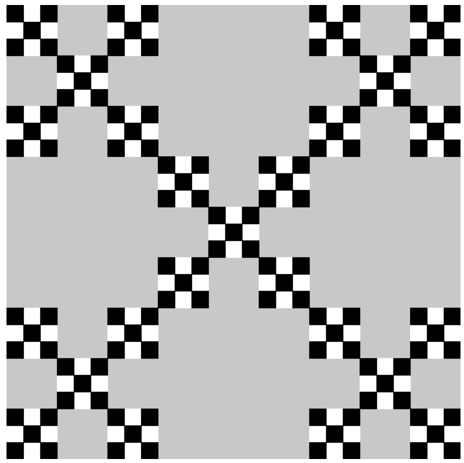
Problem of the Week
Problem A and Solution
Fancy Pattern

Problem

Elisabeth drew this fancy pattern with black, white and grey shading.

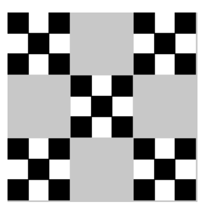


A) If the smallest black squares are 1 cm by 1 cm, what are the dimensions of the whole drawing?

B) What is the total area of the grey part of the drawing?

Solution

A) To determine the dimensions of the whole drawing, let's focus on one corner section.

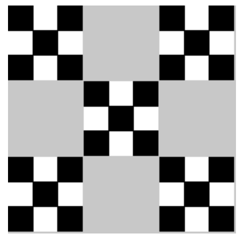


Since the white squares align horizontally and vertically with the black squares in the checkerboard pattern, then we know that the white squares are also 1 cm by 1 cm. This can also be written 1 cm $\times$ 1 cm. We will use this convention throughout the rest of the solution. In a single checkerboard, there are 3 squares across and 3 squares down. This means that the checkerboard sections themselves are 3 cm $\times$ 3 cm. And since the checkerboard sections are aligned, and there are 3 of these sections across and 3 down, then this whole corner section must be 9 cm $\times$ 9 cm.

This section is a repeated pattern in the whole drawing, and the sections are aligned horizontally and vertically. Since there are three sections across and three sections down, each of the dimensions of the whole drawing is 3 times the dimensions of this section. Therefore the dimensions of the whole drawing are 27 cm $\times$ 27 cm.

B) One way to determine the area of the grey parts of the drawing would be to draw gridlines that align with the black and white squares in the checkerboard. The gridlines can be used to identify grey squares that are also 1 cm $\times$ 1 cm. Then we can count the number of those squares to determine the grey shaded area of the drawing.

Here is another way to calculate the area. Looking at the whole drawing, it can be divided up into nine sections. Five of those sections have black and white checkerboard patterns and smaller grey squares, like this:



and 4 of those sections are larger, solid grey squares with the same dimensions as the square shown above.

In part (A) we determined that the dimensions of this section are 9 cm $\times$ 9 cm, which means that the the dimensions of the larger grey squares are also 9 cm $\times$ 9 cm. The area of one of those squares would be $9 \times 9 = 81$ cm^2. This means the total area of those four squares is $81 + 81 + 81 + 81 = 81 \times 4 = 324$ cm^2.

Within a section that contains the checkerboard patterns, there are nine subsections: five subsections that each contain the checkerboard pattern, and four subsections that are small, solid grey squares. In part (A) we determined that checkerboard subsection has dimensions 3 cm $\times$ 3 cm. So these small grey squares also have dimensions 3 cm $\times$ 3 cm, which means the area of one of those squares is $3 \times 3 = 9$ cm^2. Since there are four smaller grey squares in one of these sections, then the grey part of one of these sections is $9 + 9 + 9 + 9 = 9 \times 4 = 36$ cm^2. Since the section shown above is repeated five times in the entire drawing, the grey squares of these sections have a total area of $36 \times 5 = 180$ cm^2.

Therefore the total area of the grey part of the drawing is $180 + 324 = 504$ cm^2.

Another way to calculate the area of the grey part is to start with the area of the whole drawing and subtract the area of the black and white squares. The area of the whole drawing is $27 \times 27 = 729$ cm^2. We know the dimensions of the black and white squares are 1 cm $\times$ 1 cm, so their areas are each 1 cm^2. We can count the total number of black and white squares and see that their are 225 of them. So the total area of the grey part of the drawing is $729 - 225 = 504$ cm^2.

Teacher's Notes

The fancy pattern Elisabeth drew is an example of a *fractal*. Fractals occur in mathematics, art, and nature. The essential characteristic of a fractal is that it has a repeating pattern, within its pattern. In this example, we can break down the whole image into a square containing nine squares. The basic pattern of the whole image has the following structure:

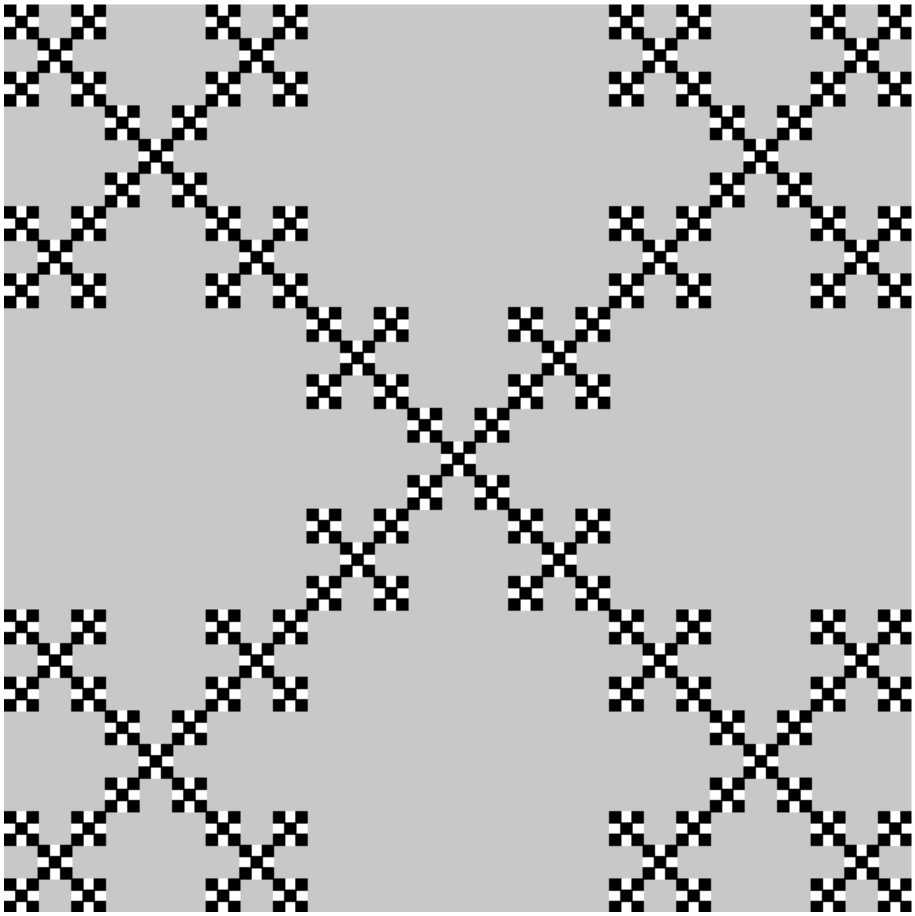
pattern	solid	pattern
solid	pattern	solid
pattern	solid	pattern

If you look at the pattern sections of the whole image, they can also be described as a square, containing nine squares, with the structure described above. Finally, each of these patterned sections can be described as a square that contains nine squares with the following similar structure:

black	white	black
white	black	white
black	white	black

The beauty of fractals, is that you can continue this structured repetition to more and more levels. You can also represent the fractal with a mathematical model. The next couple of pages show the fractal from this problem repeated to deeper levels. All of the images in this problem were actually generated by a relatively short computer program. We used the same code to create different images, by just changing the value of one variable to indicate how many levels of repetition we wanted.

Fractal with Four Levels



Fractal with Five Levels