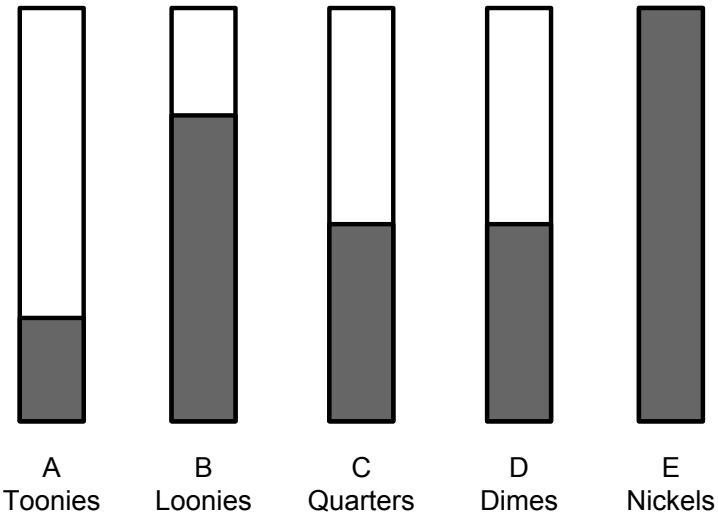
Problem of the Week
Problem A and Solution
Smart Saver

Problem

James saves all of the coins he can find. He is very organized, so he has a separate container for each type of coin. In Canada a toonie is worth \$2 (2 dollars), a loonie is worth \$1, a quarter is worth 25¢ (25 cents), a dime is worth 10¢, a nickel is worth 5¢, and \$1 is equal to 100¢.

Each container James has holds 80 coins. Here is a picture representing the containers. The shaded part represents how many coins are in each container.



A) Which container holds the most money? Justify your answer.

B) Approximately what is the difference between the amount James has saved in quarters and the amount he has saved in dimes?

C) Approximately what is the total amount of money James has saved so far?

Solution

We need to estimate what fraction of each container is filled with coins.
Container **A** appears to be approximately $\frac{1}{4}$ full.
Container **B** appears to be approximately $\frac{3}{4}$ full.
Container **C** appears to be approximately $\frac{1}{2}$ full.
Container **D** appears to be approximately $\frac{1}{2}$ full.
Container **E** appears to be completely full.

Half of 80 is 40, since $40 + 40 = 80$.

Half of 40 is 20, since $20 + 20 = 40$.

Since half of $\frac{1}{2}$ is $\frac{1}{4}$, then $\frac{1}{4}$ of 80 is 20.

Since $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, then $\frac{3}{4}$ of 80 is $20 + 20 + 20 = 60$.

With this information we can estimate how many coins are in each container, as well as the value of the coins that have been collected. To determine the amount of money in each container, we can use skip counting, or multiplication.

Container	Number of Coins	Value of Coins
A	20	$20 \times 2 = \$40$
B	60	$60 \times 1 = \$60$
C	40	$40 \times 25 = 1000¢ = \$10$
D	40	$40 \times 10 = 400¢ = \$4$
E	80	$80 \times 5 = 400¢ = \$4$

It may be easier to calculate the values of the quarters, dimes, and nickels by noting how many of each coin we need to make \$1. It takes 4 quarters, 10 dimes, or 20 nickels to make \$1. To calculate the total value of the quarters, we can skip count by 4: *4, 8, 12, 16, 20, 24, 28, 32, 36, 40*. This means that 40 is equal to 10 groups of 4. Since each of these groups of 4 is worth \$1, then the 40 quarters are worth \$10. Similarly, if we skip count by 10: *10, 20, 30, 40*, we see that the 4 groups of 10 dimes James has saved are worth \$4, and if we skip count by 20: *20, 40, 60, 80*, we see that the 4 groups of 20 nickels James has saved are worth \$4.

A) Based on the calculations shown in the table, it appears that the container holding the loonies has the most money.

B) Based on the calculations shown in the table, it appears that James has saved $10 - 4 = \$6$ more in quarters than he has in dimes.

C) James has saved approximately, $40 + 60 + 10 + 4 + 4 = \$118$, in his containers.

Teacher's Notes

Here is a question that you might want to consider. Which answers would change if the size of the containers changed? For example, suppose the containers hold 200 coins instead of 80 coins. The answer to part A) would not change, but the answers to part B) and part C) would change.

We could describe the amount of money each container holds algebraically. Let x represent the maximum number of coins each container can hold. Then,

$$\begin{aligned}
\text{number of cents in Toonies} &= \frac{200 \cdot x}{4} \\
&= 50x
\end{aligned}$$

$$\begin{aligned}
\text{number of cents in Loonies} &= \frac{100 \cdot 3x}{4} \\
&= 75x
\end{aligned}$$

$$\begin{aligned}
\text{number of cents in Quarters} &= \frac{25 \cdot x}{2} \\
&= 12.5x
\end{aligned}$$

$$\begin{aligned}
\text{number of cents in Dimes} &= \frac{10 \cdot x}{2} \\
&= 5x
\end{aligned}$$

$$\text{number of cents in Nickels} = 5x$$

So, no matter what the size of the container is, the most amount of money saved is in the Loonies container. Interestingly, there is an equal amount of money saved in the container with dimes when compared to the container with nickels.

The difference between the amount of money saved in quarters and the amount of money saved in dimes would depend on the size of the container. We can describe that difference as: $12.5x - 5x = 7.5x$

We can also generally describe the total amount of money saved so far, with an unknown container size, as an equation: $50x + 75x + 12.5x + 5x + 5x = 147.5x$