Problem of the Week
Problem D and Solution
The Inner Triangle

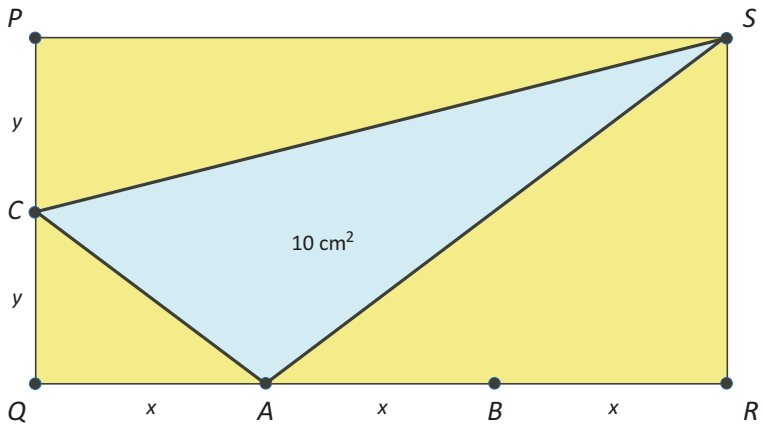
Problem

$PQRS$ is a rectangle. A and B are points on QR such that $QA = AB = BR$. C is the midpoint of PQ. The area of $\triangle ACS$ is 10 cm^2. Determine the area of rectangle $PQRS$.

Solution

Let $QA = AB = BR = x$. Then $PS = QR = 3x$ and $AR = 2x$.

Since C is the midpoint of PQ, $PC = CQ = y$. Then $SR = PQ = 2y$.



We will formulate an equation connecting the areas of the four inside shapes to the entire rectangle.

$$\begin{aligned}
\text{Area } PQRS &= \text{Area } \triangle PCS + \text{Area } \triangle SRA + \text{Area } \triangle AQC + \text{Area } \triangle ACS \\
PQ \times QR &= \frac{PC \times PS}{2} + \frac{SR \times AR}{2} + \frac{QA \times CQ}{2} + 10 \\
(2y) \times (3x) &= \frac{y \times 3x}{2} + \frac{2y \times 2x}{2} + \frac{x \times y}{2} + 10 \\
6xy &= \frac{3xy}{2} + 2xy + \frac{xy}{2} + 10 \\
\text{Multiply by 2:} \quad 12xy &= 3xy + 4xy + xy + 20 \\
4xy &= 20 \\
xy &= 5
\end{aligned}$$

The area of rectangle $PQRS$ is $6xy = 6(5) = 30$ cm^2.