



The **CENTRE** for **EDUCATION**  
in **MATHEMATICS** and **COMPUTING**

**Problem of the Week**  
**Problems and Solutions**  
**2019-2020**

**Problem A (Grade 3/4)**

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**Themes**

**Number Sense (N)**

**Geometry (G)**

**Algebra (A)**

**Data Management (D)**

**Computational Thinking (C)**

(Click on a theme name above to jump to that section)

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\*The problems in this booklet are organized into themes.  
A problem often appears in multiple themes.

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# Number Sense (N)

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## Problem of the Week

### Problem A

### Finding the Intersection

Isla starts with \$12 in her bank account. She adds \$12 to her account at the end of every two weeks from collecting recycled items. Javier starts with \$32 in his bank account. He earns \$4 at the end of every week for doing odd jobs for his neighbour, and adds that to his savings.

After how many weeks will they both have the same amount of money in their bank accounts?





## Problem of the Week

### Problem A and Solution

### Finding the Intersection

#### Problem

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After how many weeks will they both have the same amount of money in their bank accounts?

#### Solution

We can use a table to show the pattern of savings for Isla and Javier. Each week we will add \$4 to Javier's total savings. Every two weeks we will add \$12 to Isla's savings. This means every odd week, Isla's savings will not change.

Week	Isla's Savings (in \$)	Javier's Savings (in \$)
Start	12	32
1	12	36
2	24	40
3	24	44
4	36	48
5	36	52
6	48	56
7	48	60
8	60	64
9	60	68
10	72	72

After 10 weeks of savings, Isla and Javier have the same amount of money in their bank accounts. However, there may be other weeks where they have the same savings.

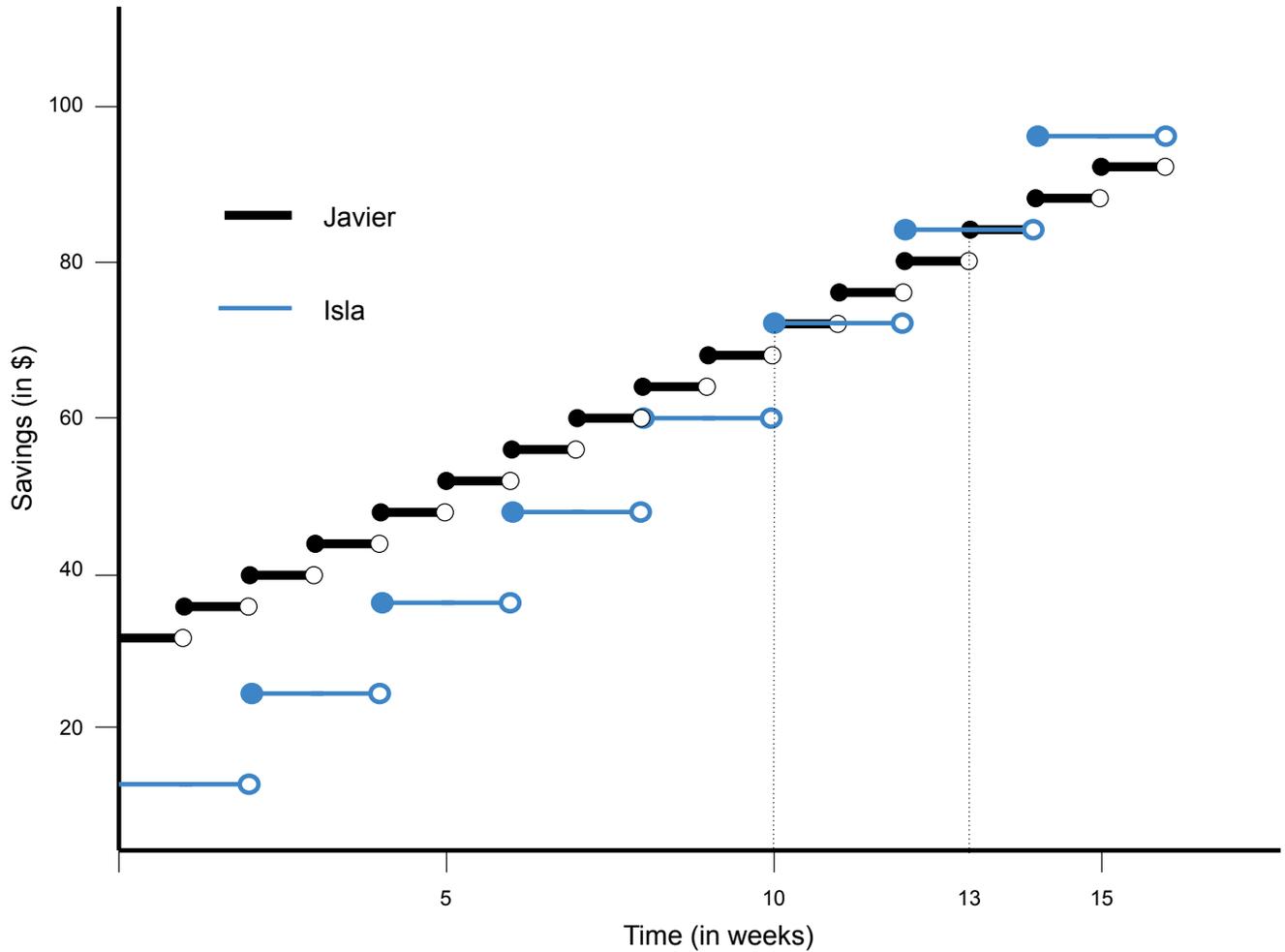
Week	Isla's Savings (in \$)	Javier's Savings (in \$)
11	72	76
12	84	80
13	84	84
14	96	88
15	96	92
16	108	96

At the end of the 13th week, Isla and Javier again have the same amount of money in their bank accounts. At the end of week 16, the difference between Isla's savings and Javier's savings is more than \$8. Since Javier only saves \$8 every two weeks and Isla saves more than Javier, his savings can no longer match Isla's. Therefore, at the end of week 13 they make the last contributions that give the same amount of money in their accounts.



### Teacher's Notes

We can use a *step function* to describe Isla and Javier's savings. If we plot their savings over time, the amount in their bank accounts does not change between deposits. We can draw a graph with time on the x-axis and amount saved in dollars on the y-axis. The result would look like this:



As time passes, we can see that the graph for each person's savings looks like equally spaced steps. Where there is a jump between steps, the filled circle indicates that the actual value is at that point, and the empty circle indicates that the step does not include the value at that point of the graph. From the graph, we can see that the savings amounts overlap at week 10 and week 13. It may also look like there is an overlap at week 8. However at that point we can see that Javier's savings are actually at the level of the next step.



# Problem of the Week

## Problem A

### Expert Predictions

Malcolm Gladwell said:

*“... researchers have settled on what they believe is the magic number for true expertise: ten thousand hours.”*

The journey towards becoming an “*expert*” student begins in Grade 1. Most students spend an average of 180 days a year in school, and 5 hours each day in the classroom. Assuming that you attend school every possible day, in what grade would you become an expert student?





## Problem of the Week

### Problem A and Solution

### Expert Predictions

#### Problem

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#### Solution

We can calculate the average number of hours per grade as  $5 \times 180 = 900$ .

Then we can use skip counting to see when we get to at least 10000:

900, 1800, 2700, 3600, 4500, 5400, 6300, 7200, 8100, 9000, 9900, 10 800

So at some point in Grade 12, on average, students will have been in school for 10 000 hours.

Alternatively, we can use division to determine the result.

$$10\,000 \div 900 = 11 \text{ remainder } 100$$

This means that students will require more than 11 years to become “experts”. So they will achieve expertise in Grade 12.



## Teacher's Notes

Anyone interested in becoming an “expert” in mathematics, can continue their studies after high school to as far as a Ph.D. Many people wonder what people can do with a degree in mathematics. It turns out that there are many options. Here are a couple of possibilities:

### Cryptography

Cryptography is the study of making and breaking codes. Systems for sending coded information securely have existed for thousands of years. In times of war, cryptography allowed governments to send messages to their allies and keep secrets from their enemies. In modern times, cryptography is essential for keeping online and other digital information safe. As technology changes, the systems used to create the methods for protecting coded information needs to change as well. Mathematicians who study cryptography may look for better ways to encode data, or they may be trying to find ways to break existing systems of encoding, or both.

### Applied Mathematics

As the name suggests, applied mathematics is the study of applying mathematics to real life situations. Applied mathematicians can create mathematical models of real life phenomena so that they can solve problems in laboratory conditions before deploying a solution on a larger scale. Studying applied mathematics is essential in the field of aerospace engineering, in medical imaging technology, and in the study of climate change. At its core applied mathematics is problem solving, and the solutions that are developed can be applied to many real world problems.



## Problem of the Week

### Problem A

### Surrounding Squares

Imran was doodling and he drew a picture that had a square in the middle, surrounded by other smaller squares. Together, the middle square and all the smaller squares form a bigger square. The smaller squares all have the same side lengths, and there are no overlaps between the squares and no gaps in the picture. The side lengths of the smaller squares are each  $\frac{1}{3}$  the side length of the middle square.

- A) How many smaller squares are in the picture?
- B) If the side length of the middle square is 6 cm, what is the area of the square formed by the whole picture?





## Problem of the Week

### Problem A and Solution

### Surrounding Squares

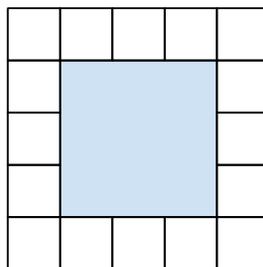
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- A) How many smaller squares are in the picture?
- B) If the side length of the middle square is 6 cm, what is the area of the square formed by the whole picture?

#### Solution

To answer the questions in this problem, the first thing we should do is draw a picture of the doodle. Since the side lengths of smaller squares are  $\frac{1}{3}$  the side length of the middle square, then three smaller squares should fit on one side of the middle square. So this is what the doodle looks like:



- A) From the diagram we can see that there are 16 smaller squares.
- B) If the side length of the middle square is 6 cm, and  $\frac{1}{3}$  of 6 is 2, then the side lengths of the smaller squares are 2 cm. Since the large square has a side length equal to five small squares, then side length of the large square is  $5 \times 2 = 10$  cm. This means the area of the large square is  $10 \times 10 = 100$  cm<sup>2</sup>. We could also calculate the area of the large square this way. The area of a small square is  $2 \times 2 = 4$  cm<sup>2</sup>. Since there are 16 small squares, the total area of the smaller squares is  $16 \times 4 = 64$  cm<sup>2</sup>. The area of the middle square is  $6 \times 6 = 36$  cm<sup>2</sup>. The total area is then  $36 + 64 = 100$  cm<sup>2</sup>.



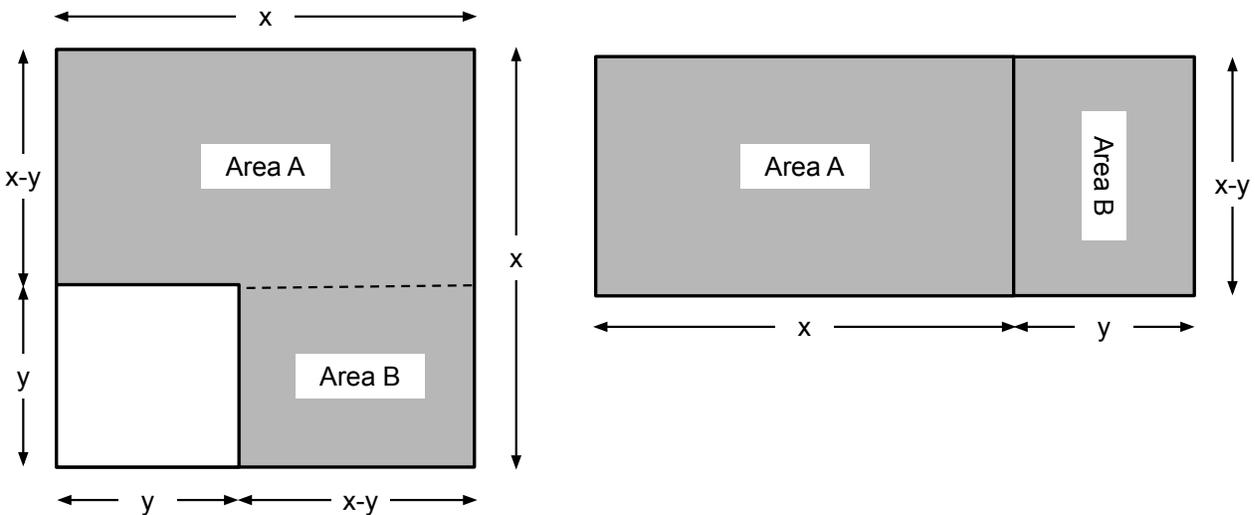
## Teacher's Notes

It is important for mathematicians to be able to draw good diagrams based on written descriptions. An accurate diagram helps visualize a problem. Diagrams often provide insight into relationships that are not necessarily clear based on the words alone.

In high school algebra, students learn about factoring polynomials. Factoring is a process of rewriting a polynomial into an equivalent form, where other polynomials are multiplied together. One factoring relationship students will learn is called the *difference of squares*. Algebraically, the rule for factoring a difference of squares looks like this:

$$x^2 - y^2 = (x - y)(x + y)$$

We can draw a diagram that helps visualize this relationship. We start by drawing a square where the sides have a length of  $x$ , and then remove a square where the sides have a length of  $y$  from one corner. Here is a diagram showing the difference of squares relationship:



The area of the shaded region in the diagram on the left is the area of the outer square ( $x^2$ ), minus the area of the white square ( $y^2$ ). This can be written algebraically as  $x^2 - y^2$ . The shaded region of the diagram on the left has been rearranged to form a rectangle in the diagram on the right. The area of this rectangle is the length of its sides multiplied together:  $(x - y)(x + y)$ .

Since these two areas are the same, we can see that  $x^2 - y^2 = (x - y)(x + y)$ .



## Problem of the Week

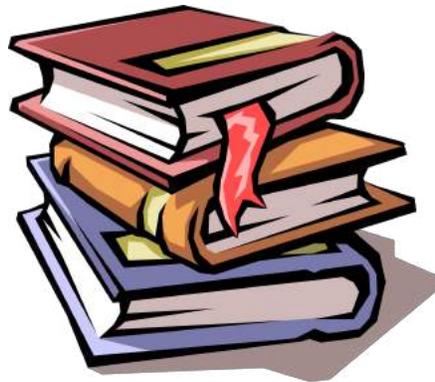
### Problem A

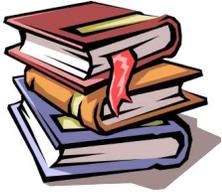
### Eight is Enough

There once was a child whose parents never taught him the number 8. He grew up to be a wonderful person, but would always count 1, 2, 3, 4, 5, 6, 7, 9, 10 and so on, skipping all numbers that include an 8. One day he was given the task of numbering the pages in two books.

- A) By his count, the last page of the first book is 110. Knowing what you do about this child, how many pages are actually in this book?
- B) By his count, the last page of the second book was 320. How many pages are actually in this book?

*Based on the poem **Eight** from the book **I'm Just No Good At Rhyming** (and other nonsense for mischievous kids and immature grown-ups) by Chris Harris and Lane Smith*





## Problem of the Week

### Problem A and Solution

### Eight is Enough

#### Problem

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#### Solution

- A) One way to solve this problem is to match each number that this child counts from 1 to 110, and see what regular decimal number matches 110. For example, we would start this way:

Missing Eights	Regular Numbers
1	1
2	2
3	3
4	4
5	5
6	6
7	7
9	8
10	9
...	...

However, this would take a long time.



We can notice that in most cases, when this child reaches a number that ends with a zero, he has actually only increased his count by 9. The exception to this rule is that he skips all of the numbers between 79 and 90. Let's start by looking at the count up to 100. We can create a table that keeps track of the actual count every time we get to a number that ends with a zero. This will reduce the work we need to do.

Child's Count (Missing Eights)	Actual Count (Regular Numbers)
10	9
20	18
30	27
40	36
50	45
60	54
70	63
90	72*
100	81

\*Note that the numbers the child counts between 70 and 90 are: 71, 72, 73, 74, 75, 76, 77, 79.

There are nine more numbers from 101 to 110 that do not include an 8. So the total number of pages in the first book is 90.

Another way to solve this problem is to consider which numbers from 1 to 100 contain an 8. There are 19 numbers, which we can list:

8, 18, 28, 38, 48, 58, 68, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 98

So, once the child numbers a page with the number 100, he has missed 19 numbers, so the page he numbers 100 is actually page number  $100 - 19 = 81$ . When he numbers pages with the numbers 101 to 110, he only misses the single number 108. Therefore, there are actually

$$110 - 19 - 1 = 90$$

pages in the book.

- B) Note we will see the same pattern for each hundred pages that is less than 800. When counting pages in the second book, we need to remember the child skips the numbers between 179 and 190 and the numbers between 279 and 290.

So when the child reaches 200, the actual count will be  $81 + 81 = 162$ , and when the child reaches 300, the actual count will be  $162 + 81 = 243$ . When the child counts from 301 to 320, this will add another 18 to the total. This means that the actual number of pages in the book is  $243 + 18 = 261$ .



## Teacher's Notes

The child in this problem is essentially using a number system that has 9 digits (0, 1, 2, 3, 4, 5, 6, 7, 8) rather than the number system we are used to, the decimal number system, which uses 10 digits. We refer to the number of digits in the number system as the base of the system. For example, the decimal number system is a base-10 number system.

We can use number systems which have more or fewer than 10 digits to represent numerical values. For example, the binary number system uses just two digits (0 and 1) and the hexadecimal number system uses 16 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). Both of these number systems are commonly used in Computer Science.

The same value looks different when represented by different number systems. For example the decimal number 17 can be represented as the binary number 10001. Both numbers represent the same abstract amount. We can convert a number represented in another base to a decimal number. Each column of a number represents a power of the number system's base. For example, the number 372 in base-10 can be expanded as follows:

$$(3 \times 10^2) + (7 \times 10^1) + (2 \times 10^0) = (3 \times 100) + (7 \times 10) + (2 \times 1) = 300 + 70 + 2 = 372$$

For a base-9 number, we can do a similar expansion, except the columns represent powers of 9 rather than powers of 10. So the number 110 in base-9 can be expanded as follows:

$$(1 \times 9^2) + (1 \times 9^1) + (0 \times 9^0) = (1 \times 81) + (1 \times 9) + (0 \times 1) = 81 + 9 + 0 = 90$$

Similarly the number 320 in base-9 can be expanded as follows:

$$(3 \times 9^2) + (2 \times 9^1) + (0 \times 9^0) = (3 \times 81) + (2 \times 9) + (0 \times 1) = 243 + 18 + 0 = 261$$



## Problem of the Week

### Problem A

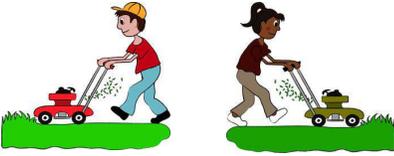
### Collecting Comics

Anaiyah and Shane worked to earn money by cutting their neighbours' lawns, so they could add to their comic book collection. They charged \$5.00 per lawn, and cut four lawns in total. After searching the internet for the best deal for comic books, they found the following prices:

- **Website A:** Pay \$2.50 for each comic book.
- **Website B:** Buy one comic book at \$3.00, get the second one half price.
- **Website C:** Buy three comic books at \$3.25 each, get the fourth one free.

If Anaiyah and Shane decide to buy as many comic books as possible, which Website offers the best deal? Justify your answer.





## Problem of the Week

### Problem A and Solution

#### Collecting Comics

#### Problem

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#### Solution

After mowing lawns, Anaiyah and Shane collected a total of  $4 \times \$5.00 = \$20.00$ . For each of the deals, we can skip count to find out how many comic books they could purchase with \$20.

For **Website A** we count by increments of \$2.50 like this:

\$2.50, \$5.00, \$7.50, \$10.00, \$12.50, \$15.00, \$17.50, \$20.00

This means that with \$20, they could purchase 8 comic books.

For **Website B** we should purchase comic books in pairs. Since half of \$3.00 is \$1.50, the total cost of the pair of comic books is  $\$3.00 + \$1.50 = \$4.50$ . For the pairs, we count by increments of \$4.50 like this:

\$4.50, \$9.00, \$13.50, \$18.00, \$22.50

This means that with \$20, they could purchase 4 pairs of comic books, which is a total of 8 comic books. However, it will only cost them \$18.00. Since  $\$20 - \$18 = \$2$ , they do not have enough money to purchase any more comic books from **Website B**.

For **Website C** we should purchase comic books in groups of four. The total cost of four comic books is  $\$3.25 + \$3.25 + \$3.25 + \$0.00 = \$9.75$ . For the groups of four, we count by increments of \$9.75 like this:

\$9.75, \$19.50, \$29.25

This means that with \$20, they could purchase two groups of four comic books, which is a total of 8 comic books. However, it will cost them \$19.50.

In each case, the maximum number of comic books Anaiyah and Shane could purchase is 8. However, on **Website B** they will pay the least amount for the 8 comic books. This means that **Website B** has the best deal if they want to purchase as many comic books as possible.



## Teacher's Notes

When making purchases where you have a choice in an item's size, identifying the *unit cost* can be very helpful. For example, if you were buying shampoo, and you had the choice of a 500 mL bottle or a 750 mL bottle, you could determine the better price based on how much each costs per mL.

We could investigate something similar with this problem, where the units are comic books rather than a measurement of volume.

- On **Website A** we are given a unit price of \$2.50.
- On **Website B** the total price of two comic books is \$4.50.  
This means the unit price is  $\$4.50 \div 2 = \$2.25$
- On **Website C** the total price of four comic books is \$9.75.  
This means the unit price is  $\$9.75 \div 4 = \$2.4375$ .

So based on the unit price, it appears that **Website B** offers the best deal. There is one catch however. If we want to purchase an odd number of comic books, then we cannot take full advantage of the pricing of **Website B** or **Website C**.

When we consider a unit price, we are thinking of the relationship between the unit and the price as a *continuous* function, meaning that we can calculate the cost of any quantity - including fractional quantities. However, the reality of the situation is that the actual pricing is a *discrete* function, meaning there are only prices for a particular set of quantities. In the case of **Website A**, we can only purchase whole numbers of comic books. In the case of **Website B**, we can only purchase even numbers of comic books. In the case of **Website A**, we can only purchase comic books in multiples of 4.

In mathematics, we must know if a function is continuous or discrete in order to work with it properly.



## Problem of the Week

### Problem A

### School Garden

Maeve and Azaadi want to make a garden in the school yard, but they do not want the other students to trample the plants. They decide to ask their friends to help find some fencing to protect the vegetables. Maeve found 5 metres of fencing in her shed. Azaadi found two pieces of fencing, one measuring 300 centimetres, the other measuring 6 metres. Their friend Thandi found another piece measuring 8 metres, and Xander donated one more piece of fencing.

The students use the fencing to surround their garden with a perimeter of 28 metres.

- A) What is the length of the fencing donated by Xander?
- B) Can Maeve and Azaadi use the donated fencing to make a rectangular garden with a perimeter of 28 metres, without cutting any of the pieces? Justify your answer.
- C) Suppose that they had made a mistake when measuring the first time, and the actual length of the fence that Thandi donated was 7 metres, but the total perimeter of the donated fencing was still 28 metres. Can they still form a rectangular garden without cutting any of the pieces? Justify your answer.





## Problem of the Week

### Problem A and Solution

### School Garden

#### Problem

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#### Solution

- A) We can convert the section measured in centimetres to metres, i.e. 300 centimetres is equal to 3 metres. Now we can add together the known lengths of fencing. The total is  $5 + 3 + 6 + 8 = 22$ .  
Since we know the total amount of fencing that surrounds the garden is 28 metres, then the amount Xander donated must be  $28 - 22 = 6$  metres.
- B) It is possible to create a rectangle out of the donated sections of fencing without cutting any pieces. Maeve and Azaadi need to find a way to form a quadrilateral with five pieces of fencing. This means that one of the sides must be formed using two pieces of fencing. The other restriction is that the opposite sides of the quadrilateral must be the same length. With these pieces of fencing, the rectangle can be formed with dimensions  $6 \text{ m} \times 8 \text{ m}$ . The two sections that are 6 metres will be opposite sides of the rectangle. The sections that are 300 centimetres and 5 metres can be joined together to form a side that is 8 metres. The other section that is 8 metres will be on the opposite side of the rectangle.



- C) In this case, if Thandi donated a section that is 7 metres long (1 metre shorter than originally thought), and the perimeter stayed the same, then Xander's section must also be 7 metres long (1 metre longer than originally thought.) However, if we consider the lengths of all possible pairs of the fence pieces, none of them are equal to the length of one of the other pieces of fence. In particular, if we consider the lengths: 3, 5, 6, 7, and 7, the possible sums of two pieces are:  $(3 + 5) = 8$ ,  $(3 + 6) = 9$ ,  $(3 + 7) = 10$ ,  $(5 + 6) = 11$ ,  $(5 + 7) = 12$ ,  $(6 + 7) = 13$ , and  $(7 + 7) = 14$ . In this case we do not even have to consider all combinations, since when we add the two smallest lengths  $(3 + 5)$ , we get a sum that is larger than the longest piece of fence. Since all of the other sums will be larger than this smallest one, then there is no combination that will work.



## Teacher's Notes

When trying to form a rectangle out of the pieces of fencing, we are looking for ways to combine these pieces so that we have two pairs of equal length. One way to look at this problem is to consider the *partitions* of the set of pieces of fencing into four subsets. Then we look at the combined lengths of the pieces in those four subsets to see if it is possible to form a rectangle.

A partition of a set is a set of non-empty subsets, where each element in the original set appears in exactly one of the subsets. Let's suppose we have five pieces of fencing and we describe these as the set  $\{A, B, C, D, E\}$ , where each letter represents one of the pieces. This means that  $\{\{A\}, \{B, C, D\}, \{E\}\}$  is an example of a partition of this set into three subsets. A set of five elements has a total of 52 partitions. However, for the problem of determining if five pieces of fencing can form a rectangle without being cut, we only need to consider the partitions into four subsets. These partitions are:

$$\{\{A, B\}, \{C\}, \{D\}, \{E\}\}$$

$$\{\{A, C\}, \{B\}, \{D\}, \{E\}\}$$

$$\{\{A, D\}, \{B\}, \{C\}, \{E\}\}$$

$$\{\{A, E\}, \{B\}, \{C\}, \{D\}\}$$

$$\{\{A\}, \{B, C\}, \{D\}, \{E\}\}$$

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$$\{\{A\}, \{B\}, \{C, D\}, \{E\}\}$$

$$\{\{A\}, \{B\}, \{C, E\}, \{D\}\}$$

$$\{\{A\}, \{B\}, \{C\}, \{D, E\}\}$$

Now, given *any* five pieces of fencing, we can consider these ten partitions. Then we would check to see if there are matching lengths in pairs of the subsets of at least one of these partitions. If we find a pair of matching pairs, then we know that these pieces can form a rectangle.

The problem gets more complex if the number of pieces of fencing increases. If we start with six pieces, there are a total of 203 partitions altogether and 65 partitions with four subsets.



## Problem of the Week

### Problem A

### Cross Training

Jelena is training at the track at her school. She does interval training, which means that she runs for some distance then stops to do other exercises. Each time around the track is called a lap. This is her training plan:

- Run half the way around the track; stop and do 10 push-ups.
- Run three quarters the way around the track; stop and do five burpees.
- Run one and a quarter the way around the track; stop and do 15 jumping jacks.

How many laps of the track has Jelena completed after doing the jumping jacks?





## Problem of the Week

### Problem A and Solution

#### Cross Training

#### Problem

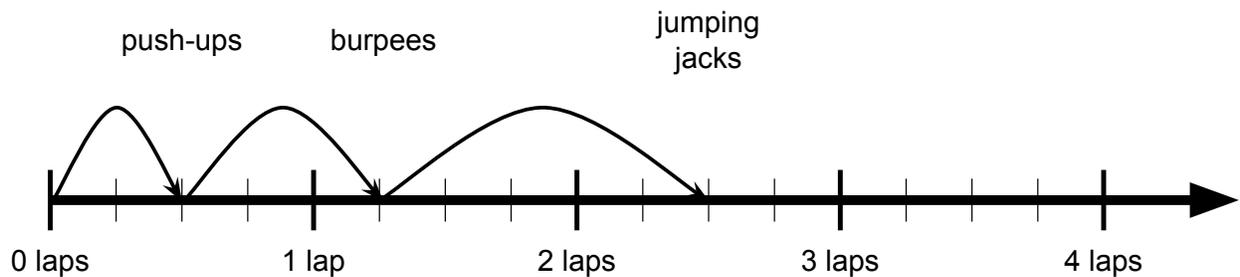
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How many laps of the track has Jelena completed after doing the jumping jacks?

#### Solution

One way to solve this problem would be to use a number line. We can break up the number line into quarters.



From this, we can see that after completing the jumping jacks, Jelena has completed 2 and a half laps of the track.

Another way to determine the answer would be to add fractions together. To add fractions, we need a common denominator. We know that  $\frac{1}{2} = \frac{2}{4}$  and that  $1 = \frac{4}{4}$  we can add:  $\frac{2}{4} + \frac{3}{4} + \frac{4}{4} + \frac{1}{4} = \frac{10}{4} = 2\frac{1}{2}$ .



## Teacher's Notes

Working with fractions is notoriously difficult for many students, even those studying high school math and beyond. Using a *number line* can help with adding fractions, especially when the sum produces a *mixed fraction*.

A number line often includes arrows at one or both ends to indicate that positive and negative numbers continue to increase in magnitude indefinitely in each direction. In particular, an arrow pointing to the right indicates that there are an *infinite* number of positive integers. There are also an infinite number of values between any two marked points on the number line. For example, there are an infinite number of values between two consecutive positive integers. Some of those values are *rational* numbers that can be represented by fractions in the form:

$$\frac{a}{b}, \text{ where } a \text{ and } b \text{ are integers, and } b \text{ is not } 0$$

There are other values, such as  $\sqrt{2}$ , that cannot be represented in this form. They are known as *irrational* numbers.

Intuitively we may think that there are many more rational numbers than integers. However, German mathematician Georg Cantor proved that we can match up each positive rational number to a whole number. This means that if we compare the set of integers and the set of rational numbers, we would find them to be the same "size" of infinity. Cantor also proved that if we consider the set of all real numbers, including irrational numbers, this set of numbers is actually bigger than the set of integers or rational numbers.



## Problem of the Week

### Problem A

### Yard Sale

North Star Community School is planning a yard sale to raise money for the Mathletes program. Each grade level is organizing items to sell. The grade 3 and 4 classes have many donations.

- Aoife donates a book, that weighs one and three-quarter kilograms.
- Soroush donates a bowling ball that weighs 7200 g.
- Keiran donates a box of laundry soap that weighs 3 kg.
- Reese donates a jug that weighs one-quarter of a kilogram.
- Sofia donates an unopened package of 6 protein bars. Each bar weighs 50 g.
- Graydon donates a sack of sugar that weighs four times as much as the jug that Reese donates.
- Nathan donates a pumpkin that weighs five and a half kilograms.

- A) What is the total weight of all donated items?
- B) The students organize the items by weight, with the heaviest item on the left side and the lightest item on the right. How are the items arranged?
- C) If each table can only hold up to a maximum of 10 kg and they keep the items in the order from part B), what is the fewest number of tables they need for the sale?





## Problem of the Week

### Problem A and Solution

#### Yard Sale

#### Problem

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- C) If each table can only hold up to a maximum of 10 kg and they keep the items in the order from part B), what is the fewest number of tables they need for the sale?

#### Solution

- A) To calculate the total weight, we can convert all of the individual weights to the same units.
- One kilogram equals 1000 g. Three quarters of a kilogram equals 750 g.
  - 3 kg is equal to  $3 \times 1000 = 3000$  g.
  - One-quarter of a kilogram equals 250 g.
  - The package of protein bars weighs  $6 \times 50 = 300$  g.
  - The sack of sugar weighs  $4 \times \frac{1}{4} = 1$  kg, and 1 kg equals 1000 g.
  - Five kilograms equals 5000 g. A half a kilogram equals 500 g.



The total weight of the items is:

$$1000 + 750 + 7200 + 3000 + 250 + 300 + 1000 + 5000 + 500 = 19\,000 \text{ g} = 19 \text{ kg}$$

B) From left to right the items would be:

bowling ball, pumpkin, laundry soap, book, sugar, protein bars, jug

C) Keeping the items in order from heaviest to lightest, putting the bowling ball and the pumpkin on the same table ( $7200 + 5500 = 12\,700$ ) would exceed the 10 kg limit. So the bowling ball needs to be on its own table.

The weight of the pumpkin and the laundry soap is  $5500 + 3000 = 8500 \text{ g}$ .

Adding the weight of the book makes the total  $8500 + 1750 = 10\,250 \text{ g}$ .

This would exceed the 10 kg limit. So the pumpkin and laundry soap must be on a table by themselves.

The weight of the remaining items is:  $1750 + 1000 + 300 + 250 = 3300 \text{ g}$ .

This is below the 10 kg limit. So they would need at most three tables.

Now, it would be possible to arrange the items on two tables, if the order of the items was not important. One way would be to have the book and jug on the same table as the bowling ball. The total weight of these three items is:  $7200 + 1750 + 250 = 9200 \text{ g}$ .

Since the total weight of all of the items is 19 000 g, the remaining items weigh  $19\,000 - 9200 = 9800 \text{ g}$ . Both of these totals are below the 10 kg maximum.



## Teacher's Notes

The third part in this problem is reminiscent of a classic problem in optimization known as the *Knapsack Problem*. The inspiration for this problem is to imagine you have a knapsack that has a limit to how much weight it can hold. You have several items, of various weights, that have some other value assigned to them. You want to pick the items that will maximize the value that is carried in the knapsack. You cannot exceed the weight limit of the knapsack. Which items should you pick?

It is tempting to simply choose the item with the highest value, and place it in the knapsack. Then choose the item with the next highest value, and place it in the knapsack. You continue this process until the next item that would be chosen would exceed the weight limit of the knapsack. This is known as the *Greedy Algorithm*. Unfortunately, this technique does not work in all cases.

Imagine you only had one table available to display items at the yard sale, and it has a weight limit of 10 kg. Suppose you had assigned prices to each of the sale items, and you want to display the items that fit on the table that have the highest combined value. Suppose you have determined that the items should be marked with the following prices:

Item	Weight (g)	Price (\$)
book	1750	1.00
bowling ball	7200	12.00
laundry soap	3000	6.00
jug	250	3.00
protein bars	300	2.00
sugar	1000	4.00
pumpkin	5500	9.00

Notice in this case, if we choose to display the item with the highest price (the bowling ball) then we could not select the laundry soap or the pumpkin to display, since adding the weight of either of these items would exceed the weight limit of the table. We could display the book and the sugar along with the bowling ball, since the total weight of those three items is 9950 g. The total value of these three items is:

$$\$12.00 + \$4.00 + \$1.00 = \$17.00$$

We could display the bowling ball with either the book or the sugar along with the jug and the protein bars. Since the sugar is priced higher than the book, we could display the bowling ball, sugar, jug, and protein bars with a total weight of 8750 g. The total value of these items is

$$\$12.00 + \$4.00 + \$3.00 + \$2.00 = \$21.00$$

If we displayed the pumpkin, the laundry soap, the sugar, and the jug, the total weight is 9750 g. We can see that this is a better choice since the total value of these items is:



$$\$9.00 + \$6.00 + \$4.00 + \$3.00 = \$22.00$$

The “Knapsack Problem” can be applied to many real life situations such as waste management and the allocation of public resources. We have to be careful when we try to find the best solution for this type of problem.



## Problem of the Week

### Problem A

### Colourful Conclusions

Mrs. Routliffe's Grade 3/4 class was learning to collect data using surveys. Kyle created the chart below to show data collected from his classmates about their favourite colours.

Favourite Colour	Red	Orange	Green	Blue	Purple	Yellow
Number of Students			 	 		

Kyle surveyed 35 students. The tallies for **Orange** and **Purple** have not been filled in.

The mode for the data set is equal to the number of tallies for **Orange**.

What are the possible values for **Orange** and **Purple**? Explain your reasoning.





## Problem of the Week

### Problem A and Solution

### Colourful Conclusions

#### Problem

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The mode for the data set is equal to the number of tallies for **Orange**.

What are the possible values for **Orange** and **Purple**? Explain your reasoning.

#### Solution

The total number of tallies is 21. Since 35 students were surveyed, the number of students who selected **Orange** or **Purple** is  $35 - 21 = 14$ .

The *mode* is the number that appears most often in the data set. If no value appears more than once, then there is no mode. Since this data set has a mode, the number of tallies for **Orange** must match the tallies for at least one of the other colours. So the possible tallies for **Orange** could match one of the known tallies (4, 6, 9, 2), and the possible tallies for **Purple** would be the difference of the **Orange** tallies from 14. Alternatively, the **Orange** tallies could match the **Purple** tallies. In this case they would each be  $14 \div 2 = 7$ . Therefore, the possible tallies for **Orange** and **Purple** are:

Orange	Purple



## Teacher's Notes

In statistics, *mean*, *median*, and *mode* are measurements of *central tendency*. These measurements are used to describe some features of the entire set with (usually) a single value. The mode measurement is different from these other two measurements in a few ways.

Although mode often does not give a “good picture” of the entire data set, there are cases where the mode is more helpful than either the median or the mean. It is possible to have multiple mode values, and those values are not necessarily close to each other. If the data set has a *bimodal* distribution, this means there are two peaks in a graph that represents the data. It is possible that those peaks are far apart. In this case, the mean and median values would appear in between those peaks, and the fact that there are clusters of data values around two distinct peaks is lost. However, if there are two modes identified, then the shape of this graph may be reflected in these values.

Also, unlike mean and median, it is possible to compute the mode of non-numeric data. This problem is a good example of that. The data values are colours, and we are able to calculate the mode of this data set. The *first-past-the-post* voting system uses mode to determine a winner in an election. In this case, the names on a ballot are the data values, and the name that matches the mode is the winner.



## Problem of the Week

### Problem A

#### Student Cookies

Cook E. Doe School had a bake sale. At the sale, a package of 36 cookies cost \$12. Four students decided to put their money together and buy one package. Kayden contributed \$2, Kayah contributed \$3, Kayla contributed \$4, and Michelle contributed \$3. Everyone wants to be fair when dividing up the package.

How many cookies should each person get based on how much each paid?





## Problem of the Week

### Problem A and Solution

### Student Cookies

#### Problem

Cook E. Doe School had a bake sale. At the sale, a package of 36 cookies cost \$12. Four students decided to put their money together and buy one package. Kayden contributed \$2, Kayah contributed \$3, Kayla contributed \$4, and Michelle contributed \$3. Everyone wants to be fair when dividing up the package.

How many cookies should each person get based on how much each paid?

#### Solution

Since 36 cookies cost \$12, we can calculate the cost of cookies per dollar as:  
 $36 \div 12 = 3$ .

So to be fair, for each dollar contributed to the purchase, the contributing student should get 3 cookies.

Kayden contributed \$2 and should receive  $2 \times 3 = 6$  cookies.

Kayah contributed \$3 and should receive  $3 \times 3 = 9$  cookies.

Kayla contributed \$4 and should receive  $4 \times 3 = 12$  cookies.

Michelle contributed \$3 and should receive  $3 \times 3 = 9$  cookies.



## Teacher's Notes

Another way to look at this problem is to consider equivalent fractions. For any fraction, there are an infinite number of fractions that are equal in value. If we start with a fraction  $\frac{a}{b}$  (assuming  $b \neq 0$ ) we can multiply the numerator and denominator by the number  $k$  (assuming  $k \neq 0$  and  $k \neq 1$ ) to produce a different fraction that is equivalent to  $\frac{a}{b}$ .

For example,  $\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$ .

In this case, we are trying to find equivalent fractions in terms of the money and in terms of the cookies. We know that Kayden contributed \$2 out of the \$12, and we know that there are 36 cookies. What we want to know is how many cookies Kayden should receive. To be fair, we want the following fractions to be equivalent:

$$\frac{2}{12} = \frac{?}{36}$$

We can see that for the denominators we must multiply  $12 \times 3$  to get 36. So to make the fractions equivalent we would need to multiply the numerator by 3 as well. This means:

$$\frac{2}{12} = \frac{2 \times 3}{12 \times 3} = \frac{6}{36}$$

so Kayden should receive 6 cookies. We could calculate the number of cookies each of the other friends should get in a similar way, and in each case it would result in multiplying the dollar amount contributed by 3.

We can also solve the problem algebraically. For example, if we wanted to calculate how many cookies Kayla should receive, we can write an equation with an unknown for the numerator of the cookie fraction. Then we solve for the unknown.

$$\begin{aligned} \frac{c}{36} &= \frac{4}{12} \\ \cancel{36} \times \frac{c}{\cancel{36}} &= \frac{4}{\cancel{12}} \times \cancel{36}^3 \\ c &= 4 \times 3 \\ c &= 12 \end{aligned}$$

So Kayla should get 12 cookies.



## Problem of the Week

### Problem A

#### Making Jewelry

Lindsay has decided to make a necklace out of red, white, and yellow beads. She decides to have a repeated pattern in the necklace. She will have five red beads, followed by three white beads, followed by one yellow bead, followed by three white beads. Then the pattern will repeat. When she is done, she connects the ends of the string of beads to form a loop. The beads in her necklace will always form a complete pattern.

- A) The beads Lindsay has are 5 mm wide. She wants to make a necklace that is at least 25 cm long. What is the smallest number of beads she needs to make the necklace where each pattern is complete?
- B) Lindsay has 44 beads of each colour. What is the longest necklace she can make where each pattern is complete?





## Problem of the Week

### Problem A and Solution

#### Making Jewelry

#### Problem

Lindsay has decided to make a necklace out of red, white, and yellow beads. She decides to have a repeated pattern in the necklace. She will have five red beads, followed by three white beads, followed by one yellow bead, followed by three white beads. Then the pattern will repeat. When she is done, she connects the ends of the string of beads to form a loop. The beads in her necklace will always form a complete pattern.

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- B) Lindsay has 44 beads of each colour. What is the longest necklace she can make where each pattern is complete?

#### Solution

The total number of beads in the pattern is:  $5 + 3 + 1 + 3 = 12$ . The length of one copy of the pattern is  $12 \times 5 \text{ mm} = 60 \text{ mm}$  which is equal to 6 cm. So we can use a table to determine the length of the necklace as it includes more copies of the pattern:

Copies of the Pattern	Number of Beads	Necklace Length (in cm)
1	12	6
2	24	12
3	36	18
4	48	24
5	60	30
6	72	36
7	84	42

- A) So Lindsay will need 60 beads (25 red, 30 white, and 5 yellow) to have a necklace that is at least 25 cm and has complete patterns.
- B) Since the pattern includes 6 white beads, this is the colour that occurs the most. If Lindsay has 44 of each colour, then we can determine the maximum number of copies of the pattern by dividing:  $44 \div 6 = 7$  remainder 2. So the pattern can be repeated at most 7 times. In this case, the longest necklace Lindsay can make will be  $7 \times 6 \text{ cm} = 42 \text{ cm}$ .



## Teacher's Notes

This week, students are solving *optimization* problems. In part (a) they are required to find a minimum value given certain problem specifications, and in part (b) they are required to find a maximum value given certain problem specifications. Optimization is an area of mathematics that explores ways to make operations more efficient. These techniques can be applied to many different situations. For example:

- In manufacturing, companies look for ways to minimize waste given the materials required for the process.
- In medicine, hospitals look for ways to improve emergency departments in a way that maximizes patient care given the limits of available staff, equipment, and other resources.
- In education, administrators need to schedule courses in a way that minimizes conflicts for students' requests given limits on classroom capacities, lab requirements, and many other factors.
- In science, people look for ways to minimize the environmental impact of an industrial process.

Studying this area of mathematics can lead to many diverse job opportunities.



## Problem of the Week

### Problem A

### Charity Drive

Every year, Spring Creek Community School has a charity drive to gather gifts for families in need. When everything is tallied, the organizers notice the following:

- There are 3 times as many stuffed toys as sweaters.
- The number of books is equal to the total number of sweaters and gift cards.
- There are 2 more board games than sweaters.
- There are 5 fewer gift cards than stuffed toys.
- There are 24 stuffed toys.

How many of each item (sweaters, board games, books, and gift cards) have been gathered?





## Problem of the Week

### Problem A and Solution

### Charity Drive

#### Problem

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- There are 24 stuffed toys.

How many of each item (sweaters, board games, books, and gift cards) have been gathered?

#### Solution

Since there are 3 times as many stuffed toys as sweaters, this means that there are one-third as many sweaters as stuffed toys. Since there are 24 stuffed toys, and  $\frac{1}{3}$  of 24 is 8, then there must be 8 sweaters. Alternatively we can calculate  $\frac{1}{3}$  of a number by dividing it by 3. So we can confirm the number of sweaters by calculating  $24 \div 3 = 8$ .

Since there are 2 more board games than sweaters, there must be  $8 + 2 = 10$  board games.

Since there are 5 fewer gift cards than stuffed toys, there must be  $24 - 5 = 19$  gift cards.

Since the number of books is equal to the total number of sweaters and gift cards, there must be  $8 + 19 = 27$  books.



## Teacher's Notes

In this problem we are given information about a relationship between the number of stuffed toys and the number of sweaters. We can write that relationship as an equation like this:

Let  $x$  represent the number of sweaters.

Let  $f(x)$  be a function that represents the number of stuffed toys.

Then,  $f(x) = 3 \cdot x$

If we are given some value for  $x$ , we can easily calculate  $f(x)$ . However we are not given the number of sweaters, but we are given the number of stuffed toys. One way to think about solving this problem is to find the *inverse function* of  $f(x)$ .

In general, if  $f(x) = y$  then  $g$  is an inverse function of  $f$  if and only if  $g(y) = x$ . We can use the notation  $f^{-1}$  to describe the inverse function of  $f(x)$ . In this case we could say that  $f^{-1}(y) = \frac{y}{3}$ . Then we can use this inverse function to determine the number of sweaters given the number of stuffed toys.

Not all functions have an inverse function; a function needs to be “one-to-one” in order to have an inverse function. A one-to-one function is a function that never produces the same value for different values of  $x$ . For example,  $f(x) = 3 \cdot x$  is one-to-one, but  $g(x) = x^2$  is not one-to-one. This is because we can find an example, such as  $g(-2) = 4$  **and**  $g(2) = 4$ , where there are two different values of  $x$  that produce the same number when we calculate  $g(x)$ .

When describing basic mathematical operations, we might informally say that addition is the opposite of subtraction, and multiplication is the opposite of division. The concept of “opposite” in this context is a precursor to understanding inverse functions.



## Problem of the Week

### Problem A

### Hockey Practice

Lucie and Liam love to play hockey. Each week, Lucie attends practice for 2 hours per day Monday to Friday, as well as  $4\frac{1}{2}$  hours per day on Saturday and Sunday. Liam attends practice for 3 hours per day on Mondays and Wednesdays, and 2 hours on Fridays. On Saturdays he attends practice from 1:00 p.m. until 4:30 p.m. On Sundays he attends practice from 1:30 p.m. until 5:00 p.m. He does not have practice on Tuesdays and Thursdays.

The answers to questions about Lucie and Liam's hockey practice schedules are:

- A) 15
- B) 19
- C) 4
- D) 34

What are possible questions that give these answers?





## Problem of the Week

### Problem A and Solution

### Hockey Practice

#### Problem

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The answers to questions about Lucie and Liam's hockey practice schedules are:

- A) 15
- B) 19
- C) 4
- D) 34

What are possible questions that give these answers?

#### Solution

Here are some calculated values about the practice times:

- From Monday to Friday, Lucie attends practice for a total of  $5 \times 2 = 10$  hours.
- On Saturday and Sunday, Lucie attends practice for a total of  $4\frac{1}{2} + 4\frac{1}{2} = 4 + 4 + \frac{1}{2} + \frac{1}{2} = 8 + 1 = 9$  hours.
- From Monday to Friday, Liam attends practice for a total of  $3 + 3 + 2 = 8$  hours.
- On Saturday and Sunday Liam attends practice for  $3\frac{1}{2}$  hours each day, for a total of  $3 + 3 + \frac{1}{2} + \frac{1}{2} = 6 + 1 = 7$  hours.

Here are some possible questions:

- A) For how many hours per week does Liam have practice? ( $8 + 7 = 15$ )
- B) For how many hours per week does Lucie have practice? ( $10 + 9 = 19$ )
- C) For how many more hours does Lucie have practice compared to Liam?  
( $19 - 15 = 4$ )
- D) For how many total hours per week do both players have practice?  
( $19 + 15 = 34$ )



## Teacher's Notes

This problem might be considered an example of *reverse engineering*, which is a process where we are given the final product and we try to determine the details of its design. Reverse engineering can be used by software engineers to improve existing code. However, software developers may want to protect against reverse engineering using *obfuscation*. This is a technique that makes it much more difficult for hackers to recreate the source code created by the original developers, given the executable code that people purchase to run on their computers. This is an attempt to maintain the valuable *intellectual property* associated with the software.

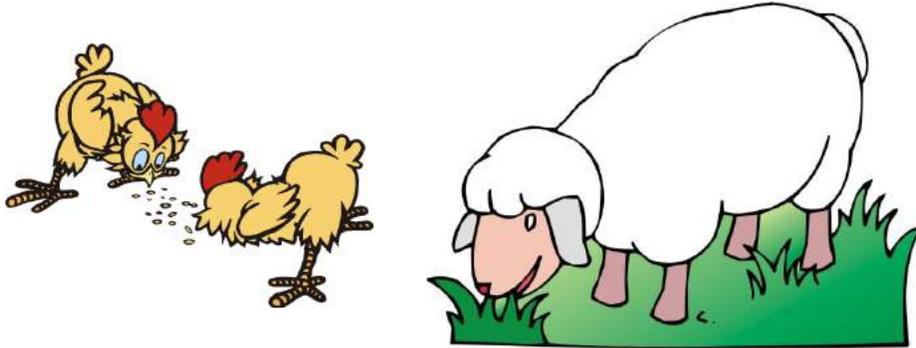


## Problem of the Week

### Problem A

#### Chicken and Sheep, Heads and Feet

Samantha has chickens and sheep on her farm. Each chicken has two legs and each sheep has four legs. Each chicken has one head and each sheep has one head. She looked out one day and counted 48 animal heads and 134 legs. How many of each animal live on the farm?





## Problem of the Week

### Problem A and Solution

### Chicken and Sheep, Heads and Feet

#### Problem

Samantha has chickens and sheep on her farm. Each chicken has two legs and each sheep has four legs. Each chicken has one head and each sheep has one head. She looked out one day and counted 48 animal heads and 134 legs. How many of each animal live on the farm?

#### Solution

One way to determine the answer is to make a table that keeps track of chickens, sheep, heads, and legs. The number of chickens plus the number of sheep must always total 48. So we could start with 0 chickens and 48 sheep. If there were 48 sheep, then there would be  $48 \times 4 = 192$  legs. Then in each row of the table that follows, we increase the number of chicken heads by 1, and decrease the number of sheep heads by 1. We also increase the number of chicken legs by 2, and decrease the number of sheep legs by 4.

Chicken Heads	Sheep Heads	Chicken Legs	Sheep Legs	Total Legs
0	$48 - 0 = 48$	$0 \times 2 = 0$	$48 \times 4 = 192$	192
1	$48 - 1 = 47$	$1 \times 2 = 2$	$47 \times 4 = 188$	190
2	$48 - 2 = 46$	$2 \times 2 = 4$	$46 \times 4 = 184$	188
3	$48 - 3 = 45$	$3 \times 2 = 6$	$45 \times 4 = 180$	186
4	$48 - 4 = 44$	$4 \times 2 = 8$	$44 \times 4 = 176$	184

From the first few rows in this table, we see there is a pattern forming for the total number of legs. Each time we increase the number of chickens by 1, the total number of legs decreases by 2. We could continue filling in the table until we get a row where the total number of legs is 134. However, this would mean filling in many more entries in the table. We need to get the total legs from 184 to 134, which is a difference of  $184 - 134 = 50$ . Since the total number of legs decreases by 2 for each additional chicken, then we would reach the correct total number of legs if we filled in  $50 \div 2 = 25$  more rows. This means we have  $4 + 25 = 29$  chickens. That row in the table would look like:

Chicken Heads	Sheep Heads	Chicken Legs	Sheep Legs	Total Legs
29	$48 - 29 = 19$	$29 \times 2 = 58$	$19 \times 4 = 76$	134

We see that there are 29 chickens and 19 sheep on the farm.



## Teacher's Notes

Here is an alternative approach to solving the problem.

We know all of the animals have at least 2 legs. Since we count 48 heads, then we can account for  $2 \times 48 = 96$  legs, no matter what type of animals are on the farm. However, we have counted  $134 - 96 = 38$  more legs than if all of the animals only had 2 legs. Those extra 38 legs must be on sheep. Each sheep contributes 2 more legs to the count. So we need to have  $38 \div 2 = 19$  sheep on the farm. This means we have  $48 - 19 = 29$  chickens.

We could also solve this problem algebraically.

Let  $s$  represent the number of sheep.

Let  $c$  represent the number of chickens.

Knowing that each animal has 1 head, that chickens have 2 legs, and that sheep have 4 legs, we can write the following equations:

$$s + c = 48 \quad (\text{equation 1})$$

$$4s + 2c = 134 \quad (\text{equation 2})$$

Then we multiply both sides of *equation 1* by 2:

$$2 \cdot (s + c) = 2 \cdot 48$$

$$2s + 2c = 96 \quad (\text{equation 3})$$

Then we subtract *equation 2* - *equation 3* and solve for  $s$ :

$$(4s - 2s) + (2c - 2c) = 134 - 96$$

$$2s = 38$$

$$\frac{2s}{2} = \frac{38}{2}$$

$$s = 19$$

Finally, we can substitute the value of  $s$  into *equation 1* and solve for  $c$ :

$$(19) + c = 48$$

$$19 - 19 + c = 48 - 19$$

$$c = 29$$

This means we have 19 sheep and 29 chickens.



## Problem of the Week

### Problem A

#### T-Shirts

A shipment of 100 shirts was received at “Shirts R Us” in a large fabric bag. Vera, the store owner, knew that she ordered four different colours of shirts. She ordered 40 red shirts and half as many yellow shirts. She ordered 30 blue shirts, but could not remember the number of green shirts she ordered. For each colour, she ordered half of the shirts long-sleeved and the other half short-sleeved. Vera pulls shirts out of the bag one at a time.

- A) What is the probability that she will pull out a red shirt from the bag first?
- B) What is the probability that she will pull out a green shirt from the bag first?
- C) What is the probability that she will pull out a short-sleeved blue shirt from the bag first?





## Problem of the Week

### Problem A and Solution

#### T-Shirts

#### Problem

A shipment of 100 shirts was received at “Shirts R Us” in a large fabric bag. Vera, the store owner, knew that she ordered four different colours of shirts. She ordered 40 red shirts and half as many yellow shirts. She ordered 30 blue shirts, but could not remember the number of green shirts she ordered. For each colour, she ordered half of the shirts long-sleeved and the other half short-sleeved. Vera pulls shirts out of the bag one at a time.

- A) What is the probability that she will pull out a red shirt from the bag first?
- B) What is the probability that she will pull out a green shirt from the bag first?
- C) What is the probability that she will pull out a short-sleeved blue shirt from the bag first?

#### Solution

We should start by calculating how many of each colour of shirt has been ordered. Since there are half as many yellow shirts as red shirts, then there are  $40 \div 2 = 20$  yellow shirts.

Now we can calculate the total number of red, yellow, and blue shirts as  $40 + 20 + 30 = 90$ . Since there were 100 shirts in the order, and the only other colour ordered was green, then there must be  $100 - 90 = 10$  green shirts.

- A) Since there are 40 red shirts in the bag, then there is a 40 out of 100 chance that the first shirt Vera pulls from the bag is red.
- B) Since there are 10 green shirts in the bag, then there is a 10 out of 100 chance that the first shirt Vera pulls from the bag is green.
- C) Since half of each colour is short-sleeved, there are  $30 \div 2 = 15$  short-sleeved blue shirts. This means there is a 15 out of 100 chance that the first shirt Vera pulls from the bag is a short-sleeved blue shirt.



## Teacher's Notes

Probability describes the likelihood that some event will happen. It is always a number between 0 and 1. When the probability is 0, we expect that the event will definitely not happen. When the probability is 1, we expect that the event definitely will happen. All other probabilities describe a relative chance of the event happening.

We see many ways of describing probability in news reports, advertisements, scientific papers, and other media. However, all of them represent a number between 0 and 1. In many cases, there is an implied fraction in the probability description. For example:

- A probability can be described in the form “**a out of b**”, where  $b$  is always greater than or equal to  $a$ . This implies the fraction  $\frac{a}{b}$ .
- A probability can be described as a percentage such as **x%**. This implies the fraction  $\frac{x}{100}$ .
- A probability can be described in the form “**1 in n chance**”, where  $n$  is a positive integer. This implies the fraction  $\frac{1}{n}$ .



## Problem of the Week

### Problem A

#### Time to Read

Jordan borrowed a book from the library on a Sunday. He must return it two weeks after he got it. He plans to read 10 pages of the book on Mondays, Tuesdays, Wednesdays, Thursdays, and Fridays. On Saturdays and Sundays he plans to read 20 pages of the book. He starts reading on the day he got the book (Sunday).

- A) If the book is 250 pages, how many pages must he read on the last Sunday to finish it before he must return it?
- B) Instead of leaving many pages to read on the last day, Jordan may decide to increase the number of pages he reads, by the same amount, every day except the last Sunday. If Jordan wants to have at least 1 but no more than 20 pages to read on the last Sunday, how many more pages should he read every day?





## Problem of the Week

### Problem A and Solution

#### Time to Read

#### Problem

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- A) If the book is 250 pages, how many pages must he read on the last Sunday to finish it before he must return it?
- B) Instead of leaving many pages to read on the last day, Jordan may decide to increase the number of pages he reads, by the same amount, every day except the last Sunday. If Jordan wants to have at least 1 but no more than 20 pages to read on the last Sunday, how many more pages should he read every day?

#### Solution

- A) Over two weeks, there are two Mondays, Tuesdays, Wednesdays, Thursdays, and Fridays. This means there is a total of  $2 \times 5 = 10$  days when Jordan reads 10 pages of the book. Over these days, he reads a total of  $10 \times 10 = 100$  pages.

There are also two Saturdays and Sundays for a total of  $2 \times 2 = 4$  days when Jordan reads 20 pages of the book. This does not include the Sunday when Jordan returns the book to the library. Over these days, he reads a total of  $20 \times 4 = 80$  pages.

So, in those two weeks he reads  $100 + 80 = 180$  pages. This means to finish the book, Jordan must read  $250 - 180 = 70$  pages on the last Sunday.

- B) We can make a table showing how many pages he reads over two weeks, starting with the number of pages read according to the original schedule. Over two weeks, for each page added per day, the total number of pages read will increase by 14. If Jordan wants to read no more than 20 pages on the last Sunday, he must read at least  $250 - 20 = 230$  pages in the previous two weeks.

Mon-Fri Pages Per Day	Sat-Sun Pages Per Day	Total Pages in 2 Weeks
10	20	180
11	21	194
12	22	208
13	23	222
14	24	236

So Jordan should read four more pages each day, to ensure he reads no more than 20 pages on the last Sunday.



## Teacher's Notes

Real life problems often come with many *constraints*. In this problem, we start off with several constraints:

- Jordan has the book for two weeks
- He reads 10 pages each day Monday through Friday
- He reads 20 pages each day Saturday through Sunday

With part (b), we add an extra constraint:

- Jordan wants to read between 1 and 20 pages on the last Sunday

In solving this problem, students are taking the necessary information from the real life description and are generating mathematical relationships. It is important that they properly consider all of the constraints. Taking real life situations and creating a mathematical model that identifies the essential elements that are required to solve the problem is known as *abstraction*. The process of creating a good and accurate mathematical model can often be the most difficult part of solving a problem.



## Problem of the Week

### Problem A

### Feeding Fractions

Surya is taking care of his friend's cat (named Yola) and dog (named Ginny) for two weeks. He feeds the animals twice a day. In the morning, Yola gets  $\frac{1}{3}$  of a can of cat food and Ginny gets  $\frac{1}{2}$  of a can of dog food. In the evening, both animals eat dry food. Surya saves the leftover canned food in the refrigerator between feedings.

The first day that he feeds the animals is Saturday the 10th. He opens up a new can of cat food and a new can of dog food on that first day. The last day he feeds the animals is on Friday the 23rd.

- A) On what days does Surya have to open both a new can of cat food and a new can of dog food? How much of each kind of food is left in cans in the refrigerator when the owners come home on Saturday the 24th?
- B) Surya recycles the cans when all the food has been served. How many cans did Surya recycle while he was taking care of Yola and Ginny?





## Problem of the Week

### Problem A and Solution

### Feeding Fractions

#### Problem

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- B) Surya recycles the cans when all the food has been served. How many cans did Surya recycle while he was taking care of Yola and Ginny?

#### Solution

We can make a table that keeps track of how much food is in the cans at the start of each day. When the can is full, we indicate this with the value 1.

Day	Cat Food	Dog Food
Saturday (10th)	1	1
Sunday (11th)	$\frac{2}{3}$	$\frac{1}{2}$
Monday (12th)	$\frac{1}{3}$	1
Tuesday (13th)	1	$\frac{1}{2}$
Wednesday (14th)	$\frac{2}{3}$	1
Thursday (15th)	$\frac{1}{3}$	$\frac{1}{2}$
Friday (16th)	1	1
Saturday (17th)	$\frac{2}{3}$	$\frac{1}{2}$
Sunday (18th)	$\frac{1}{3}$	1
Monday (19th)	1	$\frac{1}{2}$
Tuesday (20th)	$\frac{2}{3}$	1
Wednesday (21st)	$\frac{1}{3}$	$\frac{1}{2}$
Thursday (22nd)	1	1
Friday (23rd)	$\frac{2}{3}$	$\frac{1}{2}$



- A) On the days when the cat food and dog food values contain a 1 in the table, this indicates that a new can must be opened. So on the days where both the cat food and dog food values contain a 1 in the table, Surya must open both a new can of dog food and a new can of cat food. This will happen on Saturday the 10th, Friday the 16th, and Thursday the 22nd. On the last day, Surya will feed Yola  $\frac{1}{3}$  of a can of cat food leaving  $\frac{1}{3}$  of a can in the refrigerator, and will feed Ginny  $\frac{1}{2}$  of a can of dog food leaving no dog food in the refrigerator.
- B) Each time the table shows a 1 (except for the first row), this indicates that a can of food has been emptied. This means that by the 23rd, there will be 4 cans of cat food recycled. If we include the dog food can that was emptied on the last day Surya fed the animals, there will be 7 cans of dog food recycled. This is a total of 11 recycled cans.



## Teacher's Notes

The key to solving this problem is to carefully record the repetitious action. In these kinds of situations, it is very easy to introduce an *off-by-one error*.

Keeping track of the food being used in the “middle” of the problem is relatively easy. Every six days, Surya will be using two cans of cat food and three cans of dog food. So, for example, if we expanded the timeline to 7 weeks (or 49 days) we can estimate how many cans of food will be used during that time. Since 49 is close to  $6 \times 8$ , then there would be approximately  $8 \times 2 = 16$  cans of cat food and  $8 \times 3 = 24$  cans of dog food used during that time. However, determining **exactly** how many cans were used during that time, and how much food would be left at the end, involves some careful tracking.

An off-by-one error is a classic issue in computer programming for solutions that involve loops. Often the toughest aspects of using a loop correctly in a program is dealing with the special cases of before the loop starts or when the loop ends.



## Problem of the Week

### Problem A

#### What's My Number?

I am a 3-digit even number.

The sum of my three digits is 20.

I am greater than  $40 \times 10$ .

I am less than  $1000 \div 2$ .

What number am I?





## Problem of the Week

### Problem A and Solution

#### What's My Number?

#### Problem

I am a 3-digit even number.  
The sum of my three digits is 20.  
I am greater than  $40 \times 10$ .  
I am less than  $1000 \div 2$ .  
What number am I?

#### Solution

Since  $40 \times 10 = 400$ , we know the number is greater than 400. Since the number is even, the smallest possible number is 402.

Since  $1000 \div 2 = 500$ , we know the number is less than 500. Since the number is even, the largest possible number is 498.

So we are looking for an even number between 402 and 498, inclusive. We could check each of the numbers in that range, to see which one has digits that add up to 20. However, that would mean checking 49 numbers. It would be better to reduce the range of numbers to check, if possible.

Here is one way of thinking about the solution:

Since the possible numbers are from 402 to 498, the first digit of the number is 4. Since the sum of the digits is 20, then the middle and last digit must sum to  $20 - 4 = 16$ . Since the number is even, its last digit is 0, 2, 4, 6, or 8.

If the last digit is 0, then the middle digit must be  $16 - 0 = 16$ , which is not a digit from 0 to 9.

If the last digit is 2, then the middle digit must be  $16 - 2 = 14$ , which is not a digit from 0 to 9.

If the last digit is 4, then the middle digit must be  $16 - 4 = 12$ , which is not a digit from 0 to 9.

If the last digit is 6, then the middle digit must be  $16 - 6 = 10$ , which is not a digit from 0 to 9.

If the last digit is 8, then the middle digit must be  $16 - 8 = 8$ , which is a valid digit.

We have examined all possible cases and the only number satisfying all of the conditions is 488. The number we are looking for is 488.



## Teacher's Notes

Here is another way of thinking about the solution:

Since the numbers from 402 to 498 all start with the digit 4, and the sum of all three digits must be 20, then the sum of the last two digits of the number must be  $20 - 4 = 16$ .

Since the largest possible digit is 9, then the other digit of the number must be at least  $16 - 9 = 7$ . So each of the last two digits must be in the range 7 to 9.

Since we are only considering even numbers, then the solution must be a number that ends with an 8.

So we can look at even numbers, that start with a 4, end with an 8, and where the middle digit is at least 7. The possibilities are: 478, 488, and 498. Now we can check the sum of the digits of these numbers:

- $4 + 7 + 8 = 19$
- $4 + 8 + 8 = 20$
- $4 + 8 + 9 = 21$

So the only number that satisfies all of the requirements is 488.

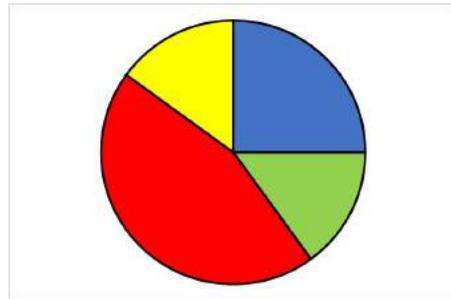
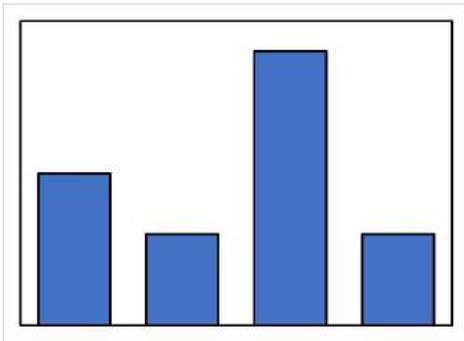


## Problem of the Week

### Problem A

#### Lost Data

Tanner randomly surveyed 40 students in his school about their ages. The ages given were six, seven, eight, and nine. After gathering the answers, he drew a bar chart and a pie chart to show the results. Unfortunately, before he labelled each chart, he lost the original data. The charts are shown below:



Tanner did remember that 6 students in the survey answered six for their age. He also remembered that  $\frac{1}{4}$  of the students surveyed answered eight for their age, and that the most popular answer was age seven. Based on this information, complete the table below:

Age	Number of Students
six	6
seven	
eight	
nine	



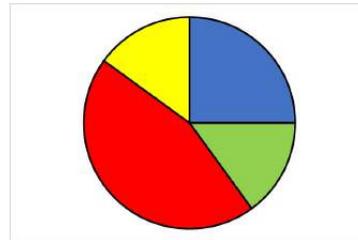
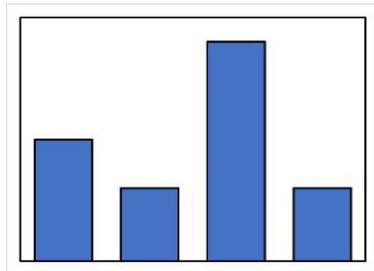
## Problem of the Week

### Problem A and Solution

#### Lost Data

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Age	Number of Students
six	6
seven	
eight	
nine	

#### Solution

We compute  $\frac{1}{4}$  of 40 is equal to 10. So 10 of the students who were surveyed are eight.

From the pie chart, we can see that there are two data values that are less than  $\frac{1}{4}$  of the total number responses, and one data value is greater than  $\frac{1}{4}$  of the total number of responses. We can also see from both charts that two of the data values are the same and less than one quarter of the total number of responses. From these two observations, along with the fact that one of the data values is 6, we can conclude that there is a second data value that is also 6.

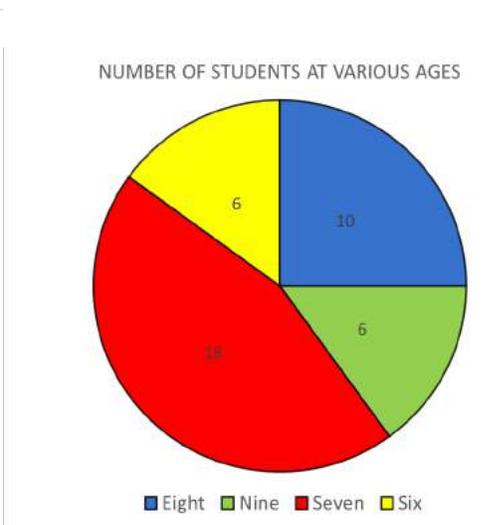
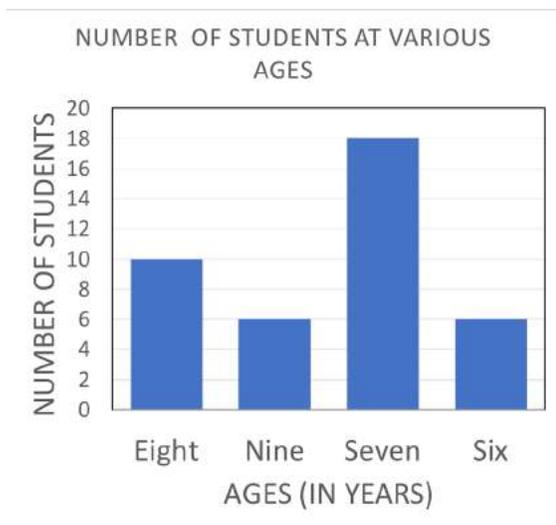


Now we can calculate the sum of three of the data values:  $6 + 6 + 10 = 22$ . Since 40 students were surveyed, the last data value must be  $40 - 22 = 18$ . This is the largest data value. Since seven was the most popular answer, there must be 18 students who are seven.

Now we know how many students answered six, seven, and eight for their ages. We also know that there is one data value (6) that we have not assigned to an age. So there must have been 6 students who answered age 9 in the survey. So the completed table is:

Age	Number of Students
six	6
seven	18
eight	10
nine	6

Note: For some reason, Tanner decided to use word labels on the horizontal axis of the bar chart, in alphabetical order. That is, the labels on the horizontal axis were eight, nine, seven and six. His completed graphs are shown below.

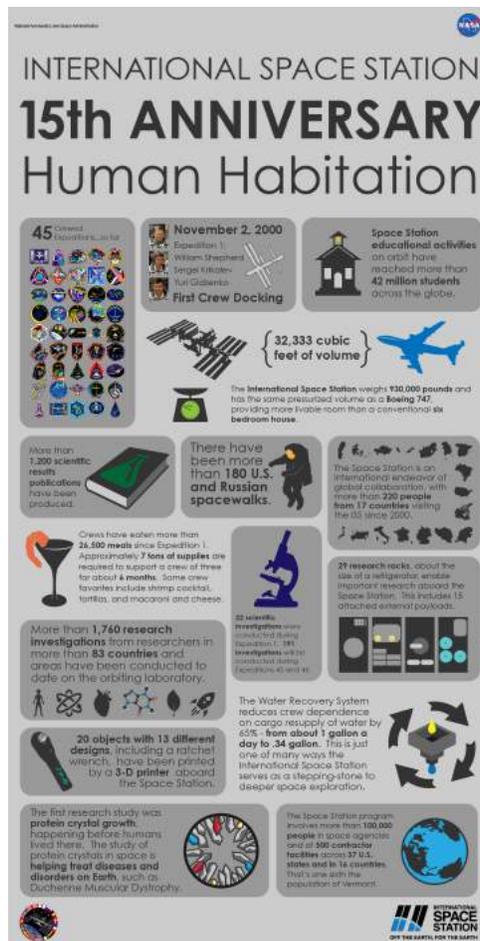




## Teacher's Notes

The charts in this problem were generated by an Excel spreadsheet with the same underlying data values: 6, 18, 10, and 6. Different graphical representations of data may make it easier to see relationships among the data values. For example, although it appears that the yellow and green sections of the pie chart are the same size, it is clearer in the bar chart (especially in the solution with grid lines) that these values are the same. However, in the pie chart, it is much easier to see that the blue section is one quarter of the total surveyed.

We see more and more sophisticated examples of using images to represent data. People have been using *infographics* as a way of capturing people's attention - often in an attempt to sell things, but also in an attempt to emphasize important information - for a long time. However with today's technology we see these kinds of images everywhere.



(Image provided by NASA / Public domain)



## Problem of the Week

### Problem A

### How Many Halves?

Robbie won 300 gumballs and he would like to share the winnings with his friends. He decides to list his friends in order based on how long he has known each of them. He wants to give away the most gumballs to the friend he has known the longest. Then he will give the next friend on the list exactly half as many gumballs as the person he has known the longest. He continues to give away exactly half as many to the next friend on his list, until the pattern cannot continue. (He will not give away half a gumball.)

- A) If he gives away 100 gumballs to the first friend on the list, how many friends will receive gumballs? Justify your answer.
- B) If the last person he gives gumballs to receives 5 gumballs, what is the largest number of gumballs that the first friend on the list can receive? How many gumballs are given away? Justify your answers.
- C) If Robbie wants to maximize the number of friends that receive gumballs, how many gumballs should the first person on the list receive? How many friends receive gumballs? Justify your answers.





## Problem of the Week

### Problem A and Solution

#### How Many Halves?

#### Problem

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- C) If Robbie wants to maximize the number of friends that receive gumballs, how many gumballs should the first person on the list receive? How many friends receive gumballs? Justify your answers.

#### Solution

- A) If the first friend receives 100 gumballs, then the next friend receives half as many, which is 50 gumballs. Half of 50 is 25 gumballs, which is the amount the next friend on the list receives. Since 25 is an odd number, it cannot be divided in half without ending up with a fraction. So the pattern ends after three friends have received gumballs from Robbie.
- B) If we know that the last friend on the list receives 5 gumballs, then as we move up the list, each friend receives twice as many as the previous one. Let's assign the last friend who receives 5 gumballs the number 1. Then we can make a table that shows the pattern of giving gumballs to his friends:



Friend	Number of Gumballs Friend Receives	Total Number of Gumballs Given
1	5	5
2	10	15
3	20	35
4	40	75
5	80	155
6	160	315

From the information on this table, we can answer the questions.

Since Robbie only has 300 gumballs to give away, if the last person receives 5 gumballs, then the largest amount of gumballs the first person on the list will receive 80 gumballs. There will be a total of 155 gumballs given away.

- C) We want to maximize the number of friends to whom Robbie gives gumballs. Using the two solutions above, we notice that the larger the last friend's amount of gumballs the fewer number of friends that can receive gumballs. One way to think about the problem is that we want the last friend to receive the smallest possible amount, which is 1 gumball. So, the second last friend would receive 2 gumballs. As in part (B) let's assign the last friend who receives 1 gumball the number 1. Then we can make a table that shows the pattern of giving gumballs to his friends:

Friend	Number of Gumballs Friend Receives	Total Number of Gumballs Given
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255

From the information on this table, we can answer the questions.

The first person on the list will receive 128 gumballs. A maximum of eight of Robbie's friends can receive gumballs.



## Teacher's Notes

In this problem, as we consider one additional friend, the number of gumballs received is reduced by half. For part A), we could write an equation for  $g(n)$ , the number of gumballs the  $n^{\text{th}}$  friend gets, as:

$$g(n) = 100 \cdot \left(\frac{1}{2}\right)^{n-1}$$

If we look at the graph of this function, we would see that it is a very steep curve. This is an example of an *exponential function*.

In general, an exponential function is a function of the form:

$$g(n) = m \cdot b^n, \quad \text{where } b > 0 \text{ and } b \neq 1.$$

When  $b > 1$ , the value of  $g(n)$  increases very quickly as  $n$  increases. We see exponential growth in real life situations such as compound interest and the spread of viruses.

When  $b < 1$  (as in this problem), the value of  $g(n)$  decreases very quickly as  $n$  increases. We see exponential decay in some chemical reactions and heat transfer.

Exponential functions, exponential growth, and exponential decay are generally seen in high school mathematics and science, and possibly some business studies courses.

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# Geometry (G)

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## Problem of the Week

### Problem A

### Time to Get Up

On the days she goes to work, Amy has a morning routine. The table below shows each activity she does and the time it takes to complete.

Activity	Time to Complete
Shower	15 minutes
Get dressed	10 minutes
Make and eat breakfast	20 minutes
Make lunch	15 minutes
Brush teeth and hair	5 minutes
Walk to work	45 minutes

Amy needs to be at work by 8:30 a.m. What is the latest time she could wake up in the morning, complete all of the activities in her routine, and get to work on time?

Justify your answer.





## Problem of the Week

### Problem A and Solution

#### Time to Get Up

#### Problem

On the days she goes to work, Amy has a morning routine. The table below shows each activity she does and the time it takes to complete.

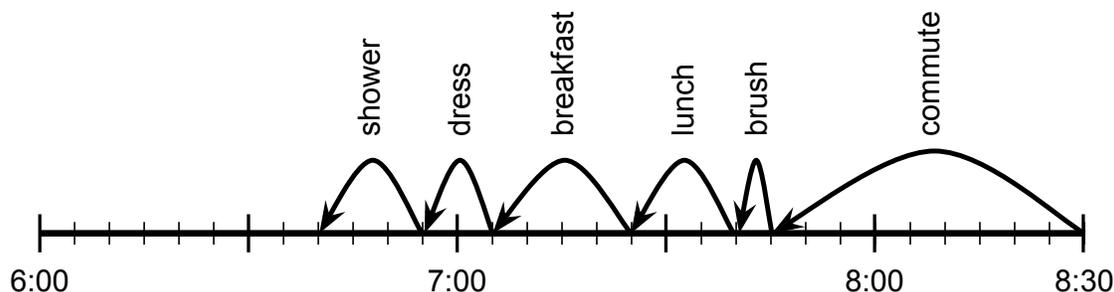
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Amy needs to be at work by 8:30 a.m. What is the latest time she could wake up in the morning, complete all of the activities in her routine, and get to work on time?

Justify your answer.

#### Solution

One way to solve this problem is to use a timeline. However in this case, we know the end time and we are trying to find the start time. This means we work backwards on the timeline. Each tick on the following timeline represents a 5 minute interval.



From the timeline, we can see that the latest Amy can wake up and make it to work on time is 6:40 a.m.

To solve the problem a second way, we can find the total time to complete all of her morning activities. Amy needs  $15 + 10 + 20 + 15 + 5 + 45 = 110$  minutes.

Converting 110 minutes to hours and minutes, Amy needs 1 hour and 50 minutes. This is 10 minutes less than 2 hours. If she needed 2 hours, she would have to be up by 6:30 a.m. Since she requires 10 minutes less than 2 hours, she can get up 10 minutes after 6:30, or at 6:40 a.m.

Therefore, the latest Amy can wake up and make it to work on time is 6:40 a.m.



## Teacher's Notes

The timeline in the solution is a variation of a number line. The spacing between tick marks on a number line normally represents integer or fraction intervals. In this case, the spacing between tick marks represents 5 minute (or  $\frac{1}{12}$  of an hour) intervals.

Number lines are helpful tools for learning to add and subtract. Adding values means moving to the right on a number line and subtracting values means moving to the left. Many people find that addition involving time is more difficult than working with simple numbers, since the values that represent later times do not continuously increase. For example, if we were using a 12-hour clock, when we add 160 minutes to 11:50, the result is 2:30. In this case, the resulting time has a smaller number of hours and minutes compared to the original time. Using a number line can make these kinds of calculations easier.

We can calculate a sum like this without a number line by using quotient and remainder values.

- Calculate the total number of minutes represented by the starting time, 11:50, by multiplying the number of hours in the time by 60 and adding the number of minutes:  
 $(60 \times 11) + 50 = 710$
- Add the number of minutes the time is increased by:  $710 + 160 = 870$
- Calculate the quotient and remainder when dividing the sum by 60:  
 $870 \div 60$  has quotient of 14 and a remainder of 30.
- The quotient represents the resulting hour and the remainder represents the minutes.
- If the new hour is more than 12, calculate the remainder when dividing the hour by 12:  
 $14 \div 12$  has a quotient of 1 and a remainder of 2
- If the remainder is not 0, it represents the new hour. In this example, the resulting time is 2:30. Note that if the remainder is 0, then the hour would be 12.

This gets trickier though, when subtracting times rather than adding times. For example suppose we have a starting time of 2:30, and we want to know what time is 160 minutes earlier. If we try to follow the same steps, we start with the following calculations:

- Original time in minutes:  $(2 \times 60) + 30 = 150$
- Subtract the number of minutes the time is decreased by:  $150 - 160 = -10$

This gives us a negative number which is not easily translated into hours and minutes. It turns out that you can use quotient and remainder calculations to determine that  $-10$  minutes represents the time of 11:50. However most students would not learn about quotient and remainder calculations involving negative numbers until after secondary school mathematics. So, using a number line is a good strategy when subtracting time.



## Problem of the Week

### Problem A

### Surrounding Squares

Imran was doodling and he drew a picture that had a square in the middle, surrounded by other smaller squares. Together, the middle square and all the smaller squares form a bigger square. The smaller squares all have the same side lengths, and there are no overlaps between the squares and no gaps in the picture. The side lengths of the smaller squares are each  $\frac{1}{3}$  the side length of the middle square.

- A) How many smaller squares are in the picture?
- B) If the side length of the middle square is 6 cm, what is the area of the square formed by the whole picture?





## Problem of the Week

### Problem A and Solution

### Surrounding Squares

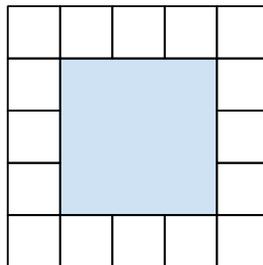
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- A) How many smaller squares are in the picture?
- B) If the side length of the middle square is 6 cm, what is the area of the square formed by the whole picture?

#### Solution

To answer the questions in this problem, the first thing we should do is draw a picture of the doodle. Since the side lengths of smaller squares are  $\frac{1}{3}$  the side length of the middle square, then three smaller squares should fit on one side of the middle square. So this is what the doodle looks like:



- A) From the diagram we can see that there are 16 smaller squares.
- B) If the side length of the middle square is 6 cm, and  $\frac{1}{3}$  of 6 is 2, then the side lengths of the smaller squares are 2 cm. Since the large square has a side length equal to five small squares, then side length of the large square is  $5 \times 2 = 10$  cm. This means the area of the large square is  $10 \times 10 = 100$  cm<sup>2</sup>. We could also calculate the area of the large square this way. The area of a small square is  $2 \times 2 = 4$  cm<sup>2</sup>. Since there are 16 small squares, the total area of the smaller squares is  $16 \times 4 = 64$  cm<sup>2</sup>. The area of the middle square is  $6 \times 6 = 36$  cm<sup>2</sup>. The total area is then  $36 + 64 = 100$  cm<sup>2</sup>.



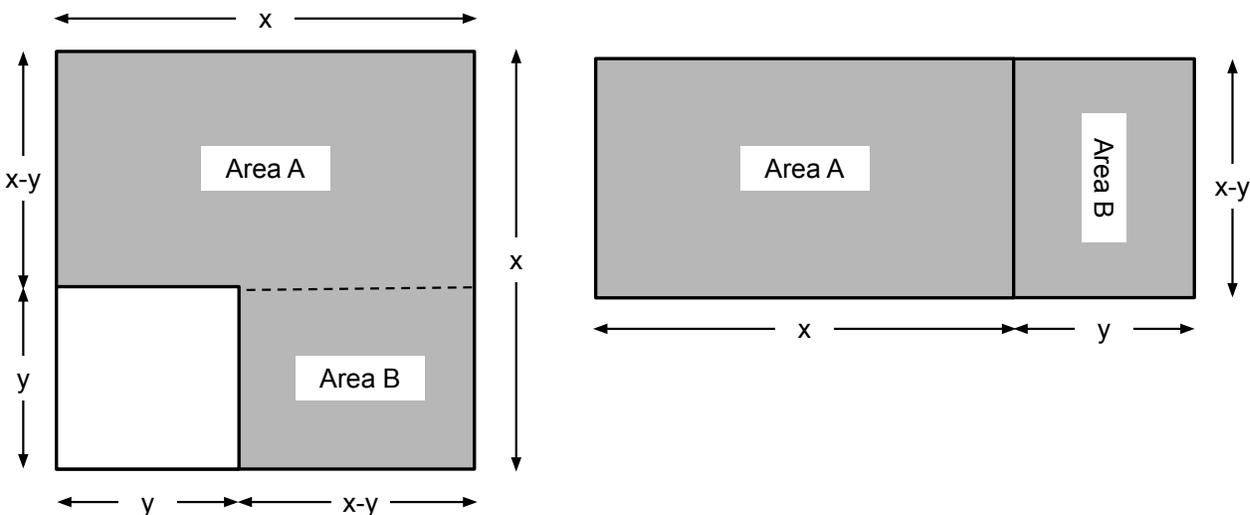
## Teacher's Notes

It is important for mathematicians to be able to draw good diagrams based on written descriptions. An accurate diagram helps visualize a problem. Diagrams often provide insight into relationships that are not necessarily clear based on the words alone.

In high school algebra, students learn about factoring polynomials. Factoring is a process of rewriting a polynomial into an equivalent form, where other polynomials are multiplied together. One factoring relationship students will learn is called the *difference of squares*. Algebraically, the rule for factoring a difference of squares looks like this:

$$x^2 - y^2 = (x - y)(x + y)$$

We can draw a diagram that helps visualize this relationship. We start by drawing a square where the sides have a length of  $x$ , and then remove a square where the sides have a length of  $y$  from one corner. Here is a diagram showing the difference of squares relationship:



The area of the shaded region in the diagram on the left is the area of the outer square ( $x^2$ ), minus the area of the white square ( $y^2$ ). This can be written algebraically as  $x^2 - y^2$ . The shaded region of the diagram on the left has been rearranged to form a rectangle in the diagram on the right. The area of this rectangle is the length of its sides multiplied together:  $(x - y)(x + y)$ .

Since these two areas are the same, we can see that  $x^2 - y^2 = (x - y)(x + y)$ .



## Problem of the Week

### Problem A

### School Garden

Maeve and Azaadi want to make a garden in the school yard, but they do not want the other students to trample the plants. They decide to ask their friends to help find some fencing to protect the vegetables. Maeve found 5 metres of fencing in her shed. Azaadi found two pieces of fencing, one measuring 300 centimetres, the other measuring 6 metres. Their friend Thandi found another piece measuring 8 metres, and Xander donated one more piece of fencing.

The students use the fencing to surround their garden with a perimeter of 28 metres.

- A) What is the length of the fencing donated by Xander?
- B) Can Maeve and Azaadi use the donated fencing to make a rectangular garden with a perimeter of 28 metres, without cutting any of the pieces? Justify your answer.
- C) Suppose that they had made a mistake when measuring the first time, and the actual length of the fence that Thandi donated was 7 metres, but the total perimeter of the donated fencing was still 28 metres. Can they still form a rectangular garden without cutting any of the pieces? Justify your answer.





## Problem of the Week

### Problem A and Solution

### School Garden

#### Problem

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#### Solution

- A) We can convert the section measured in centimetres to metres, i.e. 300 centimetres is equal to 3 metres. Now we can add together the known lengths of fencing. The total is  $5 + 3 + 6 + 8 = 22$ .  
Since we know the total amount of fencing that surrounds the garden is 28 metres, then the amount Xander donated must be  $28 - 22 = 6$  metres.
- B) It is possible to create a rectangle out of the donated sections of fencing without cutting any pieces. Maeve and Azaadi need to find a way to form a quadrilateral with five pieces of fencing. This means that one of the sides must be formed using two pieces of fencing. The other restriction is that the opposite sides of the quadrilateral must be the same length. With these pieces of fencing, the rectangle can be formed with dimensions  $6 \text{ m} \times 8 \text{ m}$ . The two sections that are 6 metres will be opposite sides of the rectangle. The sections that are 300 centimetres and 5 metres can be joined together to form a side that is 8 metres. The other section that is 8 metres will be on the opposite side of the rectangle.



- C) In this case, if Thandi donated a section that is 7 metres long (1 metre shorter than originally thought), and the perimeter stayed the same, then Xander's section must also be 7 metres long (1 metre longer than originally thought.) However, if we consider the lengths of all possible pairs of the fence pieces, none of them are equal to the length of one of the other pieces of fence. In particular, if we consider the lengths: 3, 5, 6, 7, and 7, the possible sums of two pieces are:  $(3 + 5) = 8$ ,  $(3 + 6) = 9$ ,  $(3 + 7) = 10$ ,  $(5 + 6) = 11$ ,  $(5 + 7) = 12$ ,  $(6 + 7) = 13$ , and  $(7 + 7) = 14$ . In this case we do not even have to consider all combinations, since when we add the two smallest lengths  $(3 + 5)$ , we get a sum that is larger than the longest piece of fence. Since all of the other sums will be larger than this smallest one, then there is no combination that will work.



## Teacher's Notes

When trying to form a rectangle out of the pieces of fencing, we are looking for ways to combine these pieces so that we have two pairs of equal length. One way to look at this problem is to consider the *partitions* of the set of pieces of fencing into four subsets. Then we look at the combined lengths of the pieces in those four subsets to see if it is possible to form a rectangle.

A partition of a set is a set of non-empty subsets, where each element in the original set appears in exactly one of the subsets. Let's suppose we have five pieces of fencing and we describe these as the set  $\{A, B, C, D, E\}$ , where each letter represents one of the pieces. This means that  $\{\{A\}, \{B, C, D\}, \{E\}\}$  is an example of a partition of this set into three subsets. A set of five elements has a total of 52 partitions. However, for the problem of determining if five pieces of fencing can form a rectangle without being cut, we only need to consider the partitions into four subsets. These partitions are:

$$\{\{A, B\}, \{C\}, \{D\}, \{E\}\}$$

$$\{\{A, C\}, \{B\}, \{D\}, \{E\}\}$$

$$\{\{A, D\}, \{B\}, \{C\}, \{E\}\}$$

$$\{\{A, E\}, \{B\}, \{C\}, \{D\}\}$$

$$\{\{A\}, \{B, C\}, \{D\}, \{E\}\}$$

$$\{\{A\}, \{B, D\}, \{C\}, \{E\}\}$$

$$\{\{A\}, \{B, E\}, \{C\}, \{D\}\}$$

$$\{\{A\}, \{B\}, \{C, D\}, \{E\}\}$$

$$\{\{A\}, \{B\}, \{C, E\}, \{D\}\}$$

$$\{\{A\}, \{B\}, \{C\}, \{D, E\}\}$$

Now, given *any* five pieces of fencing, we can consider these ten partitions. Then we would check to see if there are matching lengths in pairs of the subsets of at least one of these partitions. If we find a pair of matching pairs, then we know that these pieces can form a rectangle.

The problem gets more complex if the number of pieces of fencing increases. If we start with six pieces, there are a total of 203 partitions altogether and 65 partitions with four subsets.



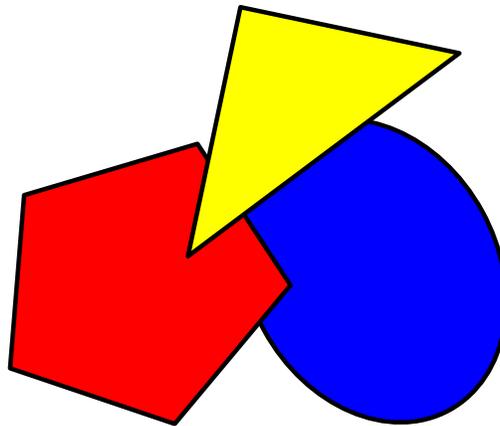
## Problem of the Week

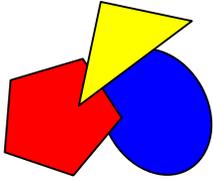
### Problem A

### Shape Sleuth

For each part, identify a possible shape that satisfies all of the clues in the part.

- A) I have six lines of symmetry. All of my angles are larger than a right angle. I have six sides and six vertices. What shape am I?
- B) I have two sets of parallel sides. I have two angles that are larger than a right angle and two angles that are smaller than a right angle. I am a quadrilateral. What shape am I?
- C) I have four vertices. I have one line of symmetry and one set of parallel sides. I have four angles, two are larger than a right angle and two are smaller than a right angle. What shape am I?
- D) I have three vertices and three angles. All of my angles are smaller than a right angle. I have three lines of symmetry. What shape am I?
- E) I have four angles, four sides and four vertices. I have two sets of parallel sides and four right angles. I have four lines of symmetry. What shape am I?





## Problem of the Week

### Problem A and Solution

### Shape Sleuth

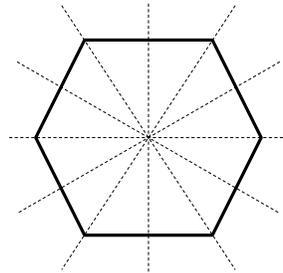
#### Problem

For each part, identify a possible shape that satisfies all of the clues in the part.

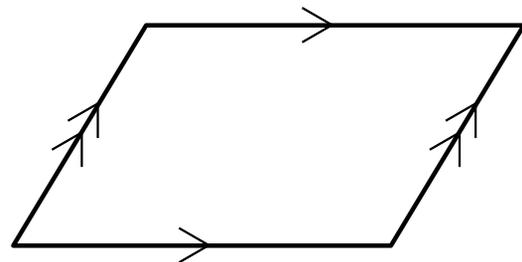
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#### Solution

- A) A regular hexagon

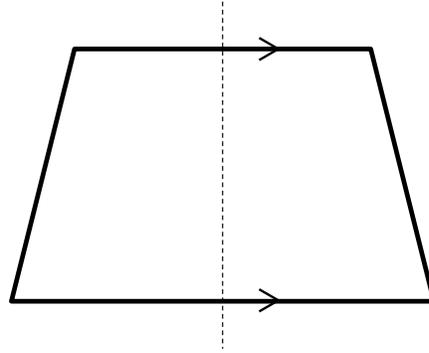


- A parallelogram or a rhombus
- B) (note that a rhombus is a special case of a parallelogram)

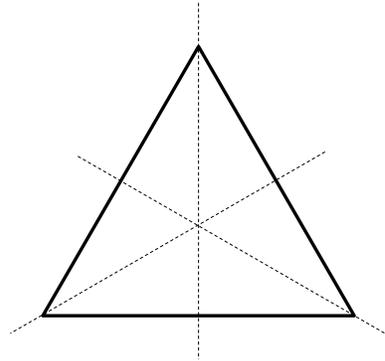




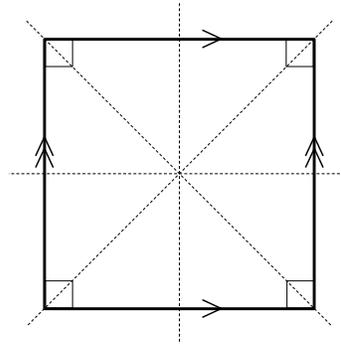
- C) An isosceles trapezoid  
(i.e. a trapezoid where the two non-parallel sides are the same length)



- D) An equilateral triangle



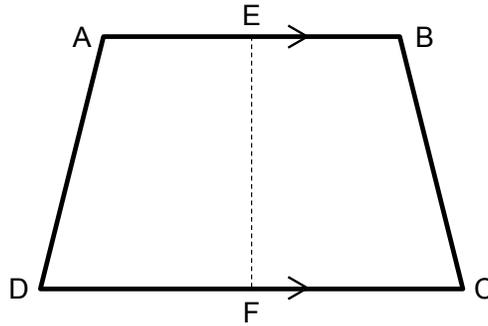
- E) A square





## Teacher's Notes

By definition, a line of symmetry divides a single shape into two identical halves. Two shapes that are identical are said to be *congruent*. For example, suppose we label the isosceles trapezoid as follows:



and we know that **EF** is a line of symmetry. This means that the quadrilateral **AEFD** is congruent to the quadrilateral **BEFC**. The symbol for congruence is  $\cong$ .

So we can state:

$$\mathbf{AEFD} \cong \mathbf{BEFC}$$

If two geometric figures are congruent, that means that all corresponding side lengths and interior angles are equal. In this case, we know that the length of **AE** is equal to the length of **BE**. We also know that the size of  $\angle\mathbf{ADF}$  is equal to the size of  $\angle\mathbf{BCF}$ . These are just two of several conclusions we can draw, knowing that these figures are congruent.

Identifying congruent figures can be very helpful when working on proofs in geometry.



## Problem of the Week

### Problem A

### Yard Sale

North Star Community School is planning a yard sale to raise money for the Mathletes program. Each grade level is organizing items to sell. The grade 3 and 4 classes have many donations.

- Aoife donates a book, that weighs one and three-quarter kilograms.
- Soroush donates a bowling ball that weighs 7200 g.
- Keiran donates a box of laundry soap that weighs 3 kg.
- Reese donates a jug that weighs one-quarter of a kilogram.
- Sofia donates an unopened package of 6 protein bars. Each bar weighs 50 g.
- Graydon donates a sack of sugar that weighs four times as much as the jug that Reese donates.
- Nathan donates a pumpkin that weighs five and a half kilograms.

- A) What is the total weight of all donated items?
- B) The students organize the items by weight, with the heaviest item on the left side and the lightest item on the right. How are the items arranged?
- C) If each table can only hold up to a maximum of 10 kg and they keep the items in the order from part B), what is the fewest number of tables they need for the sale?





## Problem of the Week

### Problem A and Solution

#### Yard Sale

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#### Solution

- A) To calculate the total weight, we can convert all of the individual weights to the same units.
- One kilogram equals 1000 g. Three quarters of a kilogram equals 750 g.
  - 3 kg is equal to  $3 \times 1000 = 3000$  g.
  - One-quarter of a kilogram equals 250 g.
  - The package of protein bars weighs  $6 \times 50 = 300$  g.
  - The sack of sugar weighs  $4 \times \frac{1}{4} = 1$  kg, and 1 kg equals 1000 g.
  - Five kilograms equals 5000 g. A half a kilogram equals 500 g.



The total weight of the items is:

$$1000 + 750 + 7200 + 3000 + 250 + 300 + 1000 + 5000 + 500 = 19\,000 \text{ g} = 19 \text{ kg}$$

B) From left to right the items would be:

bowling ball, pumpkin, laundry soap, book, sugar, protein bars, jug

C) Keeping the items in order from heaviest to lightest, putting the bowling ball and the pumpkin on the same table ( $7200 + 5500 = 12\,700$ ) would exceed the 10 kg limit. So the bowling ball needs to be on its own table.

The weight of the pumpkin and the laundry soap is  $5500 + 3000 = 8500 \text{ g}$ .

Adding the weight of the book makes the total  $8500 + 1750 = 10\,250 \text{ g}$ .

This would exceed the 10 kg limit. So the pumpkin and laundry soap must be on a table by themselves.

The weight of the remaining items is:  $1750 + 1000 + 300 + 250 = 3300 \text{ g}$ .

This is below the 10 kg limit. So they would need at most three tables.

Now, it would be possible to arrange the items on two tables, if the order of the items was not important. One way would be to have the book and jug on the same table as the bowling ball. The total weight of these three items is:  $7200 + 1750 + 250 = 9200 \text{ g}$ .

Since the total weight of all of the items is 19 000 g, the remaining items weigh  $19\,000 - 9200 = 9800 \text{ g}$ . Both of these totals are below the 10 kg maximum.



## Teacher's Notes

The third part in this problem is reminiscent of a classic problem in optimization known as the *Knapsack Problem*. The inspiration for this problem is to imagine you have a knapsack that has a limit to how much weight it can hold. You have several items, of various weights, that have some other value assigned to them. You want to pick the items that will maximize the value that is carried in the knapsack. You cannot exceed the weight limit of the knapsack. Which items should you pick?

It is tempting to simply choose the item with the highest value, and place it in the knapsack. Then choose the item with the next highest value, and place it in the knapsack. You continue this process until the next item that would be chosen would exceed the weight limit of the knapsack. This is known as the *Greedy Algorithm*. Unfortunately, this technique does not work in all cases.

Imagine you only had one table available to display items at the yard sale, and it has a weight limit of 10 kg. Suppose you had assigned prices to each of the sale items, and you want to display the items that fit on the table that have the highest combined value. Suppose you have determined that the items should be marked with the following prices:

Item	Weight (g)	Price (\$)
book	1750	1.00
bowling ball	7200	12.00
laundry soap	3000	6.00
jug	250	3.00
protein bars	300	2.00
sugar	1000	4.00
pumpkin	5500	9.00

Notice in this case, if we choose to display the item with the highest price (the bowling ball) then we could not select the laundry soap or the pumpkin to display, since adding the weight of either of these items would exceed the weight limit of the table. We could display the book and the sugar along with the bowling ball, since the total weight of those three items is 9950 g. The total value of these three items is:

$$\$12.00 + \$4.00 + \$1.00 = \$17.00$$

We could display the bowling ball with either the book or the sugar along with the jug and the protein bars. Since the sugar is priced higher than the book, we could display the bowling ball, sugar, jug, and protein bars with a total weight of 8750 g. The total value of these items is

$$\$12.00 + \$4.00 + \$3.00 + \$2.00 = \$21.00$$

If we displayed the pumpkin, the laundry soap, the sugar, and the jug, the total weight is 9750 g. We can see that this is a better choice since the total value of these items is:



$$\$9.00 + \$6.00 + \$4.00 + \$3.00 = \$22.00$$

The “Knapsack Problem” can be applied to many real life situations such as waste management and the allocation of public resources. We have to be careful when we try to find the best solution for this type of problem.



## Problem of the Week

### Problem A

### Making Jewelry

Lindsay has decided to make a necklace out of red, white, and yellow beads. She decides to have a repeated pattern in the necklace. She will have five red beads, followed by three white beads, followed by one yellow bead, followed by three white beads. Then the pattern will repeat. When she is done, she connects the ends of the string of beads to form a loop. The beads in her necklace will always form a complete pattern.

- A) The beads Lindsay has are 5 mm wide. She wants to make a necklace that is at least 25 cm long. What is the smallest number of beads she needs to make the necklace where each pattern is complete?
- B) Lindsay has 44 beads of each colour. What is the longest necklace she can make where each pattern is complete?





## Problem of the Week

### Problem A and Solution

#### Making Jewelry

#### Problem

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- B) Lindsay has 44 beads of each colour. What is the longest necklace she can make where each pattern is complete?

#### Solution

The total number of beads in the pattern is:  $5 + 3 + 1 + 3 = 12$ . The length of one copy of the pattern is  $12 \times 5 \text{ mm} = 60 \text{ mm}$  which is equal to 6 cm. So we can use a table to determine the length of the necklace as it includes more copies of the pattern:

Copies of the Pattern	Number of Beads	Necklace Length (in cm)
1	12	6
2	24	12
3	36	18
4	48	24
5	60	30
6	72	36
7	84	42

- A) So Lindsay will need 60 beads (25 red, 30 white, and 5 yellow) to have a necklace that is at least 25 cm and has complete patterns.
- B) Since the pattern includes 6 white beads, this is the colour that occurs the most. If Lindsay has 44 of each colour, then we can determine the maximum number of copies of the pattern by dividing:  $44 \div 6 = 7$  remainder 2. So the pattern can be repeated at most 7 times. In this case, the longest necklace Lindsay can make will be  $7 \times 6 \text{ cm} = 42 \text{ cm}$ .



## Teacher's Notes

This week, students are solving *optimization* problems. In part (a) they are required to find a minimum value given certain problem specifications, and in part (b) they are required to find a maximum value given certain problem specifications. Optimization is an area of mathematics that explores ways to make operations more efficient. These techniques can be applied to many different situations. For example:

- In manufacturing, companies look for ways to minimize waste given the materials required for the process.
- In medicine, hospitals look for ways to improve emergency departments in a way that maximizes patient care given the limits of available staff, equipment, and other resources.
- In education, administrators need to schedule courses in a way that minimizes conflicts for students' requests given limits on classroom capacities, lab requirements, and many other factors.
- In science, people look for ways to minimize the environmental impact of an industrial process.

Studying this area of mathematics can lead to many diverse job opportunities.

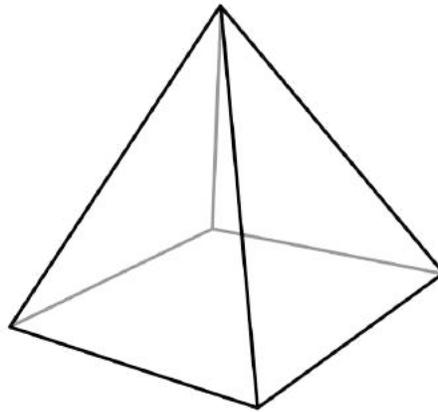


# Problem of the Week

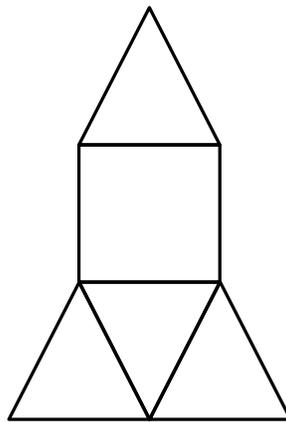
## Problem A

### Pyramid Nets

Draw a diagram of each of the nets that can be used to construct a square based pyramid.



Here is an example of one of the nets:





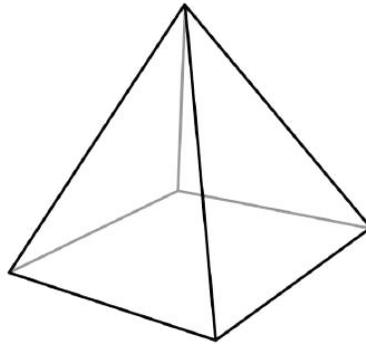
# Problem of the Week

## Problem A and Solution

### Pyramid Nets

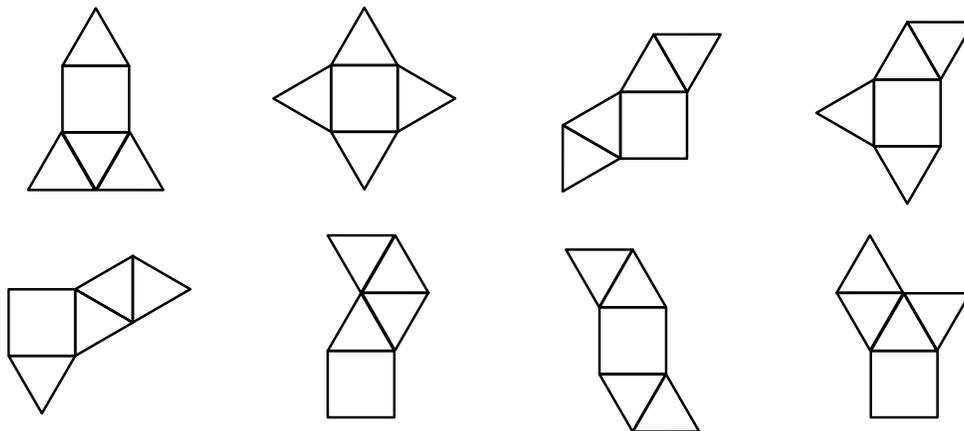
#### Problem

Draw a diagram of each of the nets that can be used to construct a square based pyramid.



#### Solution

We will define two nets as congruent (i.e. the same) if you can rotate and/or flip one of the nets to make it identical to the other. With this restriction, there are a total of eight nets that can be used to create a square based pyramid. The eight nets are shown below.

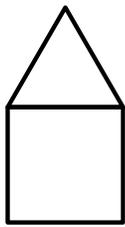




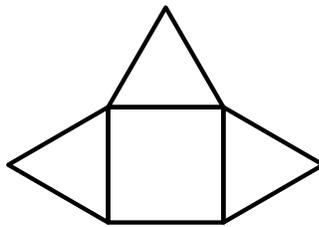
## Teacher's Notes

As mathematicians, we should convince ourselves that there are no more possible nets for the pyramid. It would take quite a bit of writing to make a full argument, but let's at least think carefully about how to count the possible nets.

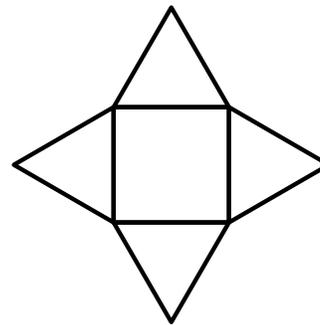
Let's start by thinking about how we start to form each net. Any possible net would have at least one, and at most four triangles sharing a side with the square. In the cases of one, three, or four triangles, ignoring any congruent nets, there is only one possible arrangement in each case as we see here:



Three additional triangles needed

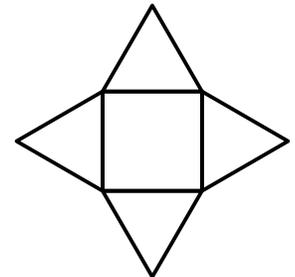


One additional triangle needed

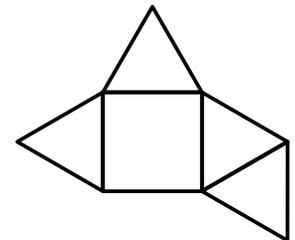


No additional triangles needed

In the case where all four triangles share a side with the square, this is the one and only net possible:

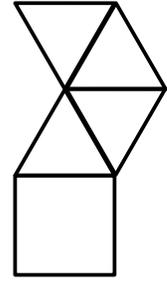


In the case where three triangles share a side with the square, it is not possible to form a pyramid if we put the fourth triangle between two other triangles. If we try to, we will see the triangles get in the way of each other. So the only possible arrangement is to add the fourth triangle on the outside edge of one of the original triangles as we see with this net:

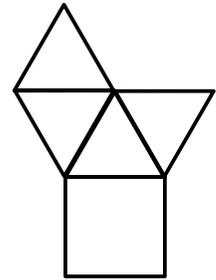




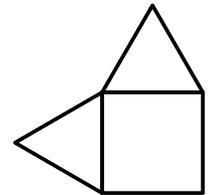
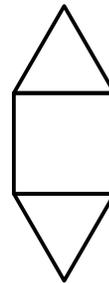
In the case where one triangle shares a side with the square, there are two cases to consider. We can arrange the other three triangles together and then add them to one side of the original triangle. Most arrangements will not work as the triangles will get in the way of each other as we try to form the pyramid. However, one arrangement will work as we see in this net:



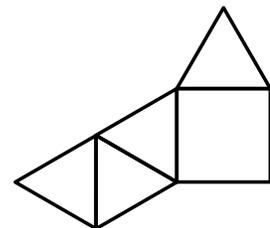
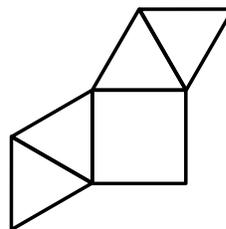
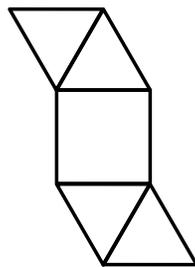
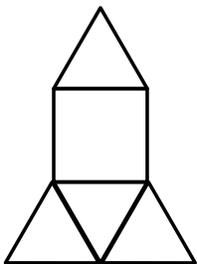
The other possibility is to have one of the additional triangles on one side and two additional triangles on the other side of the original triangle. Again, if we try it, there is only one arrangement of this kind that will work as we see in this net:



There is one more case to consider as a starting point. This is the case where we start with two triangles that share sides with the square. There are two starting arrangements: the triangles can be on opposite sides of the square or they can be on adjacent sides of the square as we see here:



With each of these starting arrangements, through trial and error we can show that there are only two possible arrangements of the additional two triangles that will correctly form a net. These lead to the last four nets as we see here:





## Problem of the Week

### Problem A

### Hockey Practice

Lucie and Liam love to play hockey. Each week, Lucie attends practice for 2 hours per day Monday to Friday, as well as  $4\frac{1}{2}$  hours per day on Saturday and Sunday. Liam attends practice for 3 hours per day on Mondays and Wednesdays, and 2 hours on Fridays. On Saturdays he attends practice from 1:00 p.m. until 4:30 p.m. On Sundays he attends practice from 1:30 p.m. until 5:00 p.m. He does not have practice on Tuesdays and Thursdays.

The answers to questions about Lucie and Liam's hockey practice schedules are:

- A) 15
- B) 19
- C) 4
- D) 34

What are possible questions that give these answers?





## Problem of the Week

### Problem A and Solution

### Hockey Practice

#### Problem

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The answers to questions about Lucie and Liam's hockey practice schedules are:

- A) 15
- B) 19
- C) 4
- D) 34

What are possible questions that give these answers?

#### Solution

Here are some calculated values about the practice times:

- From Monday to Friday, Lucie attends practice for a total of  $5 \times 2 = 10$  hours.
- On Saturday and Sunday, Lucie attends practice for a total of  $4\frac{1}{2} + 4\frac{1}{2} = 4 + 4 + \frac{1}{2} + \frac{1}{2} = 8 + 1 = 9$  hours.
- From Monday to Friday, Liam attends practice for a total of  $3 + 3 + 2 = 8$  hours.
- On Saturday and Sunday Liam attends practice for  $3\frac{1}{2}$  hours each day, for a total of  $3 + 3 + \frac{1}{2} + \frac{1}{2} = 6 + 1 = 7$  hours.

Here are some possible questions:

- A) For how many hours per week does Liam have practice? ( $8 + 7 = 15$ )
- B) For how many hours per week does Lucie have practice? ( $10 + 9 = 19$ )
- C) For how many more hours does Lucie have practice compared to Liam?  
( $19 - 15 = 4$ )
- D) For how many total hours per week do both players have practice?  
( $19 + 15 = 34$ )



## Teacher's Notes

This problem might be considered an example of *reverse engineering*, which is a process where we are given the final product and we try to determine the details of its design. Reverse engineering can be used by software engineers to improve existing code. However, software developers may want to protect against reverse engineering using *obfuscation*. This is a technique that makes it much more difficult for hackers to recreate the source code created by the original developers, given the executable code that people purchase to run on their computers. This is an attempt to maintain the valuable *intellectual property* associated with the software.



## Problem of the Week

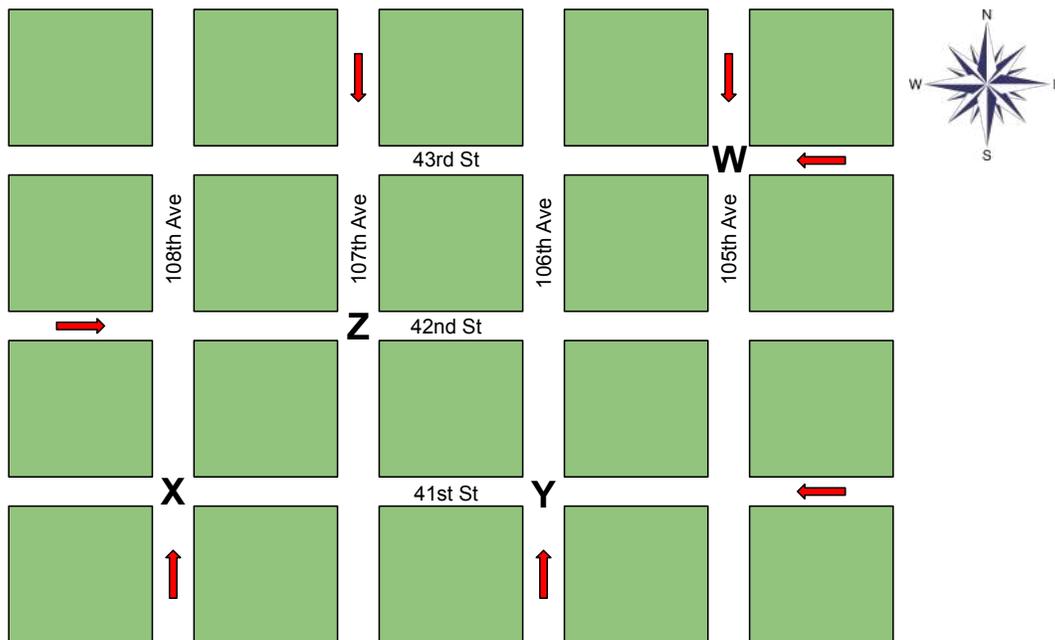
### Problem A

### Short Walks

Rhea Treever is a dog walker who has four clients that live in the city. In the neighbourhoods where the dogs live, all of the roads only allow one-way traffic.

- Even numbered streets only allow vehicles to drive east.
- Odd numbered streets only allow vehicles to drive west.
- Even numbered avenues only allow vehicles to drive north.
- Odd numbered avenues only allow vehicles to drive south.

Rhea has to walk the dogs at particular times in the day. She has made a map of where the dogs live labelled with the letters **W**, **X**, **Y**, and **Z**, which is the order in which she walks them.



Rhea wants to find the most efficient route to drive from each dog's house to the next by making the fewest turns. Describe the best route (or routes) that require the fewest turns between each of the following points.

- From point **W** to point **X**.
- From point **X** to point **Y**.
- From point **Y** to point **Z**.



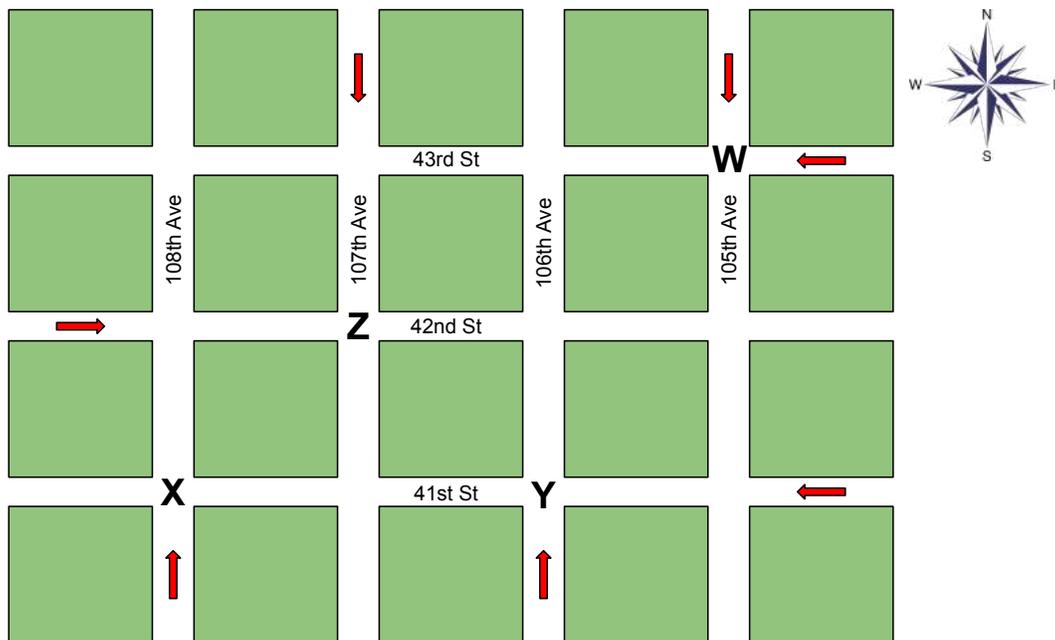
## Problem of the Week Problem A and Solution Short Walks

### Problem

Rhea Treever is a dog walker who has four clients that live in the city. In the neighbourhoods where the dogs live, all of the roads only allow one-way traffic.

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Rhea wants to find the most efficient route to drive from each dog's house to the next by making the fewest turns. Describe the best route (or routes) that require the fewest turns between each of the following points.

- A) From point **W** to point **X**.
- B) From point **X** to point **Y**.
- C) From point **Y** to point **Z**.



## Solution

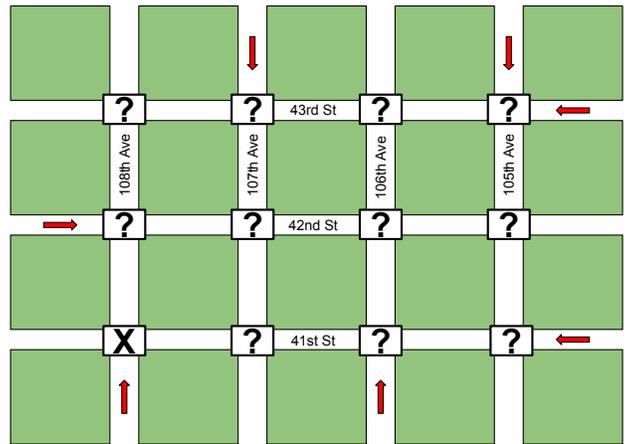
- A) It is possible to drive from point **W** to point **X** by making only one turn. Rhea drives south for two blocks to 41st Street and turns right. Then she drives three blocks west on 41st Street. It is not possible to get from one intersection to another that is on a different Street and Avenue without making at least one turn.
- B) It is possible to drive point **X** to point **Y** by making three turns. Since she cannot drive east on 41st Street, Rhea drives north for one block to 42nd Street and turns right. Then she drives east for three blocks to 105th Avenue and turns right. Then she drives south for one block to 41st Street and turns right. Then she drives one block west to point **Y**.
- C) It is possible to drive from point **Y** to point **Z** by making two turns. One possible route is to drive north on 106th Avenue for two blocks to 43rd Street and turn left. Then Rhea drives one block west to 107th Avenue and turns left. Then she drives one block to point **Z**. Another possible route is to drive west for two blocks to 108th Avenue and turn right. Then Rhea drives one block north to 42nd Street and turns right. Then she drives one block to point **Z**.



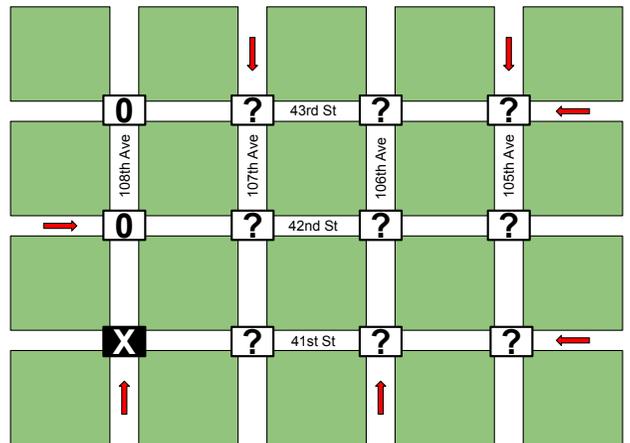
### Teacher's Notes

We should convince ourselves that the route we have chosen actually has the fewest turns. One way to do this is to methodically determine the *shortest path* from one intersection to another. Let's look at how we find the path with the fewest turns between the point **X** and point **Y** (the intersection of 41st St. and 106th Ave.).

We start by creating a diagram of the grid where we mark our starting point **X** and place a ? at every other intersection. Then we look at the intersections that can be reached from point **X** without making any turns, and replace the ? at those intersections with the number **0**. This number indicates that we found a path from our starting point to that intersection that takes 0 turns. Notice that from **X** we cannot directly visit any of the other intersections on 41st St., as we cannot travel east on this street. At this point, we will no longer consider the intersection at point **X** and we mark it as visited with a black rectangle.

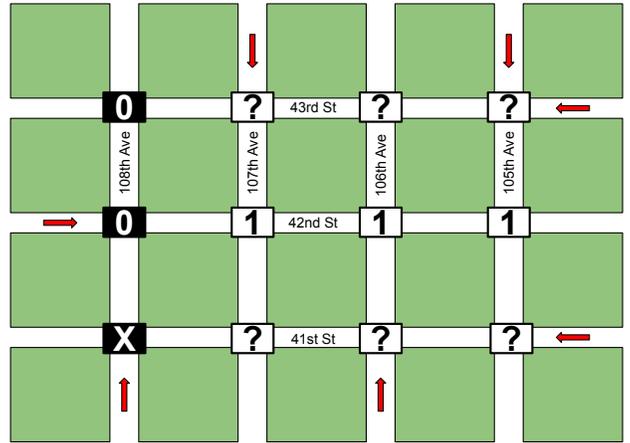


Then, one-by-one we consider the intersections that are marked with the number **0**. For example, let's start with the intersection of 42nd St. and 108th Ave. From that intersection we look for the intersections we can reach without making a turn. We can go one block north, but this intersection has already been marked with a **0**. This is the lowest number of turns we can hope for, so the number at that intersection will not change. However, from this intersection we can also get to the rest of the intersections on 42nd St. Since these intersections are marked with a ? and we would have to make a turn at 42nd St., we can replace each ? with a **1** on this street. At this point, we will mark the intersection at 42nd St. and 108th Ave as visited. Also, there is nowhere to go from 43rd St. and 108th Ave, so we mark it as visited.

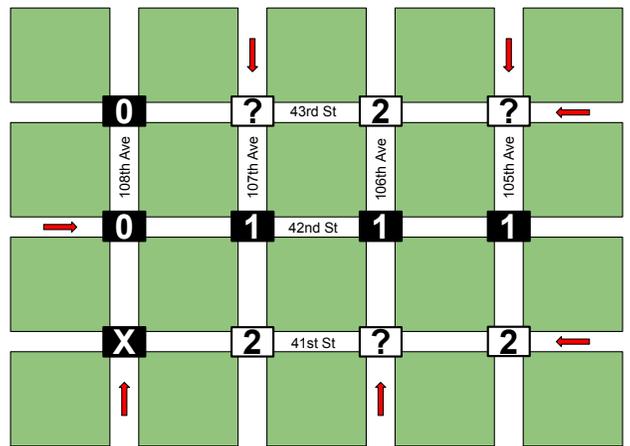




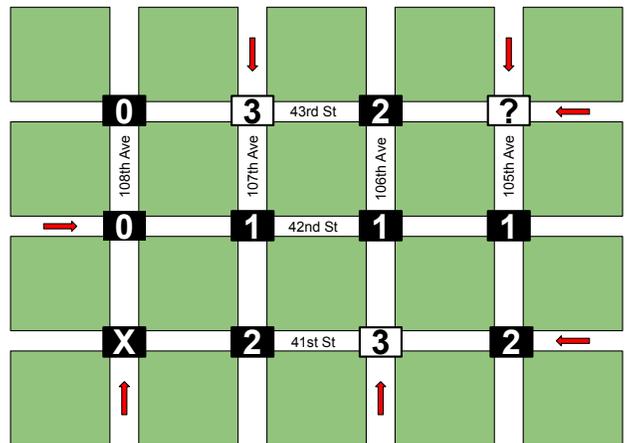
Then, one-by-one we start at intersections that are marked with numbers but not marked as visited, and we update any intersections that we can reach without making a turn. If those reachable intersections are marked with a ?, we update them with a number. If those reachable intersections have a number already, we check to see if we have a smaller number of turns possible to get to that intersection. After visiting all of the intersections along 42nd St., we can update our grid as shown:



Following the same procedure, after we visit all of the intersections marked with the number 1, the grid will be updated as shown:



Following the same procedure, after we visit all of the intersections marked with the number 2, the grid will be updated as shown:



Now we can see that we can get from the intersection at point **X** to the intersection at point **Y** using a minimum of 3 turns. This procedure is an example of a technique known as *Dijkstra's Algorithm*. This is a well known algorithm used in a field of mathematics known as *graph theory*.



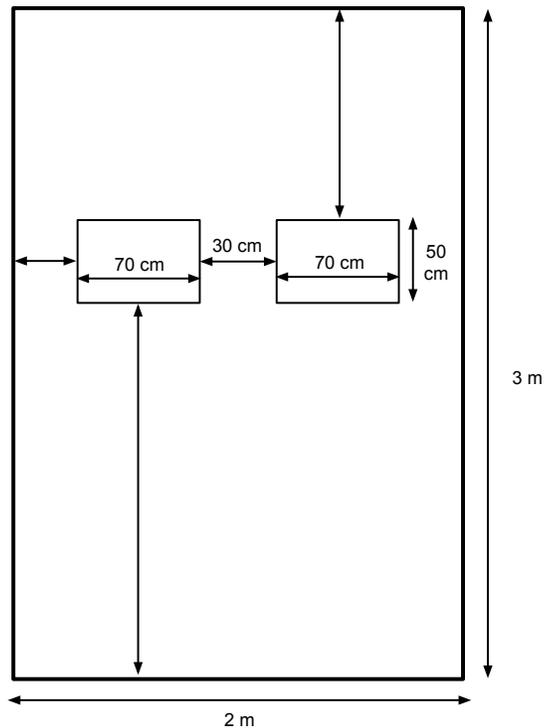
## Problem of the Week

### Problem A

#### Picture Perfect

Two pictures, each 70 cm wide and 50 cm tall, are hung on a rectangular wall so that the top of each picture is 2 m above the floor. The wall is 3 m tall and 2 m wide. The pictures are to be hung so that the horizontal distance from the outside edge of each picture to the nearest wall is the same. Two ways of arranging the pictures are being considered.

- a) If the pictures are 30 cm apart on the wall, complete the labels on the following diagram showing where the pictures will be placed on the wall. Note that the diagram is not drawn to scale.



- b) If the horizontal distance from the outside edge of each picture to the nearest wall is to be equal to the distance between the two pictures, how far apart should the pictures be?



# Problem of the Week

## Problem A and Solution

### Picture Perfect

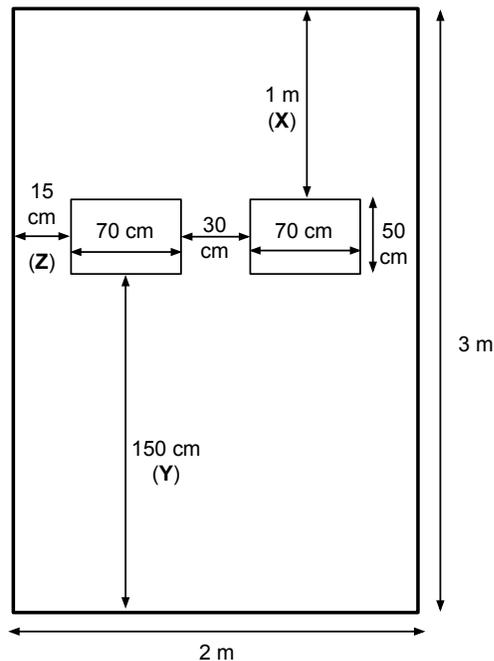
#### Problem

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- b) If the horizontal distance from the outside edge of each picture to the nearest wall is to be equal to the distance between the two pictures, how far apart should the pictures be?

#### Solution

A) Looking at the original diagram, there are three values missing. Here is the completed diagram:





Here is how we can calculate the missing values:

- (Label **X**) From the problem description we know that the top of each picture is 2 m above the floor. Since the wall is 3 m, then the distance from the ceiling to the top of the picture is  $3 - 2 = 1$  m.
  - (Label **Y** in the solution) If the distance from the top of the pictures to the floor is 2 m, this is equal to 200 cm. Since the height of each picture is 50 cm, then the distance from the floor to the bottom of the picture is  $200 - 50 = 150$  cm.
  - (Label **Z**) If the pictures are centred on the wall, then the space on the left side must match the space on the right side. Horizontally, the width of the pictures and the space between them is  $70 + 30 + 70 = 170$  cm. Since the wall is 2 m wide and this is equal to 200 cm, the leftover space is  $200 - 170 = 30$  cm. If we want the space to be equal on either side, then there must be  $30 \div 2 = 15$  cm from the left edge of the picture on the the left to the edge of the wall.
- B) The widths of the pictures take up  $70 + 70 = 140$  cm of space on the wall. This leaves  $200 - 140 = 60$  cm of horizontal space. If we want to equally distribute that space to the left, centre, and right of the pictures we need  $60 \div 3 = 20$  cm of space between the two pictures.



## Teacher's Notes

Creating and/or reading a diagram properly is a fundamental skill in mathematics. Although a diagram drawn to scale is helpful (or possibly necessary) in some cases, most of the time it is more important that the diagram includes clearly labelled, critical information, and it is unnecessary to have precise measurements. Identifying what is important information is also a useful skill. However, it is a good idea to start a diagram with **any** known information. It is possible you include values that end up being superfluous to the problem, but it is better to have access to extra information than be missing important details.

Once you have the initial information labelled, you can infer other values using deductive logic. Logical thinking and *formal logic* are important in the study of mathematics and computer science. In these contexts, we look for precise ways to state our argument that will justify conclusions. A good diagram can be very helpful in this pursuit.

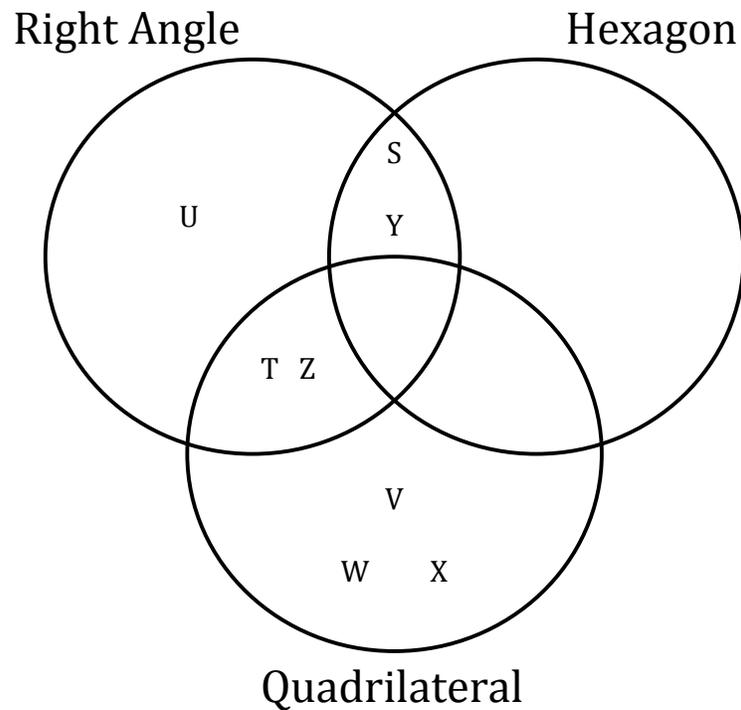


Problem of the Week  
Problem A  
Naming Conventions

The letters in this font (Kumar One Outline) contain many polygons and angles.



- A) Complete the Venn diagram (quadrilateral, right angle, hexagon) with the letters in the alphabet. The bottom row of the alphabet has been filled in already. Note that the right angle in the letter **S** is where the diagonals from the top left to the lower right meet the short diagonals going from left to right.



- B) What other shapes can you identify in the letters?



**Problem of the Week**  
**Problem A and Solution**  
**Naming Conventions**

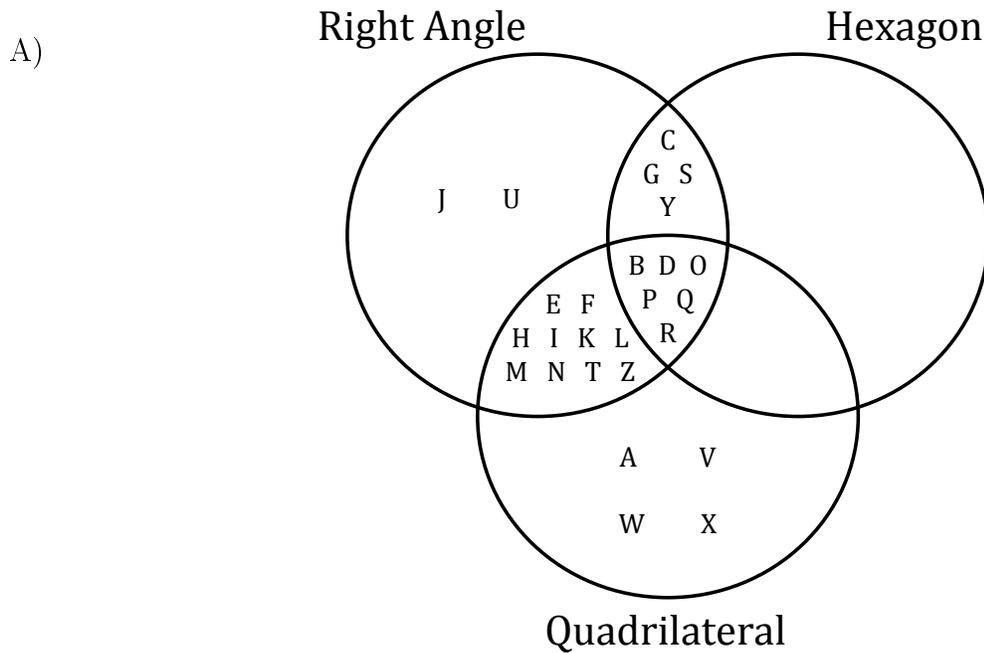
**Problem**

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 Note that the right angle in the letter **S** is where the diagonals from the top left to the lower right meet the short diagonals going from left to right.
- B) What other shapes can you identify in the letters?

**Solution**



- B) Many other shapes occur, answers may vary. Examples include a triangle (A) and pentagons (G, J, Q, U, W).



## Teacher's Notes

Font styles, or *typefaces*, have existed since the invention of the printing press in the 1400s. The typeface used by the inventor of the printing press, Johannes Gutenberg, was a style called *blackletter* that resembled the handwriting of scribes who would reproduce books by hand. Over the centuries other standard typefaces were created, such as Roman and Helvetica, that are widely used today.

With the invention of the modern computer and improvements to the video output technology it became much easier to both create and display new font styles. The font style in this problem (Kumar One Outline) was created in 2016. If you are interested, there are programs that exist that allow you to create your own typeface.

*Source: The History of Typography - Animated Short* (<https://youtu.be/wOgIkxAfJsk>)

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# Algebra (A)

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## Problem of the Week

### Problem A

### Finding the Intersection

Isla starts with \$12 in her bank account. She adds \$12 to her account at the end of every two weeks from collecting recycled items. Javier starts with \$32 in his bank account. He earns \$4 at the end of every week for doing odd jobs for his neighbour, and adds that to his savings.

After how many weeks will they both have the same amount of money in their bank accounts?





## Problem of the Week

### Problem A and Solution

### Finding the Intersection

#### Problem

Isla starts with \$12 in her bank account. She adds \$12 to her account at the end of every two weeks from collecting recycled items. Javier starts with \$32 in his bank account. He earns \$4 at the end of every week for doing odd jobs for his neighbour, and adds that to his savings.

After how many weeks will they both have the same amount of money in their bank accounts?

#### Solution

We can use a table to show the pattern of savings for Isla and Javier. Each week we will add \$4 to Javier's total savings. Every two weeks we will add \$12 to Isla's savings. This means every odd week, Isla's savings will not change.

Week	Isla's Savings (in \$)	Javier's Savings (in \$)
Start	12	32
1	12	36
2	24	40
3	24	44
4	36	48
5	36	52
6	48	56
7	48	60
8	60	64
9	60	68
10	72	72

After 10 weeks of savings, Isla and Javier have the same amount of money in their bank accounts. However, there may be other weeks where they have the same savings.

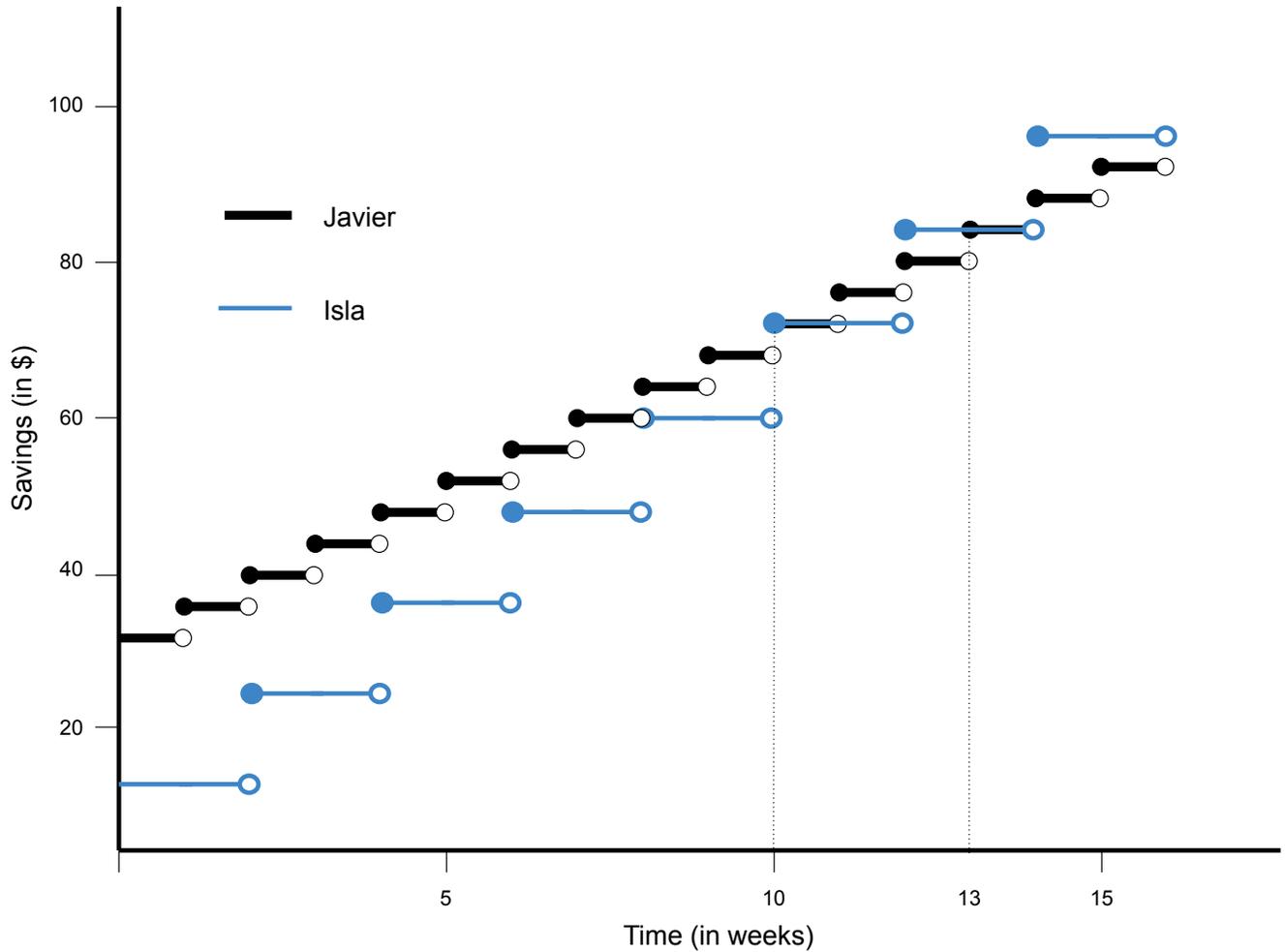
Week	Isla's Savings (in \$)	Javier's Savings (in \$)
11	72	76
12	84	80
13	84	84
14	96	88
15	96	92
16	108	96

At the end of the 13th week, Isla and Javier again have the same amount of money in their bank accounts. At the end of week 16, the difference between Isla's savings and Javier's savings is more than \$8. Since Javier only saves \$8 every two weeks and Isla saves more than Javier, his savings can no longer match Isla's. Therefore, at the end of week 13 they make the last contributions that give the same amount of money in their accounts.



### Teacher's Notes

We can use a *step function* to describe Isla and Javier's savings. If we plot their savings over time, the amount in their bank accounts does not change between deposits. We can draw a graph with time on the x-axis and amount saved in dollars on the y-axis. The result would look like this:



As time passes, we can see that the graph for each person's savings looks like equally spaced steps. Where there is a jump between steps, the filled circle indicates that the actual value is at that point, and the empty circle indicates that the step does not include the value at that point of the graph. From the graph, we can see that the savings amounts overlap at week 10 and week 13. It may also look like there is an overlap at week 8. However at that point we can see that Javier's savings are actually at the level of the next step.



## Problem of the Week

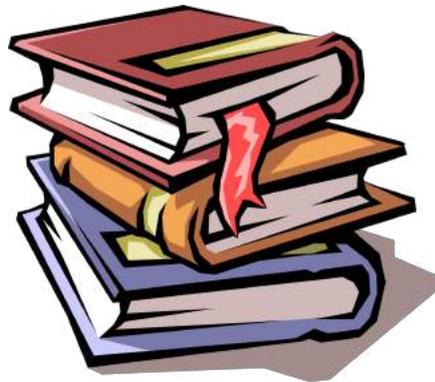
### Problem A

### Eight is Enough

There once was a child whose parents never taught him the number 8. He grew up to be a wonderful person, but would always count 1, 2, 3, 4, 5, 6, 7, 9, 10 and so on, skipping all numbers that include an 8. One day he was given the task of numbering the pages in two books.

- A) By his count, the last page of the first book is 110. Knowing what you do about this child, how many pages are actually in this book?
- B) By his count, the last page of the second book was 320. How many pages are actually in this book?

*Based on the poem **Eight** from the book **I'm Just No Good At Rhyming** (and other nonsense for mischievous kids and immature grown-ups) by Chris Harris and Lane Smith*





## Problem of the Week

### Problem A and Solution

### Eight is Enough

#### Problem

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#### Solution

- A) One way to solve this problem is to match each number that this child counts from 1 to 110, and see what regular decimal number matches 110. For example, we would start this way:

Missing Eights	Regular Numbers
1	1
2	2
3	3
4	4
5	5
6	6
7	7
9	8
10	9
...	...

However, this would take a long time.



We can notice that in most cases, when this child reaches a number that ends with a zero, he has actually only increased his count by 9. The exception to this rule is that he skips all of the numbers between 79 and 90. Let's start by looking at the count up to 100. We can create a table that keeps track of the actual count every time we get to a number that ends with a zero. This will reduce the work we need to do.

Child's Count (Missing Eights)	Actual Count (Regular Numbers)
10	9
20	18
30	27
40	36
50	45
60	54
70	63
90	72*
100	81

\*Note that the numbers the child counts between 70 and 90 are: 71, 72, 73, 74, 75, 76, 77, 79.

There are nine more numbers from 101 to 110 that do not include an 8. So the total number of pages in the first book is 90.

Another way to solve this problem is to consider which numbers from 1 to 100 contain an 8. There are 19 numbers, which we can list:

8, 18, 28, 38, 48, 58, 68, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 98

So, once the child numbers a page with the number 100, he has missed 19 numbers, so the page he numbers 100 is actually page number  $100 - 19 = 81$ . When he numbers pages with the numbers 101 to 110, he only misses the single number 108. Therefore, there are actually

$$110 - 19 - 1 = 90$$

pages in the book.

- B) Note we will see the same pattern for each hundred pages that is less than 800. When counting pages in the second book, we need to remember the child skips the numbers between 179 and 190 and the numbers between 279 and 290.

So when the child reaches 200, the actual count will be  $81 + 81 = 162$ , and when the child reaches 300, the actual count will be  $162 + 81 = 243$ . When the child counts from 301 to 320, this will add another 18 to the total. This means that the actual number of pages in the book is  $243 + 18 = 261$ .



## Teacher's Notes

The child in this problem is essentially using a number system that has 9 digits (0, 1, 2, 3, 4, 5, 6, 7, 8) rather than the number system we are used to, the decimal number system, which uses 10 digits. We refer to the number of digits in the number system as the base of the system. For example, the decimal number system is a base-10 number system.

We can use number systems which have more or fewer than 10 digits to represent numerical values. For example, the binary number system uses just two digits (0 and 1) and the hexadecimal number system uses 16 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). Both of these number systems are commonly used in Computer Science.

The same value looks different when represented by different number systems. For example the decimal number 17 can be represented as the binary number 10001. Both numbers represent the same abstract amount. We can convert a number represented in another base to a decimal number. Each column of a number represents a power of the number system's base. For example, the number 372 in base-10 can be expanded as follows:

$$(3 \times 10^2) + (7 \times 10^1) + (2 \times 10^0) = (3 \times 100) + (7 \times 10) + (2 \times 1) = 300 + 70 + 2 = 372$$

For a base-9 number, we can do a similar expansion, except the columns represent powers of 9 rather than powers of 10. So the number 110 in base-9 can be expanded as follows:

$$(1 \times 9^2) + (1 \times 9^1) + (0 \times 9^0) = (1 \times 81) + (1 \times 9) + (0 \times 1) = 81 + 9 + 0 = 90$$

Similarly the number 320 in base-9 can be expanded as follows:

$$(3 \times 9^2) + (2 \times 9^1) + (0 \times 9^0) = (3 \times 81) + (2 \times 9) + (0 \times 1) = 243 + 18 + 0 = 261$$



## Problem of the Week

### Problem A

### Making Jewelry

Lindsay has decided to make a necklace out of red, white, and yellow beads. She decides to have a repeated pattern in the necklace. She will have five red beads, followed by three white beads, followed by one yellow bead, followed by three white beads. Then the pattern will repeat. When she is done, she connects the ends of the string of beads to form a loop. The beads in her necklace will always form a complete pattern.

- A) The beads Lindsay has are 5 mm wide. She wants to make a necklace that is at least 25 cm long. What is the smallest number of beads she needs to make the necklace where each pattern is complete?
- B) Lindsay has 44 beads of each colour. What is the longest necklace she can make where each pattern is complete?





## Problem of the Week

### Problem A and Solution

#### Making Jewelry

#### Problem

Lindsay has decided to make a necklace out of red, white, and yellow beads. She decides to have a repeated pattern in the necklace. She will have five red beads, followed by three white beads, followed by one yellow bead, followed by three white beads. Then the pattern will repeat. When she is done, she connects the ends of the string of beads to form a loop. The beads in her necklace will always form a complete pattern.

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- B) Lindsay has 44 beads of each colour. What is the longest necklace she can make where each pattern is complete?

#### Solution

The total number of beads in the pattern is:  $5 + 3 + 1 + 3 = 12$ . The length of one copy of the pattern is  $12 \times 5 \text{ mm} = 60 \text{ mm}$  which is equal to 6 cm. So we can use a table to determine the length of the necklace as it includes more copies of the pattern:

Copies of the Pattern	Number of Beads	Necklace Length (in cm)
1	12	6
2	24	12
3	36	18
4	48	24
5	60	30
6	72	36
7	84	42

- A) So Lindsay will need 60 beads (25 red, 30 white, and 5 yellow) to have a necklace that is at least 25 cm and has complete patterns.
- B) Since the pattern includes 6 white beads, this is the colour that occurs the most. If Lindsay has 44 of each colour, then we can determine the maximum number of copies of the pattern by dividing:  $44 \div 6 = 7$  remainder 2. So the pattern can be repeated at most 7 times. In this case, the longest necklace Lindsay can make will be  $7 \times 6 \text{ cm} = 42 \text{ cm}$ .



## Teacher's Notes

This week, students are solving *optimization* problems. In part (a) they are required to find a minimum value given certain problem specifications, and in part (b) they are required to find a maximum value given certain problem specifications. Optimization is an area of mathematics that explores ways to make operations more efficient. These techniques can be applied to many different situations. For example:

- In manufacturing, companies look for ways to minimize waste given the materials required for the process.
- In medicine, hospitals look for ways to improve emergency departments in a way that maximizes patient care given the limits of available staff, equipment, and other resources.
- In education, administrators need to schedule courses in a way that minimizes conflicts for students' requests given limits on classroom capacities, lab requirements, and many other factors.
- In science, people look for ways to minimize the environmental impact of an industrial process.

Studying this area of mathematics can lead to many diverse job opportunities.

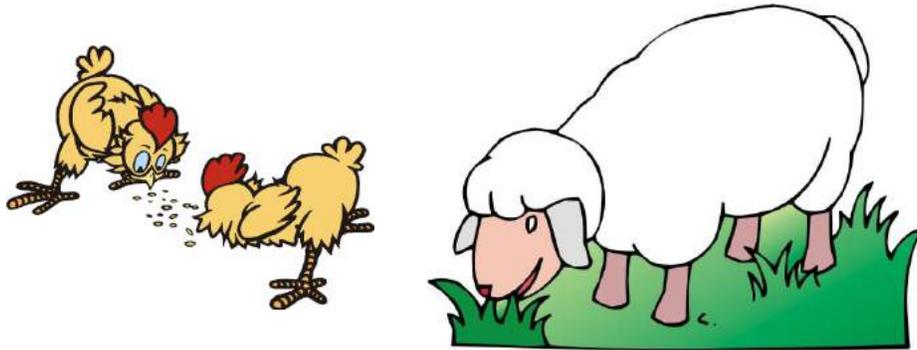


## Problem of the Week

### Problem A

#### Chicken and Sheep, Heads and Feet

Samantha has chickens and sheep on her farm. Each chicken has two legs and each sheep has four legs. Each chicken has one head and each sheep has one head. She looked out one day and counted 48 animal heads and 134 legs. How many of each animal live on the farm?





## Problem of the Week

### Problem A and Solution

### Chicken and Sheep, Heads and Feet

#### Problem

Samantha has chickens and sheep on her farm. Each chicken has two legs and each sheep has four legs. Each chicken has one head and each sheep has one head. She looked out one day and counted 48 animal heads and 134 legs. How many of each animal live on the farm?

#### Solution

One way to determine the answer is to make a table that keeps track of chickens, sheep, heads, and legs. The number of chickens plus the number of sheep must always total 48. So we could start with 0 chickens and 48 sheep. If there were 48 sheep, then there would be  $48 \times 4 = 192$  legs. Then in each row of the table that follows, we increase the number of chicken heads by 1, and decrease the number of sheep heads by 1. We also increase the number of chicken legs by 2, and decrease the number of sheep legs by 4.

Chicken Heads	Sheep Heads	Chicken Legs	Sheep Legs	Total Legs
0	$48 - 0 = 48$	$0 \times 2 = 0$	$48 \times 4 = 192$	192
1	$48 - 1 = 47$	$1 \times 2 = 2$	$47 \times 4 = 188$	190
2	$48 - 2 = 46$	$2 \times 2 = 4$	$46 \times 4 = 184$	188
3	$48 - 3 = 45$	$3 \times 2 = 6$	$45 \times 4 = 180$	186
4	$48 - 4 = 44$	$4 \times 2 = 8$	$44 \times 4 = 176$	184

From the first few rows in this table, we see there is a pattern forming for the total number of legs. Each time we increase the number of chickens by 1, the total number of legs decreases by 2. We could continue filling in the table until we get a row where the total number of legs is 134. However, this would mean filling in many more entries in the table. We need to get the total legs from 184 to 134, which is a difference of  $184 - 134 = 50$ . Since the total number of legs decreases by 2 for each additional chicken, then we would reach the correct total number of legs if we filled in  $50 \div 2 = 25$  more rows. This means we have  $4 + 25 = 29$  chickens. That row in the table would look like:

Chicken Heads	Sheep Heads	Chicken Legs	Sheep Legs	Total Legs
29	$48 - 29 = 19$	$29 \times 2 = 58$	$19 \times 4 = 76$	134

We see that there are 29 chickens and 19 sheep on the farm.



## Teacher's Notes

Here is an alternative approach to solving the problem.

We know all of the animals have at least 2 legs. Since we count 48 heads, then we can account for  $2 \times 48 = 96$  legs, no matter what type of animals are on the farm. However, we have counted  $134 - 96 = 38$  more legs than if all of the animals only had 2 legs. Those extra 38 legs must be on sheep. Each sheep contributes 2 more legs to the count. So we need to have  $38 \div 2 = 19$  sheep on the farm. This means we have  $48 - 19 = 29$  chickens.

We could also solve this problem algebraically.

Let  $s$  represent the number of sheep.

Let  $c$  represent the number of chickens.

Knowing that each animal has 1 head, that chickens have 2 legs, and that sheep have 4 legs, we can write the following equations:

$$s + c = 48 \quad (\text{equation 1})$$

$$4s + 2c = 134 \quad (\text{equation 2})$$

Then we multiply both sides of *equation 1* by 2:

$$2 \cdot (s + c) = 2 \cdot 48$$

$$2s + 2c = 96 \quad (\text{equation 3})$$

Then we subtract *equation 2* - *equation 3* and solve for  $s$ :

$$(4s - 2s) + (2c - 2c) = 134 - 96$$

$$2s = 38$$

$$\frac{2s}{2} = \frac{38}{2}$$

$$s = 19$$

Finally, we can substitute the value of  $s$  into *equation 1* and solve for  $c$ :

$$(19) + c = 48$$

$$19 - 19 + c = 48 - 19$$

$$c = 29$$

This means we have 19 sheep and 29 chickens.



## Problem of the Week

### Problem A

#### Time to Read

Jordan borrowed a book from the library on a Sunday. He must return it two weeks after he got it. He plans to read 10 pages of the book on Mondays, Tuesdays, Wednesdays, Thursdays, and Fridays. On Saturdays and Sundays he plans to read 20 pages of the book. He starts reading on the day he got the book (Sunday).

- A) If the book is 250 pages, how many pages must he read on the last Sunday to finish it before he must return it?
- B) Instead of leaving many pages to read on the last day, Jordan may decide to increase the number of pages he reads, by the same amount, every day except the last Sunday. If Jordan wants to have at least 1 but no more than 20 pages to read on the last Sunday, how many more pages should he read every day?





## Problem of the Week

### Problem A and Solution

#### Time to Read

#### Problem

Jordan borrowed a book from the library on a Sunday. He must return it two weeks after he got it. He plans to read 10 pages of the book on Mondays, Tuesdays, Wednesdays, Thursdays, and Fridays. On Saturdays and Sundays he plans to read 20 pages of the book. He starts reading on the day he got the book (Sunday).

- A) If the book is 250 pages, how many pages must he read on the last Sunday to finish it before he must return it?
- B) Instead of leaving many pages to read on the last day, Jordan may decide to increase the number of pages he reads, by the same amount, every day except the last Sunday. If Jordan wants to have at least 1 but no more than 20 pages to read on the last Sunday, how many more pages should he read every day?

#### Solution

- A) Over two weeks, there are two Mondays, Tuesdays, Wednesdays, Thursdays, and Fridays. This means there is a total of  $2 \times 5 = 10$  days when Jordan reads 10 pages of the book. Over these days, he reads a total of  $10 \times 10 = 100$  pages.

There are also two Saturdays and Sundays for a total of  $2 \times 2 = 4$  days when Jordan reads 20 pages of the book. This does not include the Sunday when Jordan returns the book to the library. Over these days, he reads a total of  $20 \times 4 = 80$  pages.

So, in those two weeks he reads  $100 + 80 = 180$  pages. This means to finish the book, Jordan must read  $250 - 180 = 70$  pages on the last Sunday.

- B) We can make a table showing how many pages he reads over two weeks, starting with the number of pages read according to the original schedule. Over two weeks, for each page added per day, the total number of pages read will increase by 14. If Jordan wants to read no more than 20 pages on the last Sunday, he must read at least  $250 - 20 = 230$  pages in the previous two weeks.

Mon-Fri Pages Per Day	Sat-Sun Pages Per Day	Total Pages in 2 Weeks
10	20	180
11	21	194
12	22	208
13	23	222
14	24	236

So Jordan should read four more pages each day, to ensure he reads no more than 20 pages on the last Sunday.



## Teacher's Notes

Real life problems often come with many *constraints*. In this problem, we start off with several constraints:

- Jordan has the book for two weeks
- He reads 10 pages each day Monday through Friday
- He reads 20 pages each day Saturday through Sunday

With part (b), we add an extra constraint:

- Jordan wants to read between 1 and 20 pages on the last Sunday

In solving this problem, students are taking the necessary information from the real life description and are generating mathematical relationships. It is important that they properly consider all of the constraints. Taking real life situations and creating a mathematical model that identifies the essential elements that are required to solve the problem is known as *abstraction*. The process of creating a good and accurate mathematical model can often be the most difficult part of solving a problem.



## Problem of the Week

### Problem A

### Feeding Fractions

Surya is taking care of his friend's cat (named Yola) and dog (named Ginny) for two weeks. He feeds the animals twice a day. In the morning, Yola gets  $\frac{1}{3}$  of a can of cat food and Ginny gets  $\frac{1}{2}$  of a can of dog food. In the evening, both animals eat dry food. Surya saves the leftover canned food in the refrigerator between feedings.

The first day that he feeds the animals is Saturday the 10th. He opens up a new can of cat food and a new can of dog food on that first day. The last day he feeds the animals is on Friday the 23rd.

- A) On what days does Surya have to open both a new can of cat food and a new can of dog food? How much of each kind of food is left in cans in the refrigerator when the owners come home on Saturday the 24th?
- B) Surya recycles the cans when all the food has been served. How many cans did Surya recycle while he was taking care of Yola and Ginny?





## Problem of the Week

### Problem A and Solution

### Feeding Fractions

#### Problem

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- B) Surya recycles the cans when all the food has been served. How many cans did Surya recycle while he was taking care of Yola and Ginny?

#### Solution

We can make a table that keeps track of how much food is in the cans at the start of each day. When the can is full, we indicate this with the value 1.

Day	Cat Food	Dog Food
Saturday (10th)	1	1
Sunday (11th)	$\frac{2}{3}$	$\frac{1}{2}$
Monday (12th)	$\frac{1}{3}$	1
Tuesday (13th)	1	$\frac{1}{2}$
Wednesday (14th)	$\frac{2}{3}$	1
Thursday (15th)	$\frac{1}{3}$	$\frac{1}{2}$
Friday (16th)	1	1
Saturday (17th)	$\frac{2}{3}$	$\frac{1}{2}$
Sunday (18th)	$\frac{1}{3}$	1
Monday (19th)	1	$\frac{1}{2}$
Tuesday (20th)	$\frac{2}{3}$	1
Wednesday (21st)	$\frac{1}{3}$	$\frac{1}{2}$
Thursday (22nd)	1	1
Friday (23rd)	$\frac{2}{3}$	$\frac{1}{2}$



- A) On the days when the cat food and dog food values contain a 1 in the table, this indicates that a new can must be opened. So on the days where both the cat food and dog food values contain a 1 in the table, Surya must open both a new can of dog food and a new can of cat food. This will happen on Saturday the 10th, Friday the 16th, and Thursday the 22nd. On the last day, Surya will feed Yola  $\frac{1}{3}$  of a can of cat food leaving  $\frac{1}{3}$  of a can in the refrigerator, and will feed Ginny  $\frac{1}{2}$  of a can of dog food leaving no dog food in the refrigerator.
- B) Each time the table shows a 1 (except for the first row), this indicates that a can of food has been emptied. This means that by the 23rd, there will be 4 cans of cat food recycled. If we include the dog food can that was emptied on the last day Surya fed the animals, there will be 7 cans of dog food recycled. This is a total of 11 recycled cans.



## Teacher's Notes

The key to solving this problem is to carefully record the repetitious action. In these kinds of situations, it is very easy to introduce an *off-by-one error*.

Keeping track of the food being used in the “middle” of the problem is relatively easy. Every six days, Surya will be using two cans of cat food and three cans of dog food. So, for example, if we expanded the timeline to 7 weeks (or 49 days) we can estimate how many cans of food will be used during that time. Since 49 is close to  $6 \times 8$ , then there would be approximately  $8 \times 2 = 16$  cans of cat food and  $8 \times 3 = 24$  cans of dog food used during that time. However, determining **exactly** how many cans were used during that time, and how much food would be left at the end, involves some careful tracking.

An off-by-one error is a classic issue in computer programming for solutions that involve loops. Often the toughest aspects of using a loop correctly in a program is dealing with the special cases of before the loop starts or when the loop ends.



## Problem of the Week

### Problem A

### How Many Halves?

Robbie won 300 gumballs and he would like to share the winnings with his friends. He decides to list his friends in order based on how long he has known each of them. He wants to give away the most gumballs to the friend he has known the longest. Then he will give the next friend on the list exactly half as many gumballs as the person he has known the longest. He continues to give away exactly half as many to the next friend on his list, until the pattern cannot continue. (He will not give away half a gumball.)

- A) If he gives away 100 gumballs to the first friend on the list, how many friends will receive gumballs? Justify your answer.
- B) If the last person he gives gumballs to receives 5 gumballs, what is the largest number of gumballs that the first friend on the list can receive? How many gumballs are given away? Justify your answers.
- C) If Robbie wants to maximize the number of friends that receive gumballs, how many gumballs should the first person on the list receive? How many friends receive gumballs? Justify your answers.





## Problem of the Week

### Problem A and Solution

#### How Many Halves?

#### Problem

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- C) If Robbie wants to maximize the number of friends that receive gumballs, how many gumballs should the first person on the list receive? How many friends receive gumballs? Justify your answers.

#### Solution

- A) If the first friend receives 100 gumballs, then the next friend receives half as many, which is 50 gumballs. Half of 50 is 25 gumballs, which is the amount the next friend on the list receives. Since 25 is an odd number, it cannot be divided in half without ending up with a fraction. So the pattern ends after three friends have received gumballs from Robbie.
- B) If we know that the last friend on the list receives 5 gumballs, then as we move up the list, each friend receives twice as many as the previous one. Let's assign the last friend who receives 5 gumballs the number 1. Then we can make a table that shows the pattern of giving gumballs to his friends:



Friend	Number of Gumballs Friend Receives	Total Number of Gumballs Given
1	5	5
2	10	15
3	20	35
4	40	75
5	80	155
6	160	315

From the information on this table, we can answer the questions.

Since Robbie only has 300 gumballs to give away, if the last person receives 5 gumballs, then the largest amount of gumballs the first person on the list will receive 80 gumballs. There will be a total of 155 gumballs given away.

- C) We want to maximize the number of friends to whom Robbie gives gumballs. Using the two solutions above, we notice that the larger the last friend's amount of gumballs the fewer number of friends that can receive gumballs. One way to think about the problem is that we want the last friend to receive the smallest possible amount, which is 1 gumball. So, the second last friend would receive 2 gumballs. As in part (B) let's assign the last friend who receives 1 gumball the number 1. Then we can make a table that shows the pattern of giving gumballs to his friends:

Friend	Number of Gumballs Friend Receives	Total Number of Gumballs Given
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255

From the information on this table, we can answer the questions.

The first person on the list will receive 128 gumballs. A maximum of eight of Robbie's friends can receive gumballs.



## Teacher's Notes

In this problem, as we consider one additional friend, the number of gumballs received is reduced by half. For part A), we could write an equation for  $g(n)$ , the number of gumballs the  $n^{\text{th}}$  friend gets, as:

$$g(n) = 100 \cdot \left(\frac{1}{2}\right)^{n-1}$$

If we look at the graph of this function, we would see that it is a very steep curve. This is an example of an *exponential function*.

In general, an exponential function is a function of the form:

$$g(n) = m \cdot b^n, \quad \text{where } b > 0 \text{ and } b \neq 1.$$

When  $b > 1$ , the value of  $g(n)$  increases very quickly as  $n$  increases. We see exponential growth in real life situations such as compound interest and the spread of viruses.

When  $b < 1$  (as in this problem), the value of  $g(n)$  decreases very quickly as  $n$  increases. We see exponential decay in some chemical reactions and heat transfer.

Exponential functions, exponential growth, and exponential decay are generally seen in high school mathematics and science, and possibly some business studies courses.

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# Data Management (D)

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## Problem of the Week

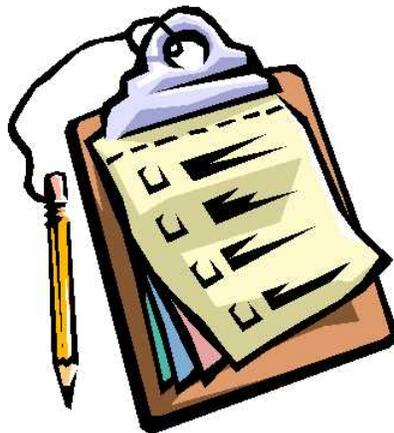
### Problem A

#### Googol App Survey

Xander and Moira surveyed the students of Meadowcrest Public School to find out their favourite Googol App. Students chose one favourite from: Googol Typing, Googol Presents or Googol Cells. Here are the results of the survey:

	Grade 3	Grade 4	Grade 5	Grade 6
Googol Typing	32	54	43	23
Googol Presents	39	21	34	44
Googol Cells	7	9	16	22

- A) For each app, how many students indicated it was their favourite?
- B) What is the favourite app at Meadowcrest Public School?
- C) What is the favourite app in each grade?
- D) How many students participated in the survey?
- E) If Meadowcrest has 615 students in grades 3 through 6, how many students did not participate in the survey?





## Problem of the Week

### Problem A and Solution

#### Googol App Survey

#### Problem

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- C) What is the favourite app in each grade?
- D) How many students participated in the survey?
- E) If Meadowcrest has 615 students in grades 3 through 6, how many students did not participate in the survey?

#### Solution

To answer some of these questions it would be helpful to add a column to the original table that shows totals.

	Grade 3	Grade 4	Grade 5	Grade 6	Total
Googol Typing	32	54	43	23	152
Googol Presents	39	21	34	44	138
Googol Cells	7	9	16	22	54

- A) From the row totals we see that 152 students prefer Googol Typing, 138 students prefer Googol Presents, and 54 students prefer Googol Cells.
- B) From the maximum of the row totals, we see that the favourite app at Meadowcrest is Googol Typing.
- C) Finding the maximum in each column we see that Grade 3 students and Grade 6 students prefer Googol Presents, and Grade 4 students and Grade 5 students prefer Googol Typing.
- D) By adding the total column, we see that  $152 + 138 + 54 = 344$  students participated in the survey.
- E) We can calculate that  $615 - 344 = 271$  students did not participate in the survey.



## Teacher's Notes

Statisticians who use surveys think carefully about how to collect good data. They consider the way questions on a survey are worded, how the data is collected, and who is answering the questions. For example, statisticians are concerned with *response rates*, as the conclusions based on surveys with higher response rates are considered more credible than those with lower response rates.

The response rate of the survey in this problem is  $\frac{344}{615} \simeq 0.559$  or approximately 56%. In their research paper **Survey response rate levels and trends in organizational research**, Yehunda Baruch and Books C. Holtom noted that in academic research surveys the average response rate from individuals was 52.7%. Response rates for other types of surveys are typically lower. They also found that the response rates in research studies have declined in recent years, and since 1995 the rates have typically been below 50%. One concern about research done based on surveys where half the population does not respond is that there may be an inherent bias in the data that is gathered. In other words, the information that is missing from those who do not respond to the surveys could be critical to making sound decisions.



## Problem of the Week

### Problem A

### Colourful Conclusions

Mrs. Routliffe's Grade 3/4 class was learning to collect data using surveys. Kyle created the chart below to show data collected from his classmates about their favourite colours.

Favourite Colour	Red	Orange	Green	Blue	Purple	Yellow
Number of Students			 	 		

Kyle surveyed 35 students. The tallies for **Orange** and **Purple** have not been filled in.

The mode for the data set is equal to the number of tallies for **Orange**.

What are the possible values for **Orange** and **Purple**? Explain your reasoning.





## Problem of the Week

### Problem A and Solution

### Colourful Conclusions

#### Problem

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Kyle surveyed 35 students. The tallies for **Orange** and **Purple** have not been filled in.

The mode for the data set is equal to the number of tallies for **Orange**.

What are the possible values for **Orange** and **Purple**? Explain your reasoning.

#### Solution

The total number of tallies is 21. Since 35 students were surveyed, the number of students who selected **Orange** or **Purple** is  $35 - 21 = 14$ .

The *mode* is the number that appears most often in the data set. If no value appears more than once, then there is no mode. Since this data set has a mode, the number of tallies for **Orange** must match the tallies for at least one of the other colours. So the possible tallies for **Orange** could match one of the known tallies (4, 6, 9, 2), and the possible tallies for **Purple** would be the difference of the **Orange** tallies from 14. Alternatively, the **Orange** tallies could match the **Purple** tallies. In this case they would each be  $14 \div 2 = 7$ . Therefore, the possible tallies for **Orange** and **Purple** are:

Orange	Purple



## Teacher's Notes

In statistics, *mean*, *median*, and *mode* are measurements of *central tendency*. These measurements are used to describe some features of the entire set with (usually) a single value. The mode measurement is different from these other two measurements in a few ways.

Although mode often does not give a “good picture” of the entire data set, there are cases where the mode is more helpful than either the median or the mean. It is possible to have multiple mode values, and those values are not necessarily close to each other. If the data set has a *bimodal* distribution, this means there are two peaks in a graph that represents the data. It is possible that those peaks are far apart. In this case, the mean and median values would appear in between those peaks, and the fact that there are clusters of data values around two distinct peaks is lost. However, if there are two modes identified, then the shape of this graph may be reflected in these values.

Also, unlike mean and median, it is possible to compute the mode of non-numeric data. This problem is a good example of that. The data values are colours, and we are able to calculate the mode of this data set. The *first-past-the-post* voting system uses mode to determine a winner in an election. In this case, the names on a ballot are the data values, and the name that matches the mode is the winner.



## Problem of the Week

### Problem A

### Birthday Predictions

Anika plans to do a survey in her school. She wants to find out the date of each person's birthday. She decides to make some predictions before she actually conducts the survey.

How should Anika answer the following questions? Justify your answers.

- A) Which month is likely to have the fewest birthdays?
- B) Which day (1 through 31) of the month is likely to have the fewest birthdays?
- C) If there are 36 students in Anika's class, how many students are likely to have their birthdays during the summer months of July and August?

After she completes the survey, Anika discovers that her predictions were incorrect. Give some reasons that might explain why the predictions failed.





## Problem of the Week

### Problem A and Solution

### Birthday Predictions

#### Problem

Anika plans to do a survey in her school. She wants to find out the date of each person's birthday. She decides to make some predictions before she actually conducts the survey.

How should Anika answer the following questions? Justify your answers.

- A) Which month is likely to have the fewest birthdays?
- B) Which day (1 through 31) of the month is likely to have the fewest birthdays?
- C) If there are 36 students in Anika's class, how many students are likely to have their birthdays during the summer months of July and August?

After she completes the survey, Anika discovers that her predictions were incorrect. Give some reasons that might explain why the predictions failed.

#### Solution

We assume that it is equally likely that an individual's birthday would land on any particular day of the year.

- A) Since February is the month with the fewest days, then we predict that February has the fewest birthdays.
- B) Since only seven months of the year have 31 days, then it is less likely to have a birthday on the 31st than any other day of the month.
- C) Since there are 12 months in the year, and there are 36 students in the class, then we predict that each month has  $36 \div 12 = 3$  birthdays. So in July and August, we predict there will be  $2 \times 3 = 6$  birthdays.

Here are a couple of reasons that the predictions might fail. The sample size is very small. It is possible with only 36 students that there are days in the month, or even a month in the year where there are no birthdays. Also, although we expect that the birthdays will be evenly distributed, it is possible that multiple students in the class share the day of the month of their birthday, or there are some months that are more popular than others, or that some students even share the same birthday.



## Teacher's Notes

How many people would we have to gather in order to have two of them share a birthday? The simple answer is that if we have at least 367 people, then there is a guarantee that two people must have the same birthday since there are only 366 possible different birthdays in a calendar year that includes a leap day.

It turns out that if you check the birthdays of 23 people, it is more likely than not that you will find that at least two of them share a birthday. This is a classic statistics problem known as the *birthday paradox*. There is no guarantee of course, but it can be proven with statistics that there is just over a 50% chance that in a group of 23 people there are two with the same birthday.



## Problem of the Week

### Problem A

#### T-Shirts

A shipment of 100 shirts was received at “Shirts R Us” in a large fabric bag. Vera, the store owner, knew that she ordered four different colours of shirts. She ordered 40 red shirts and half as many yellow shirts. She ordered 30 blue shirts, but could not remember the number of green shirts she ordered. For each colour, she ordered half of the shirts long-sleeved and the other half short-sleeved. Vera pulls shirts out of the bag one at a time.

- A) What is the probability that she will pull out a red shirt from the bag first?
- B) What is the probability that she will pull out a green shirt from the bag first?
- C) What is the probability that she will pull out a short-sleeved blue shirt from the bag first?





## Problem of the Week

### Problem A and Solution

#### T-Shirts

#### Problem

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- C) What is the probability that she will pull out a short-sleeved blue shirt from the bag first?

#### Solution

We should start by calculating how many of each colour of shirt has been ordered. Since there are half as many yellow shirts as red shirts, then there are  $40 \div 2 = 20$  yellow shirts.

Now we can calculate the total number of red, yellow, and blue shirts as  $40 + 20 + 30 = 90$ . Since there were 100 shirts in the order, and the only other colour ordered was green, then there must be  $100 - 90 = 10$  green shirts.

- A) Since there are 40 red shirts in the bag, then there is a 40 out of 100 chance that the first shirt Vera pulls from the bag is red.
- B) Since there are 10 green shirts in the bag, then there is a 10 out of 100 chance that the first shirt Vera pulls from the bag is green.
- C) Since half of each colour is short-sleeved, there are  $30 \div 2 = 15$  short-sleeved blue shirts. This means there is a 15 out of 100 chance that the first shirt Vera pulls from the bag is a short-sleeved blue shirt.



## Teacher's Notes

Probability describes the likelihood that some event will happen. It is always a number between 0 and 1. When the probability is 0, we expect that the event will definitely not happen. When the probability is 1, we expect that the event definitely will happen. All other probabilities describe a relative chance of the event happening.

We see many ways of describing probability in news reports, advertisements, scientific papers, and other media. However, all of them represent a number between 0 and 1. In many cases, there is an implied fraction in the probability description. For example:

- A probability can be described in the form “**a out of b**”, where  $b$  is always greater than or equal to  $a$ . This implies the fraction  $\frac{a}{b}$ .
- A probability can be described as a percentage such as **x%**. This implies the fraction  $\frac{x}{100}$ .
- A probability can be described in the form “**1 in n chance**”, where  $n$  is a positive integer. This implies the fraction  $\frac{1}{n}$ .

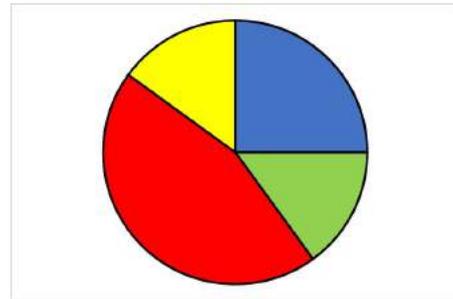
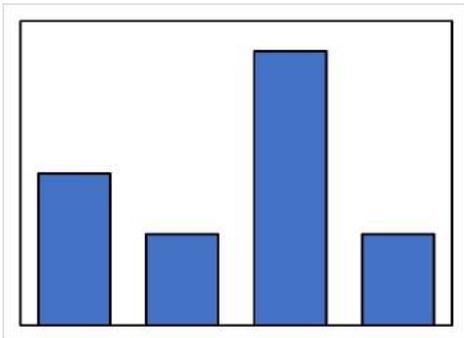


## Problem of the Week

### Problem A

#### Lost Data

Tanner randomly surveyed 40 students in his school about their ages. The ages given were six, seven, eight, and nine. After gathering the answers, he drew a bar chart and a pie chart to show the results. Unfortunately, before he labelled each chart, he lost the original data. The charts are shown below:



Tanner did remember that 6 students in the survey answered six for their age. He also remembered that  $\frac{1}{4}$  of the students surveyed answered eight for their age, and that the most popular answer was age seven. Based on this information, complete the table below:

Age	Number of Students
six	6
seven	
eight	
nine	



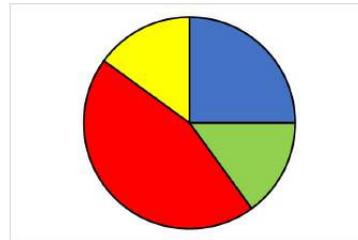
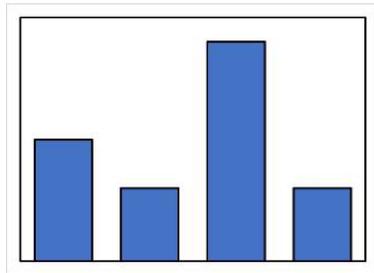
## Problem of the Week

### Problem A and Solution

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six	6
seven	
eight	
nine	

#### Solution

We compute  $\frac{1}{4}$  of 40 is equal to 10. So 10 of the students who were surveyed are eight.

From the pie chart, we can see that there are two data values that are less than  $\frac{1}{4}$  of the total number responses, and one data value is greater than  $\frac{1}{4}$  of the total number of responses. We can also see from both charts that two of the data values are the same and less than one quarter of the total number of responses. From these two observations, along with the fact that one of the data values is 6, we can conclude that there is a second data value that is also 6.

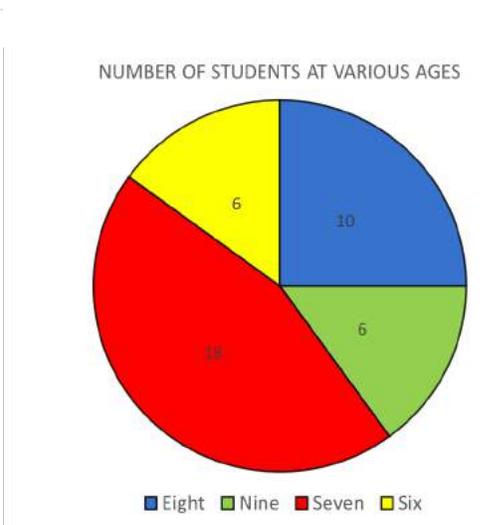
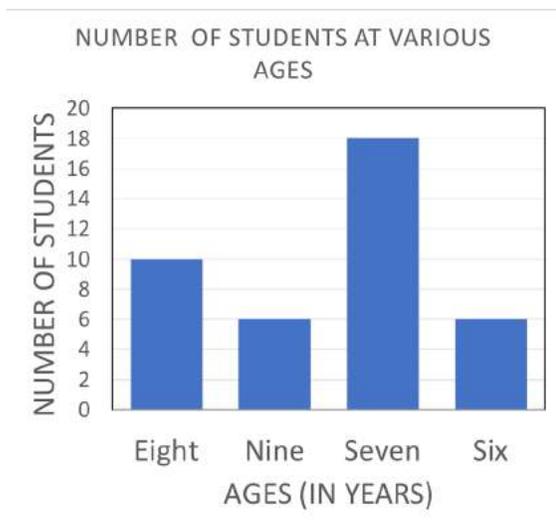


Now we can calculate the sum of three of the data values:  $6 + 6 + 10 = 22$ . Since 40 students were surveyed, the last data value must be  $40 - 22 = 18$ . This is the largest data value. Since seven was the most popular answer, there must be 18 students who are seven.

Now we know how many students answered six, seven, and eight for their ages. We also know that there is one data value (6) that we have not assigned to an age. So there must have been 6 students who answered age 9 in the survey. So the completed table is:

Age	Number of Students
six	6
seven	18
eight	10
nine	6

Note: For some reason, Tanner decided to use word labels on the horizontal axis of the bar chart, in alphabetical order. That is, the labels on the horizontal axis were eight, nine, seven and six. His completed graphs are shown below.

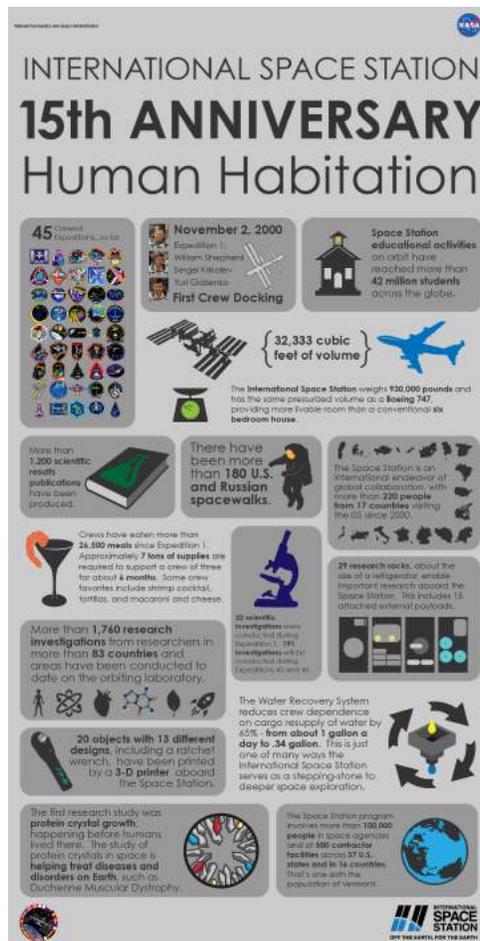




## Teacher's Notes

The charts in this problem were generated by an Excel spreadsheet with the same underlying data values: 6, 18, 10, and 6. Different graphical representations of data may make it easier to see relationships among the data values. For example, although it appears that the yellow and green sections of the pie chart are the same size, it is clearer in the bar chart (especially in the solution with grid lines) that these values are the same. However, in the pie chart, it is much easier to see that the blue section is one quarter of the total surveyed.

We see more and more sophisticated examples of using images to represent data. People have been using *infographics* as a way of capturing people's attention - often in an attempt to sell things, but also in an attempt to emphasize important information - for a long time. However with today's technology we see these kinds of images everywhere.



(Image provided by NASA / Public domain)

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# Computational Thinking (C)

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## Problem of the Week

### Problem A

### Escape Room

You need to unlock a door in an escape room by entering the correct 3-digit combination for the lock on the door. You have already tried some combinations that were incorrect. However, you have been given some feedback on each attempt. Use the following clues to crack the code.



--	--	--

CODE

5	7	1
---	---	---

One number is correct and in the correct position. The other two numbers are not in the combination.

5	0	3
---	---	---

One number is correct but not in the correct position. The other two numbers are not in the combination.

1	9	5
---	---	---

Two numbers are correct but not in the correct positions. The third number is not in the combination.

6	2	7
---	---	---

None of the numbers are in the combination.

7	6	9
---	---	---

One number is correct but not in the correct position. The other two numbers are not in the combination.



## Problem of the Week

### Problem A and Solution

#### Escape Room

### Problem

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---	---	---

Two numbers are correct but not in the correct positions. The third number is not in the combination.

6	2	7
---	---	---

None of the numbers are in the combination.

7	6	9
---	---	---

One number is correct but not in the correct position. The other two numbers are not in the combination.

### Solution

Here is one way to determine the solution.

From attempts (5, 7, 1) and (6, 2, 7) we know that either 5 or 1 is part of the correct code.

However from attempts (5, 7, 1) and (5, 0, 3), if 5 is part of the correct code then we would have a contradiction. In other words, if we assume 5 is part of the correct code, it cannot be both in the correct position and not in the correct position. This means that 1 must be part of the correct code and it is in the third position, and that 5 is not part of the correct code. We also know that either 0 or 3 is part of the correct code.

From attempt (1, 9, 5), and the fact that we already know that 5 is not part of the correct code, we know that 9 is part of the correct code, but it is not in the second position. Since from attempt (7, 6, 9) it also cannot be in the third position, then it must be in the first position.

Since we are only missing the number in the second position, and we know that 5 is not part of the correct code, then from attempt (5, 0, 3) we can conclude that 3 is part of the correct code.

So, the correct code is (9, 3, 1).



## Teacher's Notes

Solving this problem involves making logical deductions. A clear argument to crack the code can be thought of as a *proof* that  $(9, 3, 1)$  is the correct solution. Proofs are an essential part of advanced mathematics.

It is often difficult to clearly communicate the logic of an argument in a natural language. Mathematicians often use more formal notation in an effort to make their proofs unambiguous. Using established rules, we can start with some known information and make a concise argument that leads to a desired conclusion.

For example, assume that the correct code in this problem is  $(a, b, c)$ . Here is a more formal way of describing the information provided about the  $(5, 7, 1)$  attempt:

$$(a = 5) \text{ OR } (b = 7) \text{ OR } (c = 1)$$

From the  $(5, 0, 3)$  attempt, we can make this logical statement:

$$a \neq 5$$

Since we know from the  $(6, 2, 7)$  attempt that none of these numbers are in the correct code, we can make this logical statement:

$$b \neq 7$$

Since all of the information we are given must be true, we can combine these statements formally as:

$$((a = 5) \text{ OR } (b = 7) \text{ OR } (c = 1)) \text{ AND } (a \neq 5) \text{ AND } (b \neq 7)$$

Then we can use logical rules (the details of which have been omitted) that would lead to a step-by-step proof that the statement above is equivalent to the statement:

$$c = 1$$

At this point, we have one-third of the code cracked.

Formal proofs and the logical thinking behind them are very important concepts in mathematics and computer science.



## Problem of the Week

### Problem A

### Mystery Code

James wants to send an image to a friend. The image is made up of filled in black squares on a grid. The rows of the grid are labelled with numbers from 1 to 9 and the columns of the grid are labelled from *A* to *Z*. James encodes the image by providing a series of short codes. Each short code consists of four parts:

- A letter between *A* and *Z* followed by a number between 1 and 9 indicating the column and row of the first square to be filled in on the grid,
- followed by an arrow ( $\uparrow, \downarrow, \leftarrow, \rightarrow$ ) indicating the direction to fill,
- followed by a number, indicating the total number of squares to colour.

For example, the short code *B2*  $\downarrow$  7 tells you to fill in a square at position *B2* on the grid, and continue filling in 6 more squares directly below that position. And the short code *L6*  $\rightarrow$  1 tells you to fill in a square at position *L6* on the grid and no other squares. This code would have you fill in the grid as shown:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1																										
2		■																								
3		■																								
4		■																								
5		■																								
6		■										■														
7		■																								
8		■																								
9		■																								

Using the blank grid on the next page, determine the image that James sent using the following codes:

*T8*  $\uparrow$  7    *D4*  $\leftarrow$  1    *R2*  $\leftarrow$  5    *I5*  $\rightarrow$  3    *F2*  $\downarrow$  7    *L8*  $\uparrow$  6    *C3*  $\rightarrow$  1

*P3*  $\downarrow$  6    *X2*  $\downarrow$  7    *E3*  $\uparrow$  1    *W5*  $\leftarrow$  3    *H8*  $\uparrow$  6    *K2*  $\leftarrow$  3    *B2*  $\downarrow$  7





Use the first blank grid to create the image that James sent using the following codes:

$T8 \uparrow 7$     $D4 \leftarrow 1$     $R2 \leftarrow 5$     $I5 \rightarrow 3$     $F2 \downarrow 7$     $L8 \uparrow 6$     $C3 \rightarrow 1$

$P3 \downarrow 6$     $X2 \downarrow 7$     $E3 \uparrow 1$     $W5 \leftarrow 3$     $H8 \uparrow 6$     $K2 \leftarrow 3$     $B2 \downarrow 7$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1																										
2																										
3																										
4																										
5																										
6																										
7																										
8																										
9																										

An extra blank grid is provided below. Make your own image and provide a classmate with the code that would be used to create your image.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1																										
2																										
3																										
4																										
5																										
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8																										
9																										







## Teacher's Notes

Digital images are formed by many individual small squares called *pixels*. With images like pictures taken on a phone, the pixels are usually different shades of colour. Before email attachments and scanning became so popular, businesses used fax machines to send scanned documents. A fax was a document scanned by a source machine and then transmitted as data over phone lines to another machine at the receiving end. The transmitted image was normally black and white, like the image in this problem.

The image in this problem is formed by 63 black pixels and 171 white pixels, for a total of  $9 \times 26 = 234$  pixels. We could have described the image by identifying each individual pixel as being either black or white. This would require 234 bits of information. In this problem, we used 14 codes to represent the same information. Although the codes are more complicated to understand, overall they use less data to represent the same information. This concept of using less data to represent information is known as *compression*.

This particular method of compressing data is similar to a technique known as *run-length encoding*. This technique was used by fax machines to reduce the amount of data required to transmit an image over phone lines. Although fax machines are not very popular these days, run-length encoding is still used in other applications, including managing very long gene sequences in DNA research. There are many different techniques for compressing data. You are probably familiar with the results of many of them such as **jpg**, **zip**, and **mp3** files.