter **p** refers to the corresponding number of pieces into which the circle is divided.

a),b) The ways a circle can be divided by three chords is shown below. The maximum number of pieces is 7. Note that in each case, the number of pieces is 4 more than the number of intersections.



i=0, p=4 i=1, p=5 i=2, p=6 i=3, p=7

c) For four chords, the possibilities are shown below. In this case the number of pieces exceeds the number of intersections by 5 in each case. Thus we see that for 2, 3, and 4 chords, if n is the number of chords, then $p = i + n + 1$.



i=0, p=5 i=1, p=6 i=2, p=7 i=3, p=8 i=4, p=9 i=5, p=10 i=6, p=11

d) Think of placing the chords one after the other. Since each chord can intersect all the previous chords, the maximum number of intersections for six chords will be $1 + 2 + 3 + 4 + 5 = 15$. Observing the results of parts a), b), c), we predict that the number of pieces here will be $p = i + 6 + 1$. Thus, for six chords, the maximum number of pieces will be $15 + 6 + 1 = 22$, as shown below.