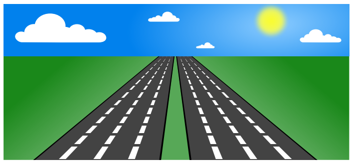


Problem of the Week
Problem E and Solution
Not That Kind of Median

Problem

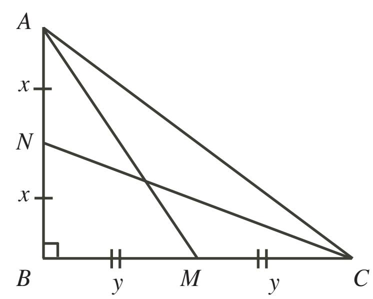
A *median* is a line segment drawn from the vertex of a triangle to the midpoint of the opposite side. In $\triangle ABC$, $\angle ABC = 90^\circ$. A median is drawn from A meeting BC at M such that $AM = 5$. A second median is drawn from C meeting AB at N such that $CN = 2\sqrt{10}$. Determine the length of the longest side of $\triangle ABC$.

Solution

Since AM is a median, M is the midpoint of BC. Then $BM = MC = y$.
Since CN is a median, N is the midpoint of AB. Then $AN = NB = x$.

$\triangle NBC$ is right angled since $\angle B = 90^\circ$. Using the Pythagorean Theorem,

$$\begin{aligned} NB^2 + BC^2 &= CN^2 \\ x^2 + (2y)^2 &= (2\sqrt{10})^2 \\ x^2 + 4y^2 &= 40 \qquad (1) \end{aligned}$$



$\triangle ABM$ is right angled since $\angle B = 90^\circ$. Using the Pythagorean Theorem,

$$\begin{aligned} AB^2 + BM^2 &= AM^2 \\ (2x)^2 + y^2 &= 5^2 \\ 4x^2 + y^2 &= 25 \qquad (2) \\ \text{Adding (1) and (2),} \quad 5x^2 + 5y^2 &= 65 \\ \text{Dividing by 5,} \quad x^2 + y^2 &= 13 \qquad (3) \end{aligned}$$

The longest side of $\triangle ABC$ is the hypotenuse AC. Using the Pythagorean Theorem,

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= (2x)^2 + (2y)^2 \\ &= 4x^2 + 4y^2 \\ &= 4(x^2 + y^2) \\ \text{Substituting from (3) above,} \quad AC^2 &= 4(13) \\ \text{Taking the square root,} \quad AC &= 2\sqrt{13} \end{aligned}$$

$\therefore$ the length of the longest side is $2\sqrt{13}$ units.

Note: The solver could actually solve a system of equations to find $x = 2$ and $y = 3$ and then proceed to solve for the longest side. The above approach was provided to expose the solver to alternate thinking about the solution of this problem.