Problem of the Week
Problem A and Solution
How Many Players?

Problem

Robbie likes to play many different card games. A deck has 52 cards made up of: aces, twos, threes, fours, fives, sixes, sevens, eights, nines, tens, jacks, queens, and kings. There are four of each of these values in the deck. When you deal the cards, you give each player the same number of cards to start. In some games you might deal all of the cards. In other games, you might deal some of the cards and have some cards left over. Answer the following questions about different card games Robbie plays.

A) **Bridge** is a game with four players. You deal all of the cards in the deck. How many cards does each player get?

B) In the game **99**, each player starts with three cards. How many cards are left over after dealing if there are six people playing the game?

C) Each player starts with eight cards in the game of **Crazy Eights**. To play the game, you must have at least 10 cards left over after dealing. What is the maximum number of players who can play **Crazy Eights** using these rules?

D) The game of **Euchre** does not use any of the cards with numbers 2, 3, 4, 5, 6, 7, or 8. How many cards does this game use?

Solution

A) We could use skip counting or repeated subtraction by fours to see how many cards each player gets from the 52 card deck. Alternatively, we can calculate the result by dividing: $52 \div 4 = 13$. Using any of these techniques, we see that each player gets 13 cards.

B) Since each of the 6 players starts with 3 cards, we can multiply $6 \times 3 = 18$ to see that there are 18 cards dealt at the beginning of the game. Since there are 52 cards in the deck, then after dealing there would be $52 - 18 = 34$ cards left over.

C) We can use repeated subtraction of 8, starting with 52, to calculate this answer:

$$52 - 8 = 44$$
$$44 - 8 = 36$$
$$36 - 8 = 28$$
$$28 - 8 = 20$$
$$20 - 8 = 12$$
$$12 - 8 = 4$$

When we subtract 8 repeatedly 5 times, we get 12. When we subtract 8 repeatedly 6 times, we get 4. Since we need at least 10 cards left after dealing the cards, we can have a maximum of 5 players in the game.

Alternatively, since we need at least 10 cards left after dealing, then we can deal a maximum of $52 - 10 = 42$ cards to the players. Since each player needs 8 cards, we can calculate $42 \div 8 = 5$ with a remainder of 2. Therefore, we can deal in a maximum of 5 players to start the game.

D) Since there are seven values of cards that are not used in this game, and there are four of each value in the deck, then there is a total of $7 \times 4 = 28$ cards not used. Since there are 52 cards in the deck, then there is a total of $52 - 28 = 24$ cards in play.

Alternatively, we could also notice that only the aces, nines, tens, jacks, queens, and kings **are** used in the game. This means that there are six values of cards that are used in this game, and there are four of each value in the deck. Therefore a total of $6 \times 4 = 24$ cards are in play.

Teacher's Notes

In part C) of this problem, Robbie needs "at least 10 cards left after dealing". In this context, **at least** implies the mathematical operator $\geq$ (i.e. greater than or equal to). We could have written the same information this way:

$$\text{cards left over} \geq 10$$

Although we often think of solving math problems as finding a precise result, in many cases we are interested in finding a range of values. Part C) asks for the maximum number of players, but if we asked for the possible number of players the answer would have been 2, 3, 4, or 5. There are many ways to describe this range of answers. In each case it is important to make sure we clearly indicate that 2 and 5 must be included in the range and that 1 and 6 are not included. We should also make it clear that our range in this case are only integer values, since when we use operators like $\geq$ or $<$ we could be including other real numbers such as fractions.

Here are a few different ways to describe our range of values:

- integers between 2 and 5 inclusive
- integers between 1 and 6 exclusive (meaning the boundaries of this range are not included)
- $2 \leq$ number of players ≤ 5, where the number of players is an integer
- Let x represent the number of players. Then $x = (1, 5], x \in \mathbb{Z}$
  Here is a breakdown of this mathematical expression. A round bracket at the beginning or end of a range indicates that value **is not** included. A square bracket at the beginning or end of a range indicates that value **is** included. The symbol $\in$ means "belongs to". The symbol $\mathbb{Z}$ refers to the "set of integers".