



Problem of the Week

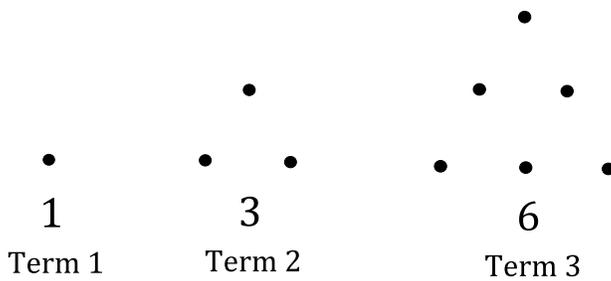
Problem B

Even Dots are Odd

Problem

The numbers 1, 3, 6, 10, 15, 21, and so on, form a *sequence*. That is, there is a rule which tells you how to find each number, or *term* in the sequence, from the previous term.

- a) These numbers are sometimes called Triangular Numbers because they can be represented by dots arranged in triangles. Here are the first three terms:



Term	Number
1	1
2	3
3	6
4	10
5	15
6	21
7	28
8	36
9	45
10	55
11	66

Draw the dot-triangles that would represent the next three terms, 10, 15, and 21.

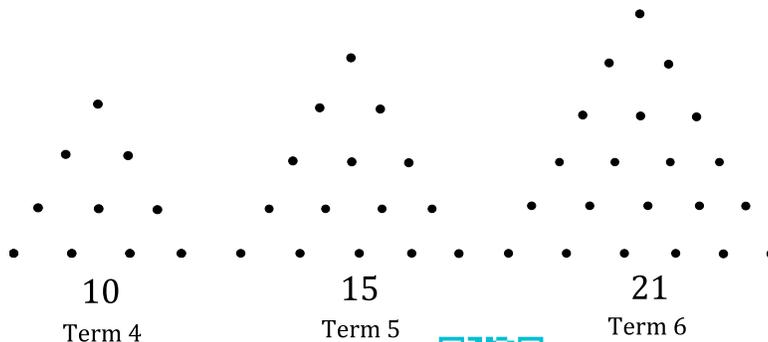
- b) Write a pattern rule for this sequence.
 c) Enter the next five numbers in the sequence in the table.

Extension:

What is the pattern of odd and even numbers in this sequence? Why does this happen?

Solution

- a) Here are the dot-triangles for terms 4, 5, and 6.





- b) Each number in the sequence is obtained by adding the number (label) of the new term to the value of the previous term. For example,

$$\text{Term } 7 = \text{Term } 6 + 7 = 21 + 7 = 28, \text{ Term } 8 = \text{Term } 7 + 8 = 28 + 8 = 36,$$

and so on. Thus the pattern rule is $\text{Term}(N+1) = \text{Term}(N) + (N+1)$.

In the dot-triangles, this corresponds to adding a row to the bottom of the previous dot triangle, with one more dot in the new row.

- c) The next five numbers are shown in the completed table above.

EXTENSION: The pattern of odd and even numbers is odd, odd, even, even, odd, odd, even, even, i.e., 2 odd, then 2 even, repeatedly. This happens because the sum of two odd numbers is even, but the sum of an even number and an odd number is odd.

Suppose we designate the parity of the term number in upper case (EVEN or ODD), and the parity of the value of the term in lower case italics (*even* or *odd*). If we start with an EVEN term number which has an even value, then the next term number will be ODD, and its value will thus be an ODD number plus an even number, which is odd (e.g., Term 4 has value 10, and next comes Term 5, which is $10 + 5 = 15$). So as the term numbers alternate between EVEN and ODD, the term values will follow this pattern:

$$\text{ODD term number} + \textit{even} \text{ value} = \textit{odd} \text{ value},$$

$$\text{EVEN term number} + \textit{odd} \text{ value} = \textit{odd} \text{ value},$$

$$\text{ODD term number} + \textit{odd} \text{ value} = \textit{even} \text{ value},$$

$$\text{EVEN term number} + \textit{even} \text{ value} = \textit{even} \text{ value},$$

$$\text{ODD term number} + \textit{even} \text{ value} = \textit{odd} \text{ value},$$

with the last statement restarting the pattern, which repeats indefinitely.

