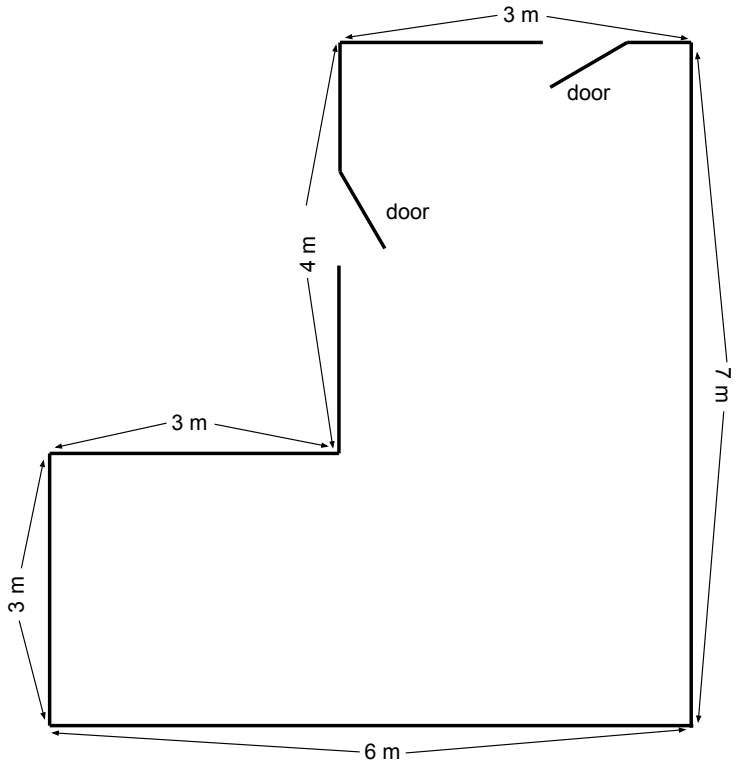
Problem of the Week
Problem A and Solution
Painting Planning

Problem

Jack and Farah want to paint some of the walls in their house. They need to calculate the area that will be painted. They have a floor plan of the part of the house they want to paint. It shows which walls have doorways, as well as the dimensions of the rooms. The walls are all 3 metres high and the doorways are all 2 metres high and 1 metre wide. They do not need to paint the doorways.

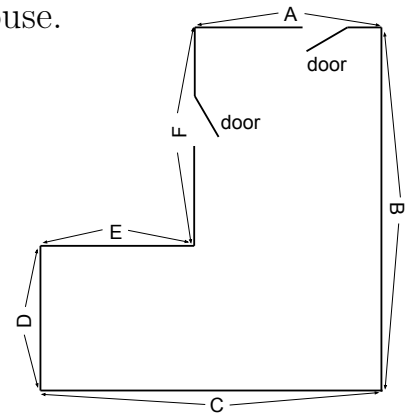


A) Draw a diagram for each of the walls in the area that needs to be painted. Show the dimensions of the walls and the doorways on each diagram.

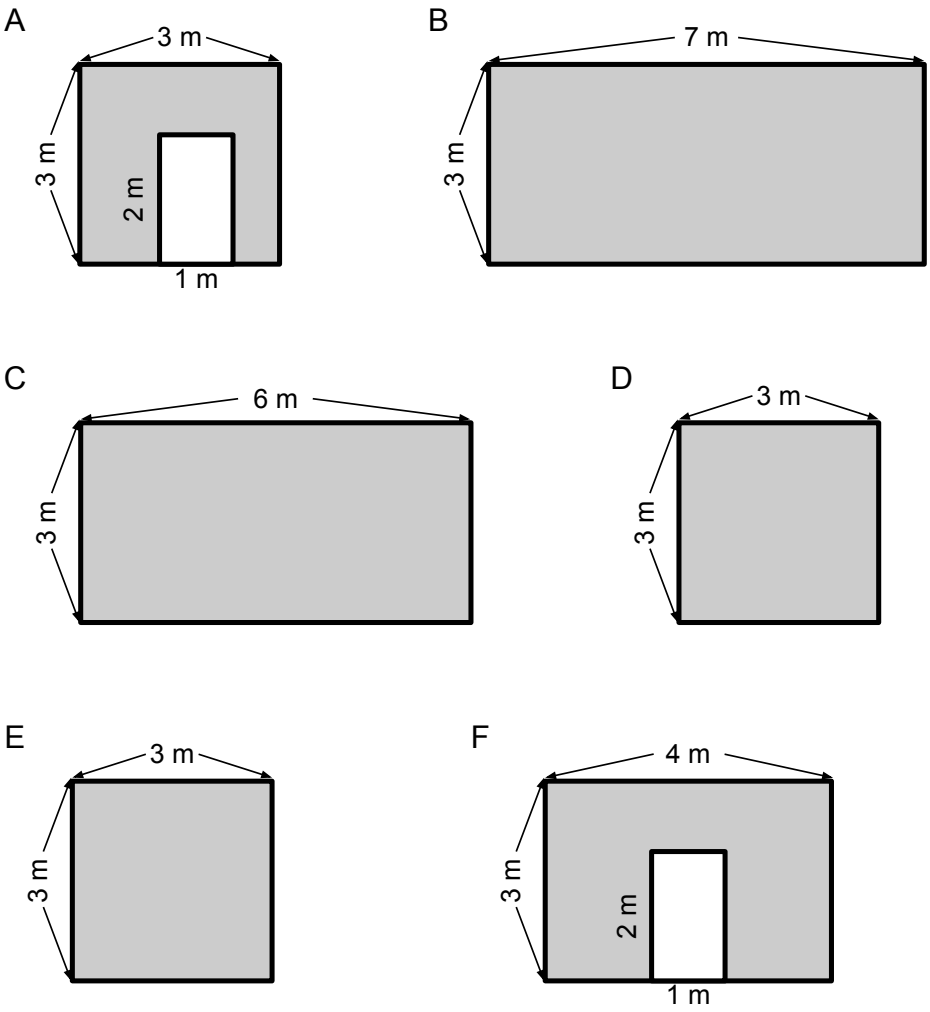
B) Calculate the total area that needs to be painted.

Solution

A) To keep things organized, consider this diagram that labels each of the walls in the floor plan of the house.



Here are diagrams showing the dimensions for each of the six walls:



B) Note that it is not necessary to know the exact location of the doors in order to calculate how much surface area requires paint. For the walls that have doors we can calculate the area of the wall, ignoring the door, and then subtract the area of the door to determine how much paint is required. The area of any of the doors is $2 \times 1 = 2$ m^2.

Here are the calculated surface areas that need to be painted:

Wall	Wall Area	Door Area	Painted Area
A	$3 \times 3 = 9$ m^2	2 m^2	$9 - 2 = 7$ m^2
B	$3 \times 7 = 21$ m^2	0 m^2	21 m^2
C	$3 \times 6 = 18$ m^2	0 m^2	18 m^2
D	$3 \times 3 = 9$ m^2	0 m^2	9 m^2
E	$3 \times 3 = 9$ m^2	0 m^2	9 m^2
F	$3 \times 4 = 12$ m^2	2 m^2	$12 - 2 = 10$ m^2

So this means there is a total of $7 + 21 + 18 + 9 + 9 + 10 = 74$ m^2 to paint.

Here is another way to calculate the surface area that requires painting. Imagine that we moved the wall labelled **F** so that it lines up with the wall labelled **D** and now the room is a rectangle with dimensions 6 m $\times$ 7 m. When we do that, we lose the surface area of the wall labelled **E**, but gain an area on at the top of the newly formed rectangle with exactly the same surface area.

Ignoring the doors, the surface area of the walls of this rectangular room would be: $3 \times 6 = 18$ m^2, $3 \times 7 = 21$ m^2, $3 \times 6 = 18$ m^2, and $3 \times 7 = 21$ m^2. This is a total of $18 + 21 + 18 + 21 = 78$ m^2. However, we still need to consider the area of the two doors that will not be painted. That total area of the doors is $2 \times 2 = 4$ m^2. If we subtract this area from the previous total we have a surface area of $78 - 4 = 74$ m^2 that needs to be painted.

Teacher's Notes

Abstraction is an essential part of mathematics. Mathematicians use various models to represent real world information. This problem uses a diagram to represent the information about a physical building.

In general, when creating a mathematical model we have to think carefully about what information needs to be included, and what can be ignored. In this case, we need to know the dimensions of the walls that are to be painted, we need to know which walls have doors, and we need to know the dimensions of the doors. However, we do not need to know exactly where the doors are located on the walls of the rooms. We could have included the details of where the doors were located, but as a general principle we want to keep the abstraction as simple as possible.

This problem allows students to practise extrapolating information from the model back to the real world situation. They are required to imagine the 3-dimensional reality that is represented by the 2-dimensional diagram. It is the job of many mathematicians to create models of the real world that they can use to solve problems in the real world.