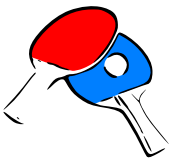

Problem of the Week
Problem A and Solution
Party Games

Problem

Arezoo is having a birthday party and she invited five friends: Gerry, Jason, Laila, Nabil, and Lisa. Everyone at the party likes to play games, so Arezoo plans to have two stations. At Station A, four people will play table tennis; at Station B two people will play tether ball. Arezoo wants to make sure everyone has a chance to play at each station at least once with each person at the party. Make a schedule so that everyone at the party plays both games at some point with every other person at the party.

Solution

If we decide who is playing tether ball, then the other four people will automatically be scheduled to play table tennis. We can make a schedule that guarantees each person plays tether ball against each other person at the party.

Playing Tether Ball	Playing Table Tennis
Arezoo and Gerry	Jason, Laila, Nabil, and Lisa
Arezoo and Jason	Gerry, Laila, Nabil, and Lisa
Arezoo and Laila	Gerry, Jason, Nabil, and Lisa
Arezoo and Nabil	Gerry, Jason, Laila, and Lisa
Arezoo and Lisa	Gerry, Jason, Laila, and Nabil
Gerry and Jason	Arezoo, Laila, Nabil, and Lisa
Gerry and Laila	Arezoo, Jason, Nabil, and Lisa
Gerry and Nabil	Arezoo, Jason, Laila, and Lisa
Gerry and Lisa	Arezoo, Jason, Laila, and Nabil
Jason and Laila	Arezoo, Gerry, Nabil, and Lisa
Jason and Nabil	Arezoo, Gerry, Laila, and Lisa
Jason and Lisa	Arezoo, Gerry, Laila, and Nabil
Laila and Nabil	Arezoo, Gerry, Jason, and Lisa
Laila and Lisa	Arezoo, Gerry, Jason, and Nabil
Nabil and Lisa	Arezoo, Gerry, Jason, and Laila

We can double check that each of the pairs playing tether ball, plays table tennis together at some other time during the party. If we search through the table, we will see that each pair does play table tennis at the same time together multiple times. So, as long as we have listed all possible pairs playing tether ball, we know that everyone plays every other person at the party at least once at each station.

Arezoo will need to schedule 15 separate times during her party to ensure that each person is guaranteed to play both tether ball and table tennis with every other person at the party.

Teacher's Notes

This is classic *combinatorics* question. We need to be careful when listing all of the pairs of people who are going to play tether ball. We want to make sure we have included every pair, without duplication.

This solution organized the pairs by listing:

- all pairs that include Arezoo,
- followed by all pairs that include Gerry, that do not include Arezoo,
- followed by all pairs that include Jason, that do not include Arezoo or Gerry,
- followed by all pairs that include Laila, that do not include Arezoo, Gerry, or Jason,
- followed by the pair that includes Nabil, that does not include Arezoo, Gerry, Jason, or Laila.

We have to stop the pattern here because there are no pairs for Lisa that do not include at least one of the other five people at the party. Hence, we have covered all pairs.

Here is another way of looking at the order of the pairs. Suppose we assigned each person a number: Arezoo (1), Gerry (2), Jason (3), Laila (4), Nabil (5), and Lisa (6). Now count (in order) the two-digit numbers that only use the digits 1 through 6, and have a second digit that is greater than the first digit. So you would include numbers like 25 and 46, but you would not include numbers like 31 or 55.

You will create a list of 15 numbers that describe the pairs at our party playing tether ball. The numbers in this list can never have the same two digits. This means we never count someone being paired with themselves. Since we always make the second digit greater than the first digit in this list, we never see two numbers with the same digits. This means the list does not count pairs like "Arezoo and Jason" and "Jason and Arezoo" separately. This counting technique ensures we have a complete list of the possible pairs, without any duplication.