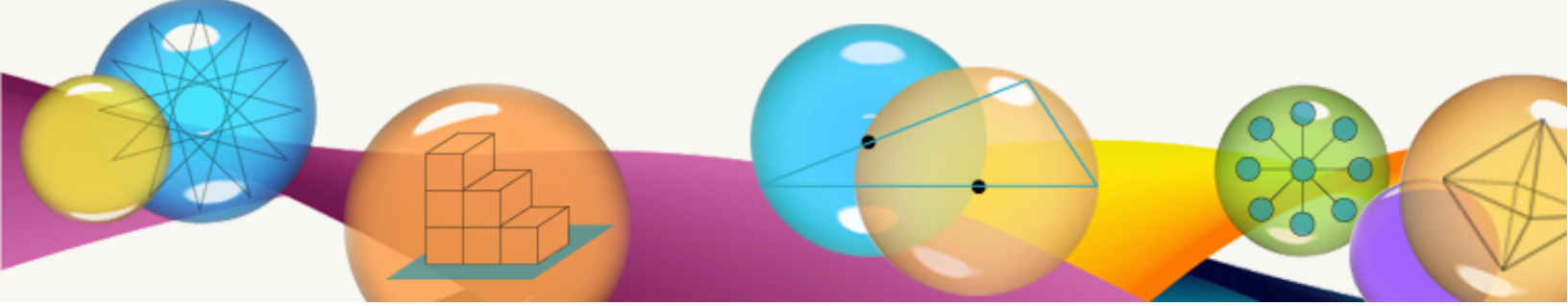


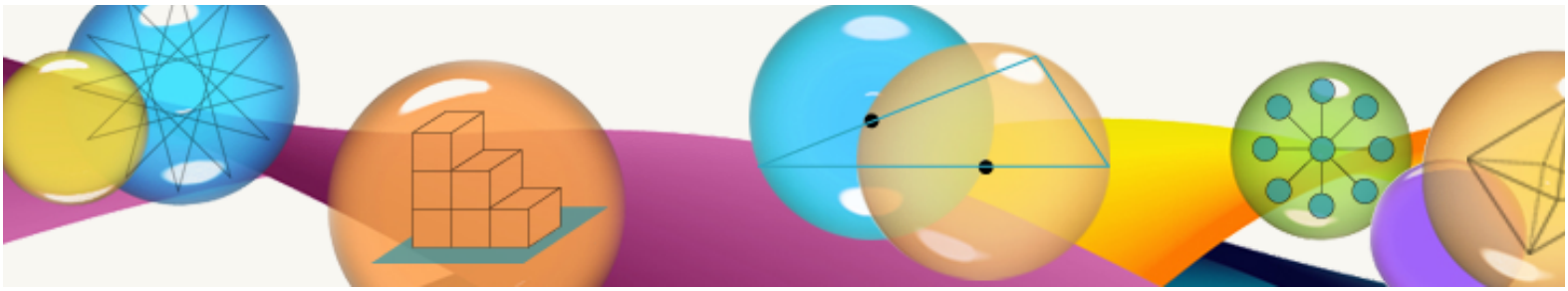
Problem of the Week
Grade 3/4 (A)

Problem and Solutions
2018 - 2019

Strands

(Click the strand name above to jump to that section)

The problems in this booklet are organized into strands. A problem often appears in multiple strands. The problems are suitable for most students in Grade 3 or higher.



Data Management & Probability

TAKE ME TO THE COVER

Problem of the Week

Problem A

Flipping for Fun

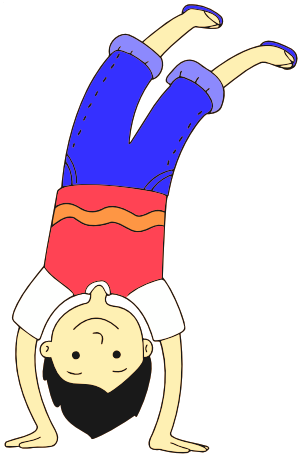
James decides to do an experiment. He flips a coin to determine how he moves. If the coin shows heads, then he moves one step to the right. If the coin shows tails, then he takes one step to the left. Assuming that the coin he is using is fair, (in other words it has an equal chance of landing on heads or tails), answer the following questions:

A) What is the probability that James will be back where he started after flipping the coin twice?

B) What is the probability that James will be within one step of where he started after flipping the coin three times?

C) What is the probability that James will be back where he started after flipping the coin four times?

D) What is the probability that James will be back where he started after flipping the coin five times?



Strand Data Management and Probability

Problem of the Week
Problem A and Solution
Flipping for Fun

Problem

James decides to do an experiment. He flips a coin to determine how he moves. If the coin shows heads, then he moves one step to the right. If the coin shows tails, then he takes one step to the left. Assuming that the coin he is using is fair, (in other words it has an equal chance of landing on heads or tails), answer the following questions:

A) What is the probability that James will be back where he started after flipping the coin twice?

B) What is the probability that James will be within one step of where he started after flipping the coin three times?

C) What is the probability that James will be back where he started after flipping the coin four times?

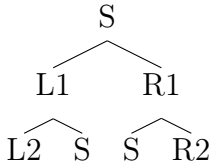
D) What is the probability that James will be back where he started after flipping the coin five times?

Solution

One way to keep track of James' movement, is to make a diagram. James will start on the square labelled **S**, and may move to one of the squares after flipping the coin.

L5	**L4**	**L3**	**L2**	**L1**	**S**	**R1**	**R2**	**R3**	**R4**	**R5**

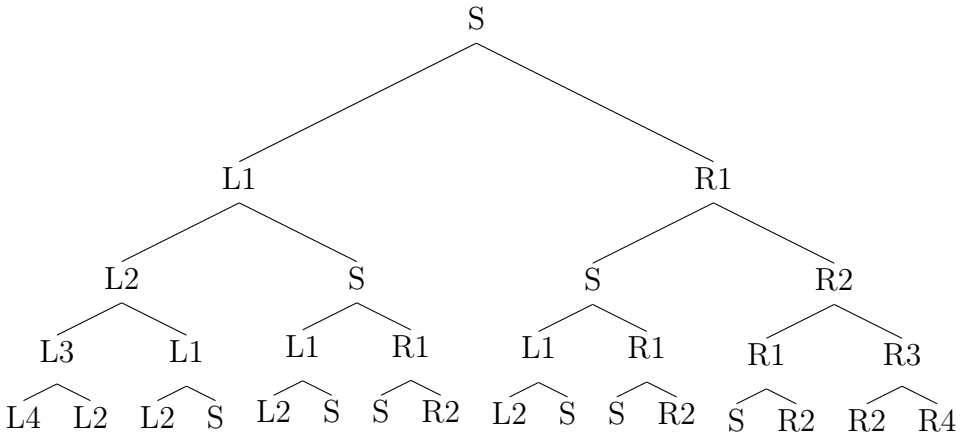
We can use a tree to show all the possibilities of where James ends up after flipping the coin. If the coin shows heads after flipping, James' position is shown by following a right branch in the tree; if the coin shows tails after flipping, James' position is shown by following the left branch in the tree. So after flipping the coin twice this tree shows James' possible positions:



For example, if the first time he flipped the coin it landed on tails, we follow the left branch of the tree and see that James is at position **L1** From there, if he flipped the coin and it also landed on tails, we follow the left branch of the tree and see that James is at position **L2**.

A) All four of the outcomes at the bottom of the tree are equally likely when flipping a fair coin. Since two out of the four outcomes result in James ending up at the starting position, the probability that he is back where he started is 2 out of 4 which is equal to 1 out of 2.

We can continue tracking where James might end up by adding to the tree. Each level in the tree shows James' possible positions after another coin flip.



The bottom row of the tree shows James' possible positions after flipping the coin four times. The second from the bottom row shows James' possible positions after flipping the coin three times. Each outcome in a row is equally likely.

B) After three flips, James is expected be at position **L1** 3 out of 8 times, and he is expected to be at position **R1** 3 out of 8 times. So a total of 6 out of 8 times, James is expected to be within one step of where he started. The probability he is within one step of his starting position is 6 out of 8 which is equal to 3 out of 4.

   Alternatively, if we did not have the tree diagram, we could simply list the possible sequences of three coin flips: `(H H H)`, `(H H T)`, `(H T H)`, `(H T T)`, `(T H H)`, `(T H T)`, `(T T H)`, and `(T T T)`. In six out of eight of these sequences, James ends up one step away from where he started.

C) From the information on the bottom row, of the tree, we see that the probability that James is back at the starting position is 6 out of 16 which is equal to 3 out of 8.

D) We could continue adding one more level to the tree to see how many outcomes are equal to **S** after flipping the coin five times. However, looking at the bottom row of the tree, we can observe that all of those outcomes are either **S** or at least two steps away from **S**. If James flips the coin once, and he is standing at the starting position, then he cannot end up at the starting position. If he is at least two steps away from the starting position, then he cannot return to the starting position after just one coin flip. So, there is no way that James can end up at the starting position after flipping the coin five times. The probability is 0.

   Another way to look at this problem is to notice that if James flips a coin an odd number of times, James always ends up on an odd numbered square. Every time James flips the coin an even number of times, he ends up on either an even numbered square, or on the **S**. So after five flips, James must land on an odd numbered square, so there is no chance that James could land on the starting point.

Teacher's Notes

This problem is actually a demonstration of a statistics concept known as a *random walk*. This is a concept that is introduced in upper year university or college courses. You can try to predict future events, by looking at what might happen if you make random choices. This concept is used to predict share prices in economics, to describe how the molecules of liquids and gasses move, and to help automate image recognition of digital images, among other applications.

Keeping track of all of the details of the possible walks that James takes, can be difficult. However, if we look at the problem one step at a time, we can figure out how many walks there could be. If James flips the coin once, there are two possible paths James could take. From each of those two paths, if we flip a coin, then there are two possible paths James could take. So after two flips we have a total of four paths. After each flip, we double the number of possible paths James could take. So after three flips, there are eight possible paths, and after four flips, there are 16 possible paths.

So rather than listing all of the possible paths, James could focus on listing the ways that he could flip a coin to get a to particular destination. For example, in part C) of the problem, James ends up at the starting position, when he flips the coins and it lands on heads and tails an equal number of times. In particular he ends up at the starting spot after these flips:

```
H T H T
H T T H
H H T T
T H T H
T H H T
T T H H
```

Problem of the Week

Problem A

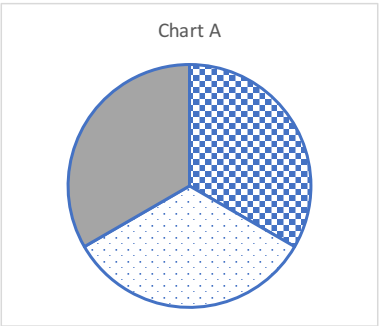
Surveys and Siblings

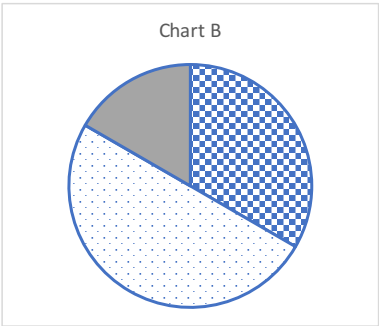
Zheng surveys his friends at school and gathers the following information:

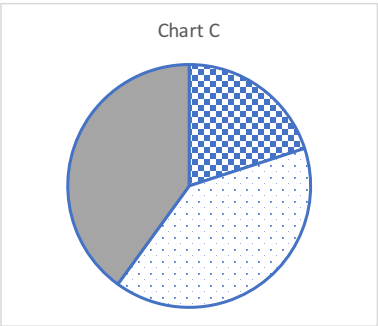
- 13 grade 3 students have no siblings
- 16 grade 3 students have exactly one sibling
- 8 grade 3 students have more than one sibling
- 7 grade 4 students have no siblings
- 14 grade 4 students have exactly one sibling
- 2 grade 4 students have more than one sibling

A) Choose the pie chart that properly shows the proportions of students in the school who have no siblings, one sibling, and more than one sibling.

Justify your answer.







B) Create a legend for the correct pie chart that shows which section represents no siblings, which section represents one sibling, and which section represents more than one sibling.

Problem of the Week

Problem A and Solution

Surveys and Siblings

Problem

Zheng surveys his friends at school and gathers the following information:

- 13 grade 3 students have no siblings
- 16 grade 3 students have exactly one sibling
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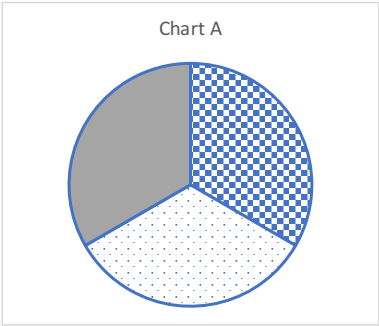
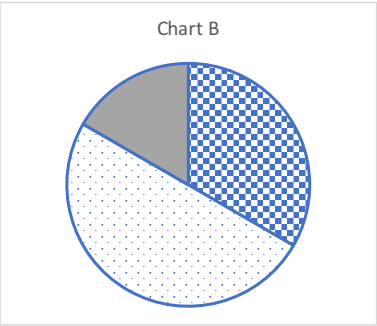
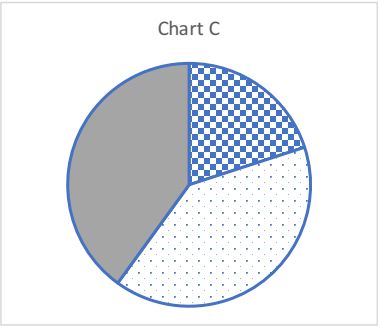
Justify your answer.







B) Create a legend for the correct pie chart that shows which section represents no siblings, which section represents one sibling, and which section represents more than one sibling.

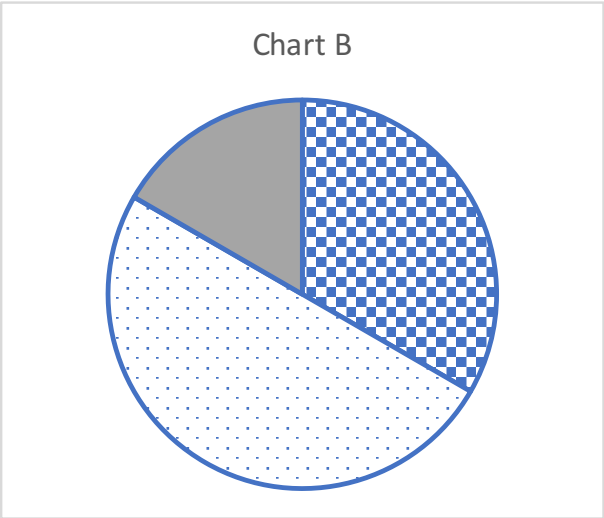
Solution

A) One way to determine which chart is correct is to collate and analyze the original data. Zheng surveyed a total of $13 + 16 + 8 + 14 + 2 = 60$ students.

We can also calculate the following data:

- A total of $13 + 7 = 20$ students have no siblings.
- A total of $16 + 14 = 30$ students have one sibling.
- A total of $8 + 2 = 10$ students have more than one sibling.

Examining the charts, we can see that the sections of Chart A are all approximately the same size. However, in our data analysis, there are clearly many more students who have one sibling than have more than one sibling. So Chart A cannot be correct. We also notice that half of the students (30 out of 60) have exactly one sibling. In Chart C, none of the sections fill half of the pie. So Chart C cannot be correct. Chart B has one section that fills half of the pie. The other two sections of Chart B are clearly not the same size. Since the number of students who have no siblings and the number of students who have more than one sibling are not equal, this means that Chart B is correct.



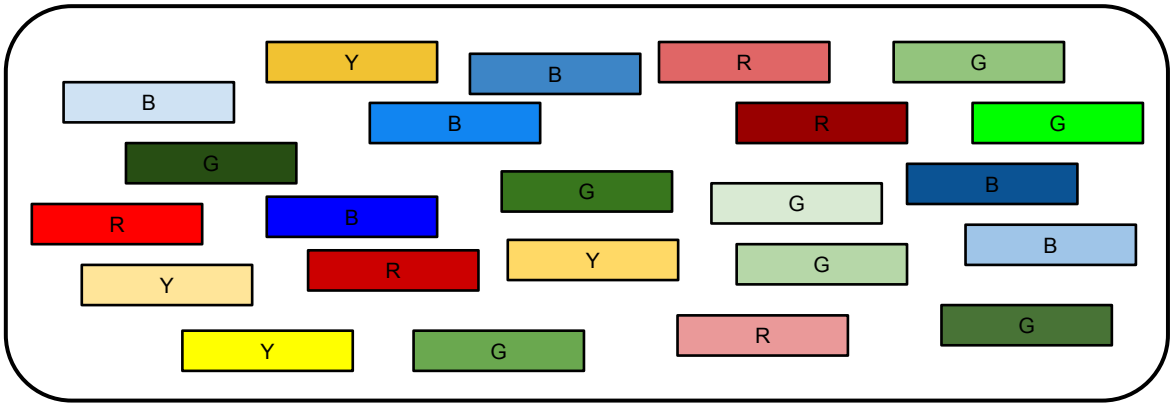
B) Since half of the students in the survey said they had one sibling, and since the dotted section of the chart fills half the pie, then that section must be representing the students with one sibling. Since the fewest number of students in the survey indicated that they had more than one sibling, then the smallest section of the chart which has a solid fill, is representing that group. This means that the checker-board filled section of the chart must be representing the students with no siblings.

Teacher's Notes

For this problem, we are given possible answers and expected to determine which one is correct. In mathematics and computer science, producing an answer is only one part of a good solution. Complete solutions to problems usually involve being able to explain how you got the solution and how you know that your solution is correct. Producing a logical argument to support your answer is an important skill in many situations.

Problem of the Week
Problem A
Colourful Conundrum

Ian has a pencil case full of coloured pencils. He has many shades of four main colours: red (R), blue (B), green (G), and yellow (Y). The diagram below represents the contents of the pencil case:



A) Use the chart below to tally the coloured pencils and then create a pictograph using the key provided:

Key:  = 2 pencils

Main Colour	Tally	Pictograph
red		
blue		
green		
yellow		

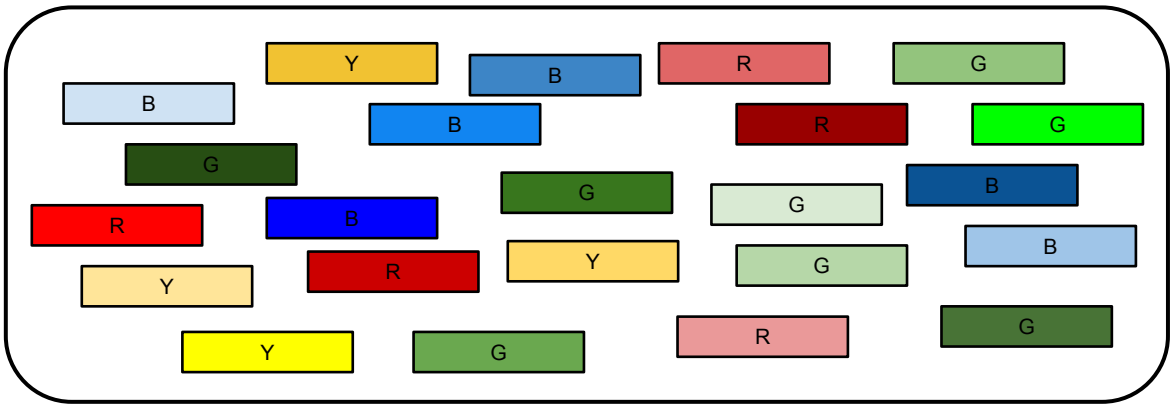
B) How many pencils will you have to draw from the pencil case to guarantee you will get two pencils of the same main colour?

Strands Data Management and Probability, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Colourful Conundrum

Problem

Ian has a pencil case full of coloured pencils. He has many shades of four main colours: red (R), blue (B), green (G), and yellow (Y). The diagram below represents the contents of the pencil case:



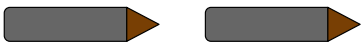
A) Use the chart below to tally the coloured pencils and then create a pictograph using the key provided:

B) How many pencils will you have to draw from the pencil case to guarantee you will get two pencils of the same main colour?

Solution

A) Here is the completed tally and pictograph for the pencil case data:

Key:  = 2 pencils

Main Colour	Tally	Pictograph
red	𝍸	
blue	𝍸 |	
green	𝍸 |||	
yellow	||||	

B) Since there are four main colours, then it is possible to pick four pencils from the case that all have different main colours. So to guarantee you will get two pencils of the same main colour, you will need to pick more than four from the case. Since there are not five main colours, if you pick five pencils, at least two of the pencils will be the same colour. So five is the smallest number of pencils you could draw that would guarantee two of the same main colour.

Teacher's Notes

The solution for part **B** is an example of the *Pigeonhole Principle* at work. The following analogy is often used to describe the Pigeonhole Principle.

> Imagine that you have n pigeons and that you have k pigeonholes where the birds roost. All of the pigeons come home to roost at night. If $n > k$ (i. e. the number of pigeons is greater than the number of pigeonholes), then at least one of the pigeonholes contains more than one pigeon.
>
> We can show that this is true by trying to prove the opposite. Let's assume that no pigeonhole contains more than one pigeon. As each pigeon flies home to roost, it enters an empty pigeon hole. However, after k pigeons have come home all of the pigeonholes will contain a bird. If $n > k$, then there is at least one more pigeon that needs somewhere to roost. It will need to share one of the pigeonholes.

Note that the Pigeonhole Principle does not guarantee that each pigeonhole contains a pigeon. In theory (although maybe not in reality), all of the pigeons could be roosting in one pigeonhole and there would be many empty spots. The only guarantee is that there will be at least one pigeonhole that contains more than one pigeon.

The key to using the Pigeonhole Principle in a mathematics problem is to determine what are the *pigeons* and what are the *pigeonholes*. In part **B** of this problem we can imagine that we have one container, each of which is labelled with one of the four main colours. As we draw pencils out of the case, we will put them in the container labelled with a matching main colour. The question becomes, how many pencils do we need to draw from the pencil case before at least one of the containers has two pencils? In this case, the containers are like the pigeonholes and the coloured pencils are like the pigeons. Since there are four containers, then if we have more than four coloured pencils, one of the containers must have at least two pencils. In particular, when we draw five pencils from the case we must have two that are the same main colour. Since five is the smallest integer that is greater than four, then five is the fewest number of pencils we must draw to guarantee two have the same main colour. However, we do not know which two are the same main colour, and it is possible we have three, four, or even all five pencils that are the same main colour.

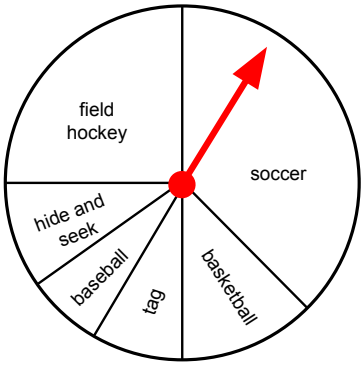
Problem of the Week

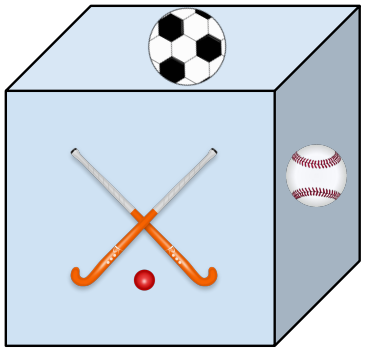
Problem A

Recess Randomness

Davis and Ali surveyed their friends and discovered that the six most popular recess activities are: baseball, basketball, field hockey, hide and seek, tag, and soccer.

They want to randomly choose an activity for their friends to do at recess, so they decide to make a spinner and a game cube showing each activity. The pictures below represent the spinner that Davis designed and the game cube that Ali designed. The same six activities appear on the spinner and the cube.





Which of the following statements is true?

a) Either the spinner or cube can be used to select the activity fairly.

b) Only the spinner can be used to select the activity fairly.

c) Only the cube can be used to select the activity fairly.

d) Neither the spinner nor the cube can be used to select the activity fairly.

Justify your answer.

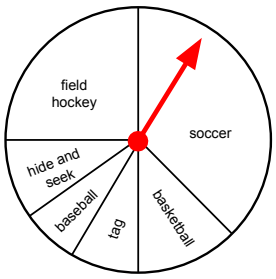
Strand Data Management and Probability

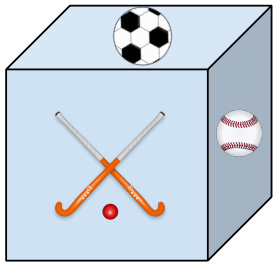
Problem of the Week
Problem A and Solution
Recess Randomness

Problem

Davis and Ali surveyed their friends and discovered that the six most popular recess activities are: baseball, basketball, field hockey, hide and seek, tag, and soccer.

They want to randomly choose an activity for their friends to do at recess, so they decide to make a spinner and a game cube showing each activity. The pictures below represent the spinner that Davis designed and the game cube that Ali designed. The same six activities appear on the spinner and the cube.





Which of the following statements is true?

A) Either the spinner or cube can be used to select the activity fairly.

B) Only the spinner can be used to select the activity fairly.

C) Only the cube can be used to select the activity fairly.

D) Neither the spinner nor the cube can be used to select the activity fairly.

Justify your answer.

Solution

Let us assume that the game cube is a fair die, which means it is expected to have an equal chance of landing on any of the sides. If the friends want to have a random but equal chance for each activity, then you can say that **c** is true.

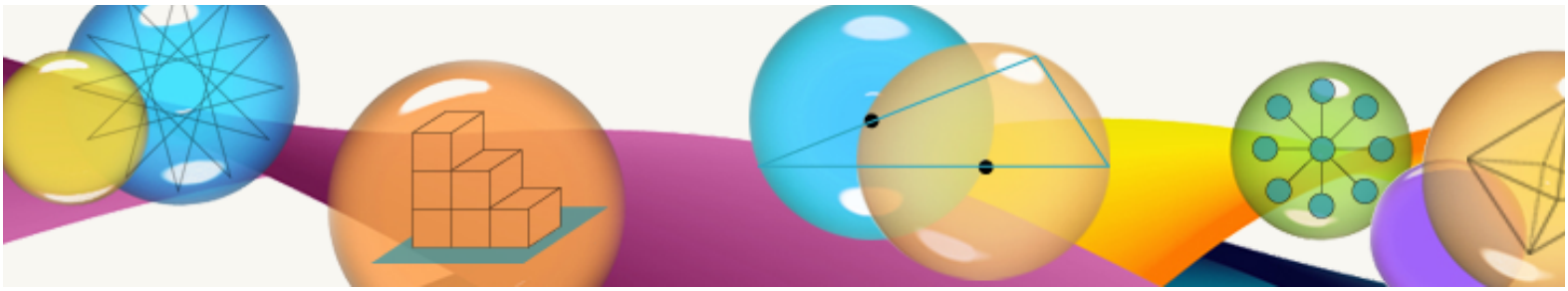
It might be the case that the survey results indicated that some activities are more popular than others and the friends would like to participate in the more popular activities more often. If the size of the spinner sections reflect the relative popularity of each activity, then you can say that **b** is true. However if the size of the sections does not reflect the relative popularity of each activity, then you can say that **d** is true. For example, if twice as many friends like basketball compared to field hockey, neither the spinner nor the cube would be a good way to select the activities in a way that reflects each one's relative popularity.

Teacher's Notes

Getting random results is not necessarily a simple process. Using a fair die or flipping a coin are standard ways of generating random results, but we do not always have these tools available. Computers often need random numbers for their programs. Random values can be used in applications such as games to add to the entertainment value or in simulations that scientists can use to help make predictions related to real life events.

Unfortunately, it is quite difficult to have a computer generate a truly random number. Normally a program generates the number. Since the program follows a particular set of rules to generate these values, they are not truly random. These values are called *pseudo-random* numbers. This means it is possible for someone to be able to discover a pattern in the numbers the computer is generating.

In many cases, pseudo-random numbers are sufficient for computer applications. However, there are situations where the ability to see a pattern in supposedly random events can be a problem. One example of this involves a television game show called *Press Your Luck*. The show involved a large, electronic board that had several squares. The game had a highlighted square moving around the board to random squares one at a time. A contestant would hit a button to stop the movement and would land on some random location. Stopping on some squares would mean good results like winning money, but there were bad squares as well which would mean contestants could lose their turn and possibly lose everything that they had won up to that point. One person watched many episodes of the game show before becoming a contestant. He managed to memorize the pattern that the highlighted square followed, so he could avoid stopping on a bad square. He eventually won over $100000 by discovering a pattern in supposedly random events.



Geometry

&

Spatial Sense

TAKE ME TO THE COVER

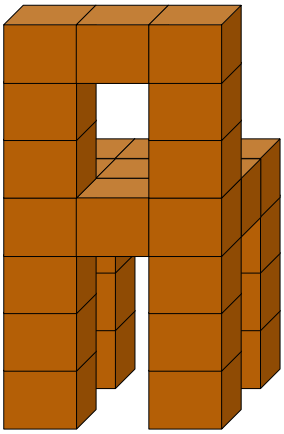
Problem of the Week

Problem A

Block Furniture

The Suite Deal Furniture company is famous for their best selling block furniture that is built out of interlocking wooden cubes.

A) Use the diagram below, to determine the number of blocks required to build the chair.



B) Suite Deal wants to make a larger chair, by increasing the leg height by 1 and the seat width and depth by 1. How many blocks are required to build the larger version?

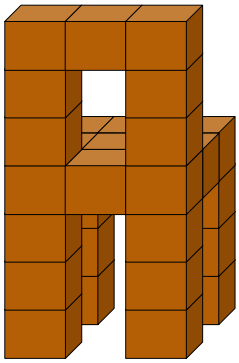
Strands Geometry and Spatial Sense, Patterning and Algebra

Problem of the Week
Problem A and Solution
Block Furniture

Problem

The Suite Deal Furniture company is famous for their best selling block furniture that is built out of interlocking wooden cubes.

A) Use the diagram below, to determine the number of blocks required to build the chair.



B) Suite Deal wants to make a larger chair, by increasing the leg height by 1 and the seat width and depth by 1. How many blocks are required to build the larger version?

Solution

A) From what we can see in the diagram, the seat of the chair is 3×3 blocks. This is a total of 9 blocks for the seat. Each of the four legs of the chair are 3 blocks tall, so this is a total of $4 \times 3 = 12$ blocks for the legs. We can count the blocks that form the back of the chair and see that there are a total of 7 blocks for the back. This means it takes a total of $9 + 12 + 7 = 28$ blocks to build the chair in the diagram.

B) If the seat width and depth of the chair are increased by 1, then the seat is formed using $4 \times 4 = 16$ blocks. If the leg height of the chair is increased by 1, then each leg would be 4 blocks tall, so you would have $4 \times 4 = 16$ blocks for the legs. The height of the back of the chair is unchanged, but when the seat width increases by 1, you will need 4 blocks across the top of the back. This is one more block than the original for a total of 8 blocks for the back. So, the total number of blocks required is $16 + 16 + 8 = 40$ blocks.

Alternatively we can count the extra blocks we would need to build the bigger chair. We would need to add 1 block to each leg for a total of 4 blocks. We would need to add 7 blocks to the seat, since a 4×4 square has 7 more units than a 3×3 square. And, we would need to add one more block to the back of the chair to handle the new seat width. This means we would need a total of $28 + 4 + 7 + 1 = 40$ blocks for the bigger chair.

Teacher's Notes

This problem illustrates the literal relationship between *linear* and *square* functions. To calculate the square of a number, we could build a square out of blocks. For example, if we want to know the value of 3^2 we can build a square like the seat of the chair. The seat has dimensions 3×3 and there are 9 blocks that form the seat. Thus $3^2 = 9$. Similarly, we can calculate the cube of a number by building a cube out of blocks. For example, the value of 4^3 is equal to the total number of blocks required to build a $4 \times 4 \times 4$ cube. If we count the number of blocks in that cube we would see that we needed 64 in total. Thus $4^3 = 64$.

Another thing we can notice from this problem is that a small change in a linear value leads to a much bigger change in the square value. When we increased the width of the seat from 3 to 4, the number of blocks required to build the square seat increased from 9 to 16. More generally, if we consider any number x that is greater than or equal to 1, and know that another number y is double the size of x, then we can prove that y^2 is always four times as big as x^2. We can check this result with a few examples:

If $x = 5$, then $y = 10$
Therefore, $x^2 = 25$ and $y^2 = 100$
Since $4 \times 25 = 100$ we can say $4 \times x^2 = y^2$

or

If $x = 30$, then $y = 60$
Therefore $x^2 = 900$ and $y^2 = 3600$
Since $4 \times 900 = 3600$ we can say $4 \times x^2 = y^2$

We can prove the general case with algebra:

If $y = 2x$, then $y^2 = (2x)^2$
$(2x)^2 = (2x) \times (2x) = 4x^2$
Therefore, y^2 is always 4 times the size of x^2.

Problem of the Week

Problem A

Secret Message

Andrea wants to send secret messages to her friends. She creates a grid and fills it with the letters of the alphabet in order. She uses letters to identify the columns of the grid and numbers to identify the rows of the grid. Then she sends a message by using the positions of the letters and spaces in the grid.

To make it harder for people to decode her messages, she uses different sized grids for different messages, and sometimes she leaves some blank spaces at the beginning of the grid. For example, this is a grid with 8 columns and 3 blank spaces at the beginning:

	A	B	C	D	E	F	G	H
1				a	b	c	d	e
2	f	g	h	i	j	k	l	m
3	n	o	p	q	r	s	t	u
4	v	w	x	y	z			

A) Using the grid above, decode the message: `H2D1G3C2C1D2F3F4A2H3A3`

B) Without seeing the particular grid Andrea uses, she can still send her friends secret messages that they can decode if they know how many columns are in the grid and how many blank spaces are at the beginning of the grid. We call this kind of information the key. So for the grid above, the key is 83.

If the key is 62, the grid would have six columns and two blank spaces at the beginning. How do you encode the message: **this is secret** using the key 62?

C) Decode the message `D1D4A6B4A5A4C2D4B1B4D2A6` using the key 51.

Strands Geometry and Spatial Sense, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Secret Message

Problem

Andrea wants to send secret messages to her friends. She creates a grid and fills it with the letters of the alphabet in order. She uses letters to identify the columns of the grid and numbers to identify the rows of the grid. Then she sends a message by using the positions of the letters and spaces in the grid.

To make it harder for people to decode her messages, she uses different sized grids for different messages, and sometimes she leaves some blank spaces at the beginning of the grid. For example, this is a grid with 8 columns and 3 blank spaces at the beginning:

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1				a	b	c	d	e
2	f	g	h	i	j	k	l	m
3	n	o	p	q	r	s	t	u
4	v	w	x	y	z			

A) Using the grid above, decode the message: `H2D1G3C2C1D2F3F4A2H3A3`

B) Without seeing the particular grid Andrea uses, she can still send her friends secret messages that they can decode if they know how many columns are in the grid and how many blank spaces are at the beginning of the grid. We call this kind of information the key. So for the grid above, the key is 83.

If the key is 62, the grid would have six columns and two blank spaces at the beginning. How do you encode the message: **this is secret** using the key 62?

C) Decode the message `D1D4A6B4A5A4C2D4B1B4D2A6` using the key 51.

Solution

A) The decoded message is: **math is fun**

B) Here is the grid that matches the key 62:

	A	B	C	D	E	F
1			a	b	c	d
2	e	f	g	h	i	j
3	k	l	m	n	o	p
4	q	r	s	t	u	v
5	w	x	y	z		

We could use any of `A1`, `B1`, `E5`, or `F5` for the spaces in the message. The letters can only be encoded in one way. So here is one solution for the encoding of **this is secret**: `D4D2E2C4A1E2C4B1C4A2E1B4A2D4`.

C) Here is the grid that matches the key 51:

	A	B	C	D	E
1		a	b	c	d
2	e	f	g	h	i
3	j	k	l	m	n
4	o	p	q	r	s
5	t	u	v	w	x
6	y	z			

So the decoded message is: **cryptography**

Teacher's Notes

Cryptography is the study of encoding private data. It is a practice that goes back thousands of years. In most cryptographic schemes, encoding and decoding a message requires a *key*. One of the trickiest parts of sending secret messages is that both the sender and the receiver need to know a key in order to encode and decode the message. The question becomes how do you transmit the key without someone finding out what it is.

Today, most data online is secured using *public key encryption*. In this case, the receiver has two keys: a public key and a private key. The receiver provides the sender with a public key that the sender uses to encode the message. However, without the private key, it would be almost impossible to decode the message - even for the sender.

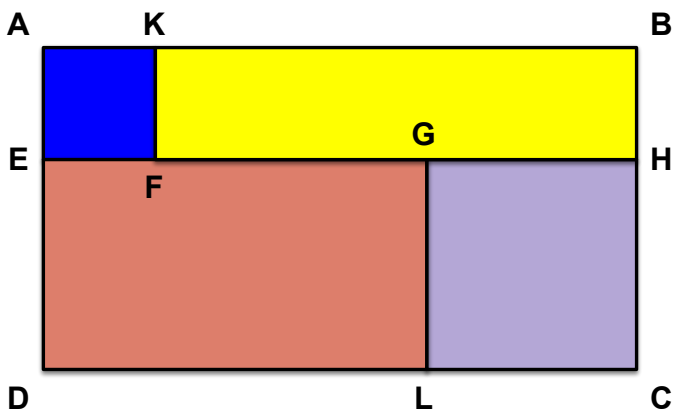
Here is one way to think of public key cryptography. If I want you to be able to send me a secret message, I can send you a box with an open (public) lock. You can put the message in the box and lock it. After it is locked, nobody except the person with a (private) key can read the message. Now you can safely send me a secret message.

Problem of the Week

Problem A

Area Issues

The employees of Sew Inspired need to make a quilt for a special puzzle project. They draw a pattern for the four pieces of cloth they will cut for each section. The pattern includes two square pieces (**AKFE** and **GHCL**), and two rectangular pieces (**KBHF** and **EGLD**).



The area of the piece **AKFE** is 4 square units. The area of the piece **GHCL** is 9 square units. The points **E**, **F**, **G**, and **H** are on the same straight line, and the line segment **FG** is 5 units long.

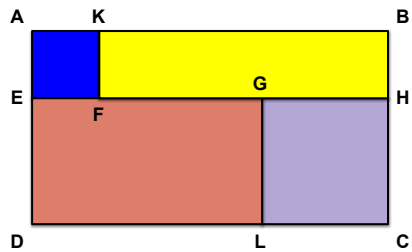
What is the area of the rectangular section **ABCD**?

Strands Measurement, Geometry and Spatial Sense, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Area Issues

Problem

The employees of Sew Inspired need to make a quilt for a special puzzle project. They draw a pattern for the four pieces of cloth they will cut for each section. The pattern includes two square pieces (**AKFE** and **GHCL**), and two rectangular pieces (**KBHF** and **EGLD**).



The area of the piece **AKFE** is 4 square units. The area of the piece **GHCL** is 9 square units. The points **E**, **F**, **G**, and **H** are on the same straight line, and the line segment **FG** is 5 units long.

What is the area of the rectangular section **ABCD**?

Solution

One way to calculate the area of rectangle **ABCD**, is to determine the lengths of its sides. We know the area of the square **AKFE** is 4 square units, and we know that the lengths of the sides of a square must be the same. So if the length of one side of a square is n, then the area of the square must be $n \times n$. By trial and error we can see that $2 \times 2 = 4$, so each side of the square **AKFE** must be 2 units. Similarly, since the area of the square **GHCL** is 9 square units, we can see that $3 \times 3 = 9$. So each side of the square **GHCL** must be 3 units.

Another way to determine the lengths of the sides of square **AKFE**, would be to start with 4 unit squares (using blocks or cut out of paper for example) and determine how to arrange them into a larger square. The only possible arrangement is a 2×2 square.

The opposite sides of a rectangle must be the same length, and the bottom of the square **AKFE** is on the same line as the top of square **GHCL**. This means the length side **AD** is equal to the sum of the lengths of the sides of the two squares. So the length of side **AD** is $2 + 3 = 5$ units. Also, since the length of line segment **FG** is 5 units, the length of line **EH** must be $2 + 5 + 3 = 10$ units. The length of this line is the same as the length of the side **AB** or the side **DC**.

Now we can calculate the area of rectangle **ABCD**. The area of this rectangle is the product of the length of side **AD** and the length of side **AB**. So the area of rectangle **ABCD** is $5 \times 10 = 50$ square units.

Teacher's Notes

Exploring the areas of rectangles or squares is a good way to practice multiplication of positive numbers. The result of calculating $a \times b$ is equivalent to calculating the area of a rectangle with side lengths a and b.

Similarly, exploring the relationship between the area of a square and the lengths of its sides is a good way to learn about *square roots*. However, this exploration only produces half of the answer.

A square root of a number is defined as follows:

$$\text{If } y \text{ is a square root of } x\text{, then } y \cdot y = x.$$

Since the product of two negative numbers is positive, the value of y could be positive or negative. So the square root of 9 is either 3 or -3. When we use the radical symbol, $\sqrt{}$, we are looking for the principal (or positive) square root. Although -3 and 3 are both square roots of 9, the $\sqrt{9} = 3$.

If x is a *perfect square* (i.e. it is the the result of multiplying a rational number by itself), then the result of calculating $\sqrt{x}$ is also a rational number. A *rational number* is any number that can be represented as a fraction $\frac{a}{b}$, where a and b are integers, and b is not 0.

For example, $\sqrt{144}$ is exactly 12. There are also non-integers values that have rational square roots. For example, $\sqrt{\frac{9}{16}}$ is exactly $\frac{3}{4}$.

There are many numbers, however, that are not perfect squares. Square roots of these numbers are known as *irrational numbers*. An irrational number cannot be represented precisely as a fraction. If we try to write an irrational number in decimal form, we end up with a result where the digits after the decimal place never end, and never repeat a pattern. So, we cannot compute the square root of an irrational number on a calculator exactly. The result will appear as a limited number of digits after the decimal place. The calculator rounds the answer to a fixed number of decimal places.

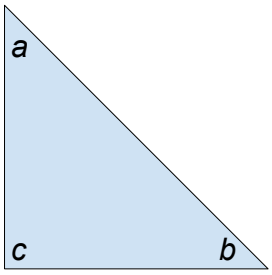
Problem of the Week

Problem A

Transforming Triangles

A) Start with a square piece of paper. Fold it in half along the diagonal to form a triangle and then open it up again. There should be a crease in the paper. Cut along the crease to make two triangles.

Label the angles of each of the triangles with the letters a, b, and c, where c is at the corner with the largest angle like this:



What are the sizes of the angles a, b, and c? Justify your answer.

B) Take one of triangles and fold it in half so that the corner labelled a touches the corner labelled b and then open it up again. There should be a crease in the paper. Cut along the crease to make two triangles.

Compare one of the smaller triangles to the larger triangle left uncut from part A). What features of the triangles are different? What features of the triangles are the same?

C) The two smaller triangles from part B) have equal sides and angles. Arrange these triangles into a single shape where sides from each triangle with matching lengths are lined up and touching without overlapping. What shapes can you form?

Problem of the Week

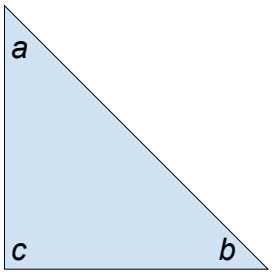
Problem A and Solution

Transforming Triangles

Problem

A) Start with a square piece of paper. Fold it in half along the diagonal to form a triangle and then open it up again. There should be a crease in the paper. Cut along the crease to make two triangles.

Label the angles of each of the triangles with the letters a, b, and c, where c is at the corner with the largest angle like this:



What are the sizes of the angles a, b, and c? Justify your answer.

B) Take one of triangles and fold it in half so that the corner labelled a touches the corner labelled b and then open it up again. There should be a crease in the paper. Cut along the crease to make two triangles.

Compare one of the smaller triangles to the larger triangle left uncut from part A). What features of the triangles are different? What features of the triangles are the same?

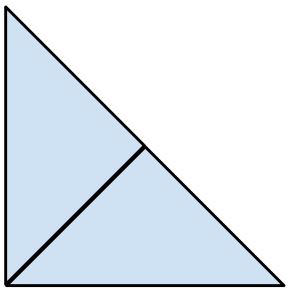
C) The two smaller triangles from part B) have equal sides and angles. Arrange these triangles into a single shape where sides from each triangle with matching lengths are lined up and touching without overlapping. What shapes can you form?

Solution

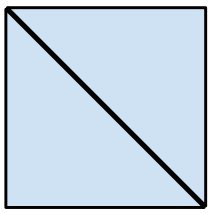
A) We started the construction with a square and we know that all angles in a square are right angles (90°). Since angle c is untouched from the original square, it must also be a right angle. Angles a and b are formed by folding the paper in half and cutting along the crease. Since the two angles on either side of the crease are the same size and together they formed a right angle, then angles a and b must each be exactly half of that right angle. This means that angle a and angle b are each $90° \div 2 = 45°$.

B) The lengths of sides of the larger and smaller triangles are clearly different. However, the angles of the small triangle and the large triangle can be matched up. Both the small and large triangles have one right angle and two 45° angles.

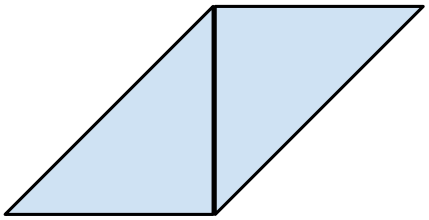
C) You can recreate a bigger triangle by lining up the sides of the smaller triangles that were created by cutting along the crease that was formed in part B).



You can also line up the longest sides of each of the smaller triangles. In this case you will create a square.



There is a third way of aligning the two triangles that forms a parallelogram.



Teacher's Notes

If two triangles have matching side lengths and matching angles then they are said to be *congruent*. In this problem, when we form two triangles by cutting a square in half along its diagonal, we are creating two congruent triangles.

If two triangles have matching angles, but they do not have matching side lengths, then they are said to be *similar*. In this problem, the larger and smaller triangles we construct are similar triangles.

Although the side lengths are not the same, there is a mathematical relationship between the lengths of the corresponding sides of similar triangles. The ratios of the lengths of the corresponding sides of similar triangles are equal.

You can try to convince yourself of this by comparing the lengths of the sides of the larger triangle to the smaller triangle in this problem:

- measure the lengths of the longest side of the larger and smaller triangles
- measure the lengths of one of the shorter sides of the larger and smaller triangles
- compare the following ratios:

$$\frac{\text{length of longest side of large triangle}}{\text{length of largest side of small triangle}}$$

to

$$\frac{\text{length of shortest side of large triangle}}{\text{length of shortest side of small triangle}}$$

These number should be approximately the same. Any difference can be accounted for by the fact that our measurements are not necessarily accurate.

Problem of the Week
Problem A
Scrabble Scoring

Scrabble^{TM} is a board game where players make words using tiles containing individual letters. Each of the letters have a point value which are shown in the following table:

A	B	C	D	E	F	G	H	I	J	K	L	M
1	3	3	2	1	4	2	4	1	8	5	1	3
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	1	3	10	1	1	1	1	4	4	8	4	10

A player places tiles on the board to form a word. The board is divided up into squares. Some of the squares have special values. The words are placed in a straight line horizontally (across, reading from left to right) or vertically (reading from the top down).

- Blue squares are double letter squares, which means if you put a tile on that square the letter is worth twice as many points.
- Green squares are triple letter squares, which means if you put a tile on that square the letter is worth three times as many points.
- Red squares are double word squares, which means if you put a tile on that square, the whole word is worth twice as many points.
- Orange squares are triple word squares, which means if you put a tile on that square, the whole word is worth three times as many points.
- Double and triple letter scores are calculated before the double and triple word scores.

Here is a diagram of part of the Scrabble^{TM} board.

	A	B	C	D	E	F
1	G				G	
2		R				
3			R			B
4				R		
5	G				R	
6			B			O

A) How many points would the word MATH be worth if you started the word in square B1 and placed it horizontally across the board?

B) How many points would the word ZEBRA be worth if you started the word in square D2 and placed it vertically down the board?

C) How many points would the word HAPPY be worth if you started the word in square F2 and placed it vertically down the board?

Strands Number Sense and Numeration, Geometry and Spatial Sense

Problem of the Week
Problem A and Solution
Scrabble Scoring

Problem

Scrabble^{TM} is a board game where players make words using tiles containing individual letters. Each of the letters have a point value which are shown in the following table:

A	B	C	D	E	F	G	H	I	J	K	L	M
1	3	3	2	1	4	2	4	1	8	5	1	3

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	1	3	10	1	1	1	1	4	4	8	4	10

A player places tiles on the board to form a word. The board is divided up into squares. Some of the squares have special values. The words are placed in a straight line horizontally (across, reading from left to right) or vertically (reading from the top down).

- Blue squares, marked **B** on the board below, are double letter squares, which means if you put a tile on that square the letter is worth twice as many points.
- Green squares, marked **G** on the board below, are triple letter squares, which means if you put a tile on that square the letter is worth three times as many points.
- Red squares, marked **R** on the board below, are double word squares, which means if you put a tile on that square, the whole word is worth twice as many points.
- Orange squares, marked **O** on the board below, are triple word squares, which means if you put a tile on that square, the whole word is worth three times as many points.
- Double and triple letter scores are calculated before the double and triple word scores.

Here is a diagram of part of the Scrabble^{TM} board.

	A	B	C	D	E	F
1	**G**				**G**	
2		**R**				
3			**R**			**B**
4				**R**		
5	**G**				**R**	
6			**B**			**O**

A) How many points would the word MATH be worth if you started the word in square B1 and placed it horizontally across the board?

B) How many points would the word ZEBRA be worth if you started the word in square D2 and placed it vertically down the board?

C) How many points would the word HAPPY be worth if you started the word in square F2 and placed it vertically down the board?

Solution

A) If we place the letters for **MATH** horizontally on the board starting at square B1 then the **H** will be on a green square. This means that letter is worth three times its normal value. That is, the **H** is worth $3 \times 4 = \mathbf{12}$ points when placed on the green square located in this placement of the word. So the score for this word is: $3 + 1 + 1 + \mathbf{12} = 17$ points.

B) If we place the letters for **ZEBRA** vertically down the board starting at square D2 then the word covers a red square. This means the word is worth twice its normal value. The normal score for the word would be $10 + 1 + 3 + 1 + 1 = 16$. The red square means that the word would be worth twice as much, so the score for the word at this location on the board is $2 \times 16 = 32$ points.

C) If we place the letters for **HAPPY** vertically down the board starting at square F2 then the **A** will be on a blue square. This means that letter is worth twice its normal value. That is, in this placement of the word, the letter **A** is worth $2 \times 1 = \mathbf{2}$ points. The word also covers an orange square, which means the word is worth three times its normal value. We calculate the score including the double value of the letter **A** before applying the triple word score. The score for the word without considering the triple word square, but including its double letter score, is $4 + \mathbf{2} + 3 + 3 + 4 = 16$. When we consider the triple word square, the result is $3 \times 16 = 48$ points.

Teacher's Notes

When we apply the rule that double and triple letter scores are calculated before the double and triple word scores, we are enforcing an *order of operations* to our calculations.

We often use the mnemonic **BEDMAS** to describe the standard order of operations for mathematical operators. This stands for **B**rackets, **E**xponents, **D**ivision and **M**ultiplication in the order that they appear, and **A**ddition and **S**ubtraction in the order that they appear.

We use brackets in places where we want some operation to take precedence over another operation that would otherwise happen first according to BEDMAS. For example, in part B) of this problem, we want to add the letter values together before we apply the double word score. We could calculate the result like this:

$$(10 + 1 + 3 + 1 + 1) \times 2 = (16) \times 2 = 32$$

Without the brackets the calculation

$$10 + 1 + 3 + 1 + 1 \times 2$$

would equal

$$10 + 1 + 3 + 1 + 2 = 17$$

since according to BEDMAS the multiplication is done before the addition.

In part C), we need to multiply the point value of the letter **A** by 2 since it is on a double letter square. Then we add the rest of the letter values together before multiplying that sum by 3. So we could calculate the result for part C) like this:

$$(4 + 2 \times 1 + 3 + 3 + 4) \times 3 = (4 + 2 + 3 + 3 + 4) \times 3 = (16) \times 3 = 48$$

Note that we do not need brackets around 2×1 since the multiplication will be done automatically before the addition in the calculation. However, sometimes we use brackets (even when they are not required) to make calculations clearer. It may be easier to understand the calculation for part C) if we used this expression:

$$(4 + (2 \times 1) + 3 + 3 + 4) \times 3$$

just to emphasize that the double letter score must be calculated before the sum of the letters is calculated.

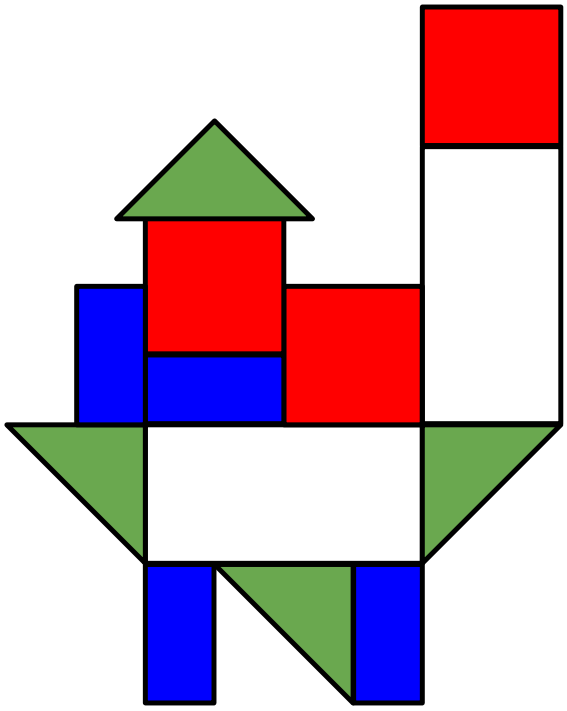
Problem of the Week

Problem A

Area Mystery

The following image was formed by arranging various rectangles, squares, and triangles. The parts of the image with the same shape and shading have the same dimensions. For example, the rectangles that are filled have identical widths and lengths. The rectangles that are not filled have identical widths and lengths. The dimensions of the filled rectangles are different than the dimensions of the rectangles that are not filled.

The filled squares each have an area of 4 cm^2. Determine the area of the whole image. Justify your answer.



Strand Geometry and Spatial Sense

Problem of the Week

Problem A and Solution

Area Mystery

Problem

The following image was formed by arranging various rectangles, squares, and triangles. The parts of the image with the same shape and shading have the same dimensions. For example, the rectangles that are filled have identical widths and lengths. The rectangles that are not filled have identical widths and lengths. The dimensions of the filled rectangles are different than the dimensions of the rectangles that are not filled.

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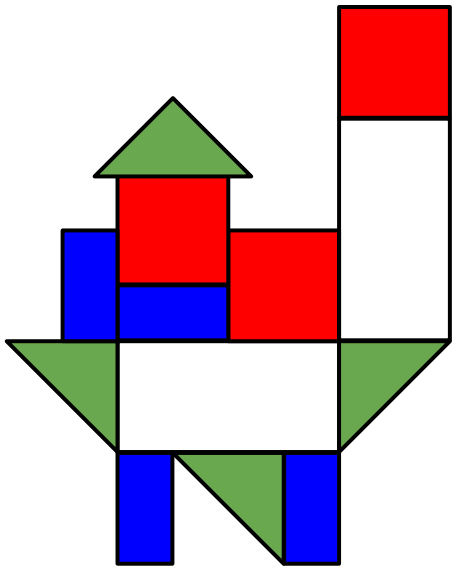


Solution

We know that the area of each square is 4 cm^2. Since there are three squares in the image, the total area of the squares is $3 \times 4 = 12$ cm^2.

We can see from the image that two squares side by side fit exactly along the length of the longer side of an unfilled rectangle. We can also see that the length of the shorter side of the unfilled rectangle is equal to the length of a side of the square. This means that the area of an unfilled rectangle must be equal to the area of two squares or 8 cm^2. Since there are two unfilled rectangles in the image, the total area of the unfilled rectangles is $2 \times 8 = 16$ cm^2.

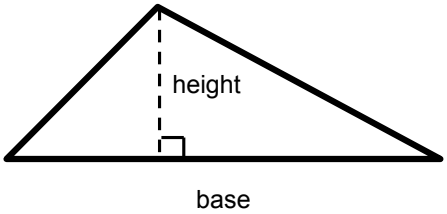
We can see from the image that two short sides of each triangle have the same length. We can also see that these shorter sides are the same length as the sides of the square. If we arranged two triangles so that their longer sides were aligned, then we would form a square. From this we can deduce that the area of one triangle is equal to half the area of one square. This means that area of a triangle is 2 cm^2. Since there are four triangles in the image, the total area of the triangles is $4 \times 2 = 8$ cm^2.

We can see from the image that sum of the lengths of two short sides of filled rectangles plus the length of a short side of a triangle is equal to the length of the longer side of an unfilled rectangle. Since the length of the short side of a triangle is equal to the length of the side of a square, and since the length of the long side of an unfilled rectangle is equal to two square side lengths, then the lengths of two short sides of filled rectangles must be equal to the length of the side of a square. We can also see from the image that the length of the long side of a filled rectangle is equal to the length of a side of a square. From this we can deduce that the area of a filled rectangle is half the area of one square. This means that the area of a filled rectangle is 2 cm^2. Since there are four filled rectangles in the image, the total area of the filled rectangles is $4 \times 2 = 8$ cm^2.

Therefore, total area of the image is: $12 + 16 + 8 + 8 = 44$ cm^2.

Teachers's Notes

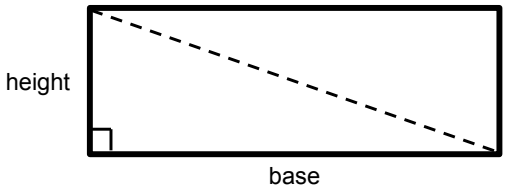
There are many formulae that can be used to calculate the area of a triangle. Probably the most well-known formula is:



Area of a triangle $= \frac{1}{2} \times$ base $\times$ height

In the diagram above, we choose the *base* of a triangle to be the longest side. The *height* of this triangle is the *perpendicular distance* from the base to the opposite corner of the triangle. The perpendicular distance is the length of a line segment from the base to the opposite corner of the triangle. This line segment must form a right angle with the base of the triangle.

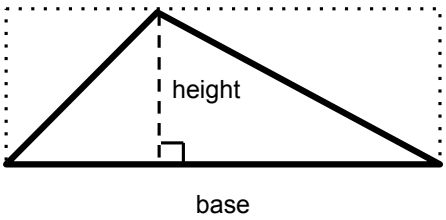
This formula seems to show a relationship with the formula for finding the area of a rectangle. We normally describe the dimensions of a rectangle as *length* and *width*. But we could also describe the dimensions as *base* and *height*. So the area of a rectangle can be written as:



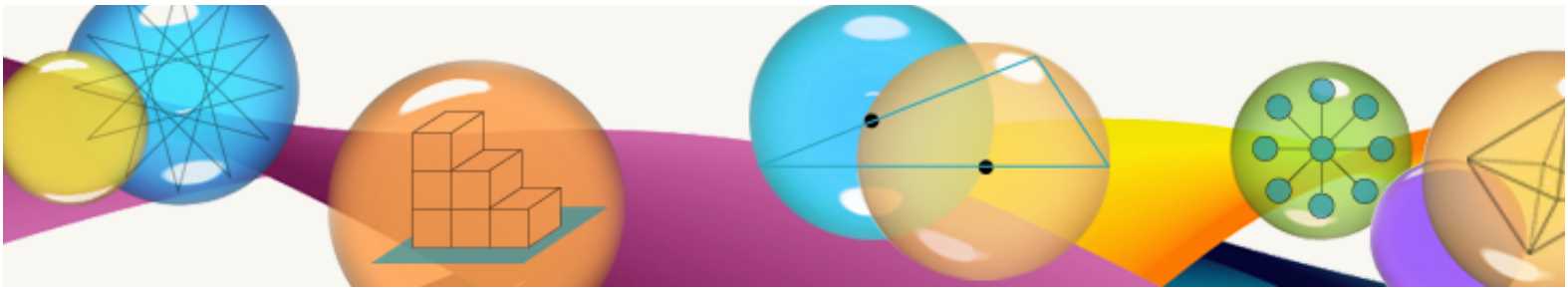
Area of a rectangle $=$ base $\times$ height

The relationship between these areas is easy to see for right-angled triangles. If we draw a diagonal in a rectangle we form two, *congruent*, right-angled triangles. Congruent triangles have matching side lengths, angles, and areas. So, as shown in the diagram above, the area of one of these triangles is half the area of the rectangle.

We can also see the relationship with a triangle that does not have a right angle. We start by forming a rectangle around the triangle, as shown by the dotted lines in the diagram on the right.



In this diagram, we can see the larger rectangle is divided into two smaller rectangles separated by the dashed line that indicates the height of the triangle. We can also see that the two sides of the triangle, that are not the base, are diagonals in those two smaller rectangles. The diagonals divide the smaller rectangles into two pairs of congruent triangles. From this, we can see that the area of our original triangle is half the area of the large rectangle.



Measurement

TAKE ME TO THE COVER

Problem of the Week

Problem A

Age Action

Marty is ten years old. His grandfather is seven times his age. Marty's mother is half the age of his grandfather.

A) How old is Marty's mother?

B) How many decades old is Marty's grandfather?

Strands Number Sense and Numeration, Measurement

Problem of the Week

Problem A and Solution

Age Action

Problem

Marty is ten years old. His grandfather is seven times his age. Marty's mother is half the age of his grandfather.

A) How old is Marty's mother?

B) How many decades old is Marty's grandfather?

Solution

A) Since Marty's grandfather is seven times his age, his grandfather's age is $7 \times 10 = 70$ years. Since Marty's mother is half his grandfather's age, his mother's age is $70 \div 2 = 35$ years.

B) Since one decade is 10 years, Marty's grandfather is $70 \div 10 = 7$ decades old.

Teacher's Notes

When describing quantities, we have many choices for units of measurement. In this example, the grandfather's age was described in years and decades. However, we could have described Marty's age in weeks, day, hours, or even seconds. When we select a unit of measurement, we want to choose units that fit the value being measured. It is often easier for us to understand relatively small quantitative values. For example, we might describe a new baby as being five weeks old. However, we are unlikely to say something like "Marty is 536 weeks old".

We also choose units based on the precision we require in the measurement. For example, we would not describe the distance between the cities Halifax and Yellowknife in millimetres. But we do use millimetres to describe amounts of rainfall.

It is also important that we choose units that are well known to those with whom we are communicating. We might talk about a fortnight to a tennis fan. North American news reports describe some lengths in terms of a number of football fields. An equestrian would know if a horse that is 12 hands tall is big or small. However there are many people in the world who would not know that a fortnight is 14 days, an American football field is 100 yards, and a hand is 4 inches.

Here is a table showing the pattern:

Trail Distance (in metres)	Vertical Distance (in metres)
400	50
800	100
1200	150
1600	200
2000	250
2400	300
2800	350
3200	400
3600	450
4000	500
4400	550
4800	600
5200	650
5600	700

So after walking 5.6 km on the trail, you have travelled approximately 700 m metres vertically up the mountain.

Teacher's Notes

The pattern shown in this problem can be described in the form of an equation of a line. In Cartesian geometry, the equation of a line can be written with this format:

$$y = mx + b$$

where m is defined as the *slope* of the line, and b is defined as the *y-intercept* of the line. The y-intercept describes the point on the line where it crosses the y-axis. In other words, it is the point on the line that satisfies the equation when $x = 0$.

The slope of a line describes a constant relationship between any two points on the line. By definition, if we choose two points that are on the same line: (x_1, y_1) and (x_2, y_2), and compare the ratio of the difference of the y values and the difference of the x values, that ratio will always be the same. We normally work with the rise (the difference in the y values) over the run (the difference in the x values), and call this value the slope. In particular:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

There are two special cases we should consider. Since lines extend infinitely, all lines will cross the y-axis at some point, unless they are parallel to the y-axis. In this case, it is not possible to find a y-intercept. For a line that is parallel to the y-axis, each point on the line will always have the same value of x. We could write the equations of lines like this in the form: $x = c$ where c is a number. The slope of a line that is parallel to the y-axis is undefined since we cannot divide by zero.

Another special case to consider are lines that are parallel to the x-axis. For these lines, each point on the line will always have the same y value. We could write the equations of lines like this in the form: $y = b$. The slope of a line that is parallel to the x-axis is equal to 0.

This problem literally describes a slope. We can think of the vertical distance we travel as being the y value, and the actual distance we walk along the trail as being the x value. The constant ratio in this case is:

$$m = \frac{rise}{run} = \frac{50}{400} = \frac{1}{8}$$

We can think of the start of the trail as being when $x = 0$ and $y = 0$. This means the y-intercept of the line describing this pattern is 0. So an equation describing the relationship between the trail distance and the vertical distance is:

$$y = \frac{1}{8}x$$

To solve this problem, we want to know the value of y when $x = 5600$.

By substitution, $y = \frac{1}{8}(5600)$ or $y = 400$. So the vertical change is 400 m.

Problem of the Week

Problem A

Backpack Limit

Selma is going on a trip with her family. The airline allows each person's carry-on luggage to have a maximum mass of 10 kg. The clothes she wants to bring have a mass of 1.5 kg. She has three books that she wants to read. Two books are the same size, and each has a mass of 2 kg. The third book is 1500 g. She wants to bring her laptop and tablet as well. The laptop's mass is 2.3 kg. Her tablet's mass is 800 g. The shoes she needs for the trip are 1200 g. Her personal toiletries kit is 0.6 kg.

Can Selma bring everything she wants for the trip in her carry-on? Justify your answer.

Strands Measurement, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Backpack Limit

Problem

Selma is going on a trip with her family. The airline allows each person's carry-on luggage to have a maximum mass of 10 kg. The clothes she wants to bring have a mass of 1.5 kg. She has three books that she wants to read. Two books are the same size, and each has a mass of 2 kg. The third book is 1500 g. She wants to bring her laptop and tablet as well. The laptop's mass is 2.3 kg. Her tablet's mass is 800 g. The shoes she needs for the trip are 1200 g. Her personal toiletries kit is 0.6 kg.

Can Selma bring everything she wants for the trip in her carry-on? Justify your answer.

Solution

It would be easier to calculate the total mass of the objects Selma wants to take if the individual masses were all measured in the same units. If we choose grams as the unit, all the masses in kilograms must be multiplied by 1000. Here are the objects' masses all in grams:

clothes: 1500, first book: 2000, second book: 2000, third book: 1500,
laptop: 2300, tablet: 800, shoes: 1200, toiletries kit: 600

The sum of these masses is:
$1500 + 2000 + 2000 + 1500 + 2300 + 800 + 1200 + 600 = 11900$ g

This is equal to 11.9 kg. According to the airline rules, this is too much weight for the carry-on luggage.

Teacher's Notes

Proper handling and conversion of units of measurement is very import in the real world. The metric system makes it easy to convert between units of the same category since you may simply multiply or divide by powers of 10 to do the conversions. However, not everyone in the world uses the metric system. Where different measurement systems are used, there may be problems.

In 1999, NASA sent a satellite to Mars that was intended to orbit the planet and send back climate information. The *Mars Climate Orbiter* was lost when its orbit became unstable and it burned up in the atmosphere surrounding Mars. The cause of the accident was human error. The force calculated in one part of the operating system was assumed to be in *pounds*, but in another part it was assumed to be in *newtons*. This incompatibility ended up with an incorrect amount of power given to the thrusters which led to the eventual destruction of the satellite. *(Source: wired.com)*.

In 1983, an Air Canada flight ran out of fuel on its way from Toronto to Edmonton. The flight specifications had indicated that the aircraft required an amount of fuel in kilograms. However, the ground crew pumped the amount given in pounds. Since one pound is less than one-half a kilogram, the flight had much less fuel than it required. Luckily the pilots landed the aircraft safely, with no engine power, at a closed air force base in Manitoba. *(Source: wikipedia.org)*

Problem of the Week

Problem A

My Beating Heart

A) My resting heart rate is 21 beats in 15 seconds. Predict my number of heart beats in 1 minute.

B) After skipping rope for 1 minute, my heart rate is now 30 beats in 15 seconds. Now, predict my number of heart beats in 1 minute.

C) If I skip for 10 minutes and my heart rate continues to be 30 beats in 15 seconds, predict how many times my heart will beat in 10 minutes.

Strands Number Sense and Numeration, Measurement

Problem of the Week

Problem A and Solution

My Beating Heart

Problem

A) My resting heart rate is 21 beats in 15 seconds. Predict my number of heart beats in 1 minute.

B) After skipping rope for 1 minute, my heart rate is now 30 beats in 15 seconds. Now, predict my number of heart beats in 1 minute.

C) If I skip for 10 minutes and my heart rate continues to be 30 beats in 15 seconds, predict how many times my heart will beat in 10 minutes.

Solution

A) One way to solve this problem is by making a table keeping track of how times your heart beats every 15 seconds.

Time Elapsed in seconds	Heart Beats
15	21
30	42
45	63
60	84

Since 60 seconds equals one minute, we can expect that you will have had 84 heart beats over that time.

B) Since $4 \times 15 = 60$ seconds, if we know your heart rate for 15 seconds we can calculate your heart rate for 1 minute by multiplying by 4. So, we can predict that your heart beats $4 \times 30 = 120$ times in 1 minute after skipping.

C) We expect that the number of times your heart beats in 10 minutes will be 10 times the number of beats you have in 1 minute. So after 10 minutes of skipping we expect your heart beats $10 \times 120 = 1200$ times.

Teacher's Notes

To solve this problem, we used a concept from probability theory known as *expected value*. We assumed that the actual measurement of our heart beat over a short period of time would allow us to predict the number of heart beats over a longer period of time. Our predictions in this case are reasonable if the conditions remain stable.

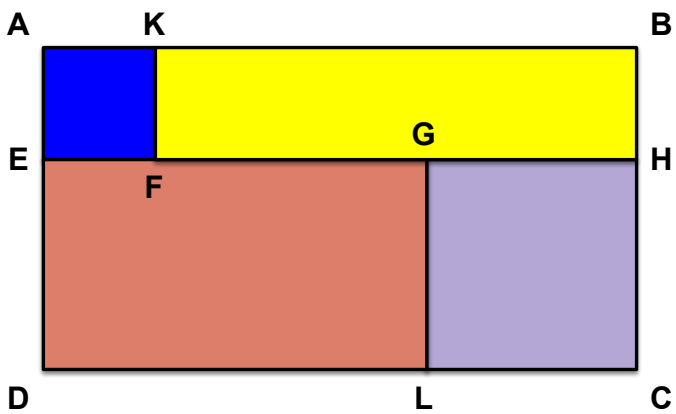
Although expected value is useful for predicting the future, there are other factors that affect predictions. Statisticians would also consider the *variance* of a particular situation. Variance describes the range of possible values that could actually occur. When the variance is low, then our predictions are more likely to be correct or at least close to correct. When the variance is high, then our prediction may be correct, but the actual outcome may be much different than the expected value.

Problem of the Week

Problem A

Area Issues

The employees of Sew Inspired need to make a quilt for a special puzzle project. They draw a pattern for the four pieces of cloth they will cut for each section. The pattern includes two square pieces (**AKFE** and **GHCL**), and two rectangular pieces (**KBHF** and **EGLD**).



The area of the piece **AKFE** is 4 square units. The area of the piece **GHCL** is 9 square units. The points **E**, **F**, **G**, and **H** are on the same straight line, and the line segment **FG** is 5 units long.

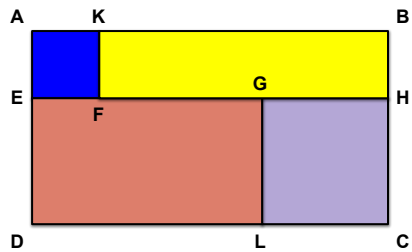
What is the area of the rectangular section **ABCD**?

Strands Measurement, Geometry and Spatial Sense, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Area Issues

Problem

The employees of Sew Inspired need to make a quilt for a special puzzle project. They draw a pattern for the four pieces of cloth they will cut for each section. The pattern includes two square pieces (**AKFE** and **GHCL**), and two rectangular pieces (**KBHF** and **EGLD**).



The area of the piece **AKFE** is 4 square units. The area of the piece **GHCL** is 9 square units. The points **E**, **F**, **G**, and **H** are on the same straight line, and the line segment **FG** is 5 units long.

What is the area of the rectangular section **ABCD**?

Solution

One way to calculate the area of rectangle **ABCD**, is to determine the lengths of its sides. We know the area of the square **AKFE** is 4 square units, and we know that the lengths of the sides of a square must be the same. So if the length of one side of a square is n, then the area of the square must be $n \times n$. By trial and error we can see that $2 \times 2 = 4$, so each side of the square **AKFE** must be 2 units. Similarly, since the area of the square **GHCL** is 9 square units, we can see that $3 \times 3 = 9$. So each side of the square **GHCL** must be 3 units.

Another way to determine the lengths of the sides of square **AKFE**, would be to start with 4 unit squares (using blocks or cut out of paper for example) and determine how to arrange them into a larger square. The only possible arrangement is a 2×2 square.

The opposite sides of a rectangle must be the same length, and the bottom of the square **AKFE** is on the same line as the top of square **GHCL**. This means the length side **AD** is equal to the sum of the lengths of the sides of the two squares. So the length of side **AD** is $2 + 3 = 5$ units. Also, since the length of line segment **FG** is 5 units, the length of line **EH** must be $2 + 5 + 3 = 10$ units. The length of this line is the same as the length of the side **AB** or the side **DC**.

Now we can calculate the area of rectangle **ABCD**. The area of this rectangle is the product of the length of side **AD** and the length of side **AB**. So the area of rectangle **ABCD** is $5 \times 10 = 50$ square units.

Teacher's Notes

Exploring the areas of rectangles or squares is a good way to practice multiplication of positive numbers. The result of calculating $a \times b$ is equivalent to calculating the area of a rectangle with side lengths a and b.

Similarly, exploring the relationship between the area of a square and the lengths of its sides is a good way to learn about *square roots*. However, this exploration only produces half of the answer.

A square root of a number is defined as follows:

$$\text{If } y \text{ is a square root of } x\text{, then } y \cdot y = x.$$

Since the product of two negative numbers is positive, the value of y could be positive or negative. So the square root of 9 is either 3 or -3. When we use the radical symbol, $\sqrt{}$, we are looking for the principal (or positive) square root. Although -3 and 3 are both square roots of 9, the $\sqrt{9} = 3$.

If x is a *perfect square* (i.e. it is the the result of multiplying a rational number by itself), then the result of calculating $\sqrt{x}$ is also a rational number. A *rational number* is any number that can be represented as a fraction $\frac{a}{b}$, where a and b are integers, and b is not 0.

For example, $\sqrt{144}$ is exactly 12. There are also non-integers values that have rational square roots. For example, $\sqrt{\frac{9}{16}}$ is exactly $\frac{3}{4}$.

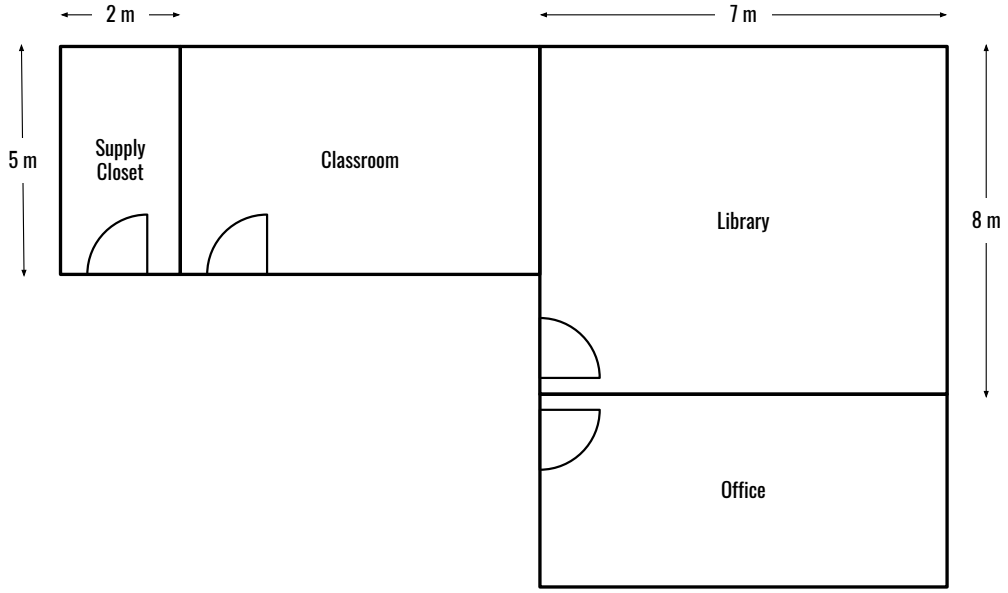
There are many numbers, however, that are not perfect squares. Square roots of these numbers are known as *irrational numbers*. An irrational number cannot be represented precisely as a fraction. If we try to write an irrational number in decimal form, we end up with a result where the digits after the decimal place never end, and never repeat a pattern. So, we cannot compute the square root of an irrational number on a calculator exactly. The result will appear as a limited number of digits after the decimal place. The calculator rounds the answer to a fixed number of decimal places.

Problem of the Week

Problem A

School Expansion

Emmy Noether Public School needs to add a new wing to its main building. The contractor has the plans for the addition, but some measurements were missing. The new rooms are rectangular and are connected as shown.



The good news is that the contractor remembered some extra details.

- When describing the dimensions of a rectangle, the contractor uses *length* to describe the longer side, and *width* to describe the shorter side.
- The width of the classroom is the same as the length of the supply closet.
- The length of the classroom is three times longer than the width of the supply closet.
- The width of the library and the length of the office are the same.
- The width of the office is half the length of the library.

The contractor needs to order tiles for the new rooms.

A) What are the missing dimensions of the classroom and the office?

B) What is the total area needed to be covered by the tiles?

Strands Measurement, Number Sense and Numeration

Problem of the Week
Problem A and Solution
School Expansion

Problem

Emmy Noether Public School needs to add a new wing to its main building. The contractor has the plans for the addition, but some measurements were missing. The new rooms are rectangular and are connected as shown.

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- The width of the office is half the length of the library.

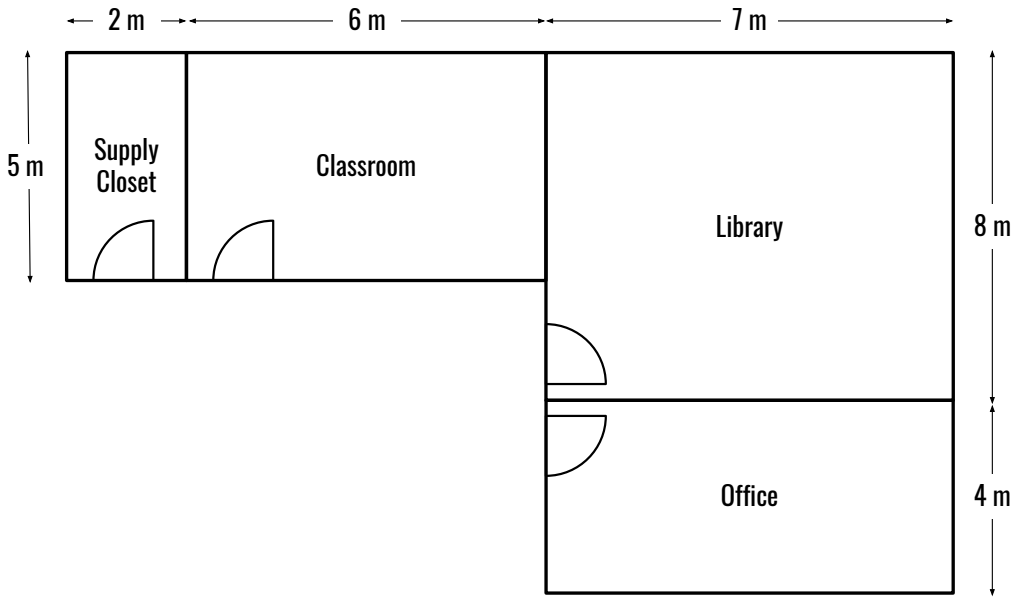
The contractor needs to order tiles for the new rooms.

A) What are the missing dimensions of the classroom and the office?

B) What is the total area needed to be covered by the tiles?

Solution

A) Since the classroom length is three times longer than the supply closet width, its length is $2 \times 3 = 6$ m. Since the office width is half the length of the library, its width is $8 \div 2 = 4$ m. Here is a diagram with all of the dimensions:



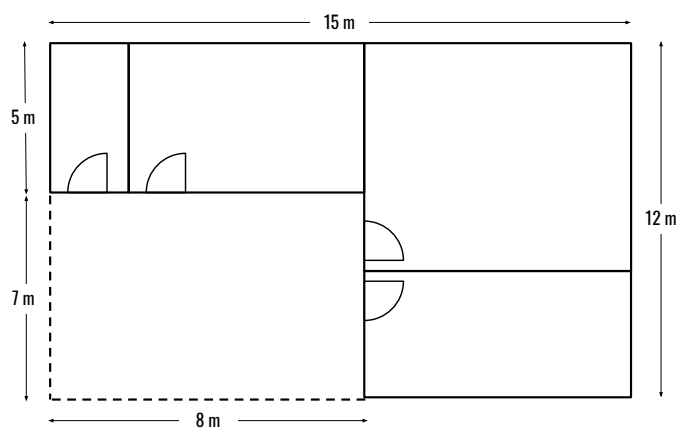
B) There are many ways to calculate the area needed to be covered. One way is to calculate the area of each room and add those numbers together.

- The area of the supply closet is $5 \times 2 = 10$ m^2.
- The area of the classroom is $5 \times 6 = 30$ m^2.
- The area of the library is $7 \times 8 = 56$ m^2.
- The area of the office is $7 \times 4 = 28$ m^2.

So the total area the contractor needs to tile is $10 + 30 + 56 + 28 = 124$ m^2.

Another way to find the total area is to calculate the area classroom based on the area of the supply closet and to calculate the office based on the area of the library. Since the length of the classroom is three times longer than the supply closet width, but the other dimensions are the same, the area of the classroom is three times the area of the supply closet. We can imagine that we could put three closets side-by-side and they would fill in the same area as the classroom. This means that the area of the classroom is $10 \times 3 = 30$ m^2. Similarly we can imagine that the area of the office would fill half of the area of the library. This means that the area of the office is $56 \div 2 = 28$ m^2.

A third way we could calculate the area that needs to be covered in tiles is to square off the diagram, as shown below.



The length across the top is $2 + 6 + 7 = 15$ m. The width along the side is $8 + 4 = 12$ m. The total area of that rectangle is $15 \times 12 = 180$ m^2.

However, there is a section of that rectangle that is not part of the building and hence needs no tiles. That section has a length equal to $2 + 6 = 8$ m and a width equal to $12 - 5 = 7$ m. The section of the diagram with no tile has an area of $8 \times 7 = 56$ m^2.

So the tiled area is the difference between these the areas of these two rectangles: $180 - 56 = 124$ m^2.

Teacher's Notes

Throughout most of history women were not actively encouraged to pursue academia, especially in the fields of mathematics and science. Emmy Noether (the namesake of the school in this problem) was born in Germany in 1882. Her father was a mathematician and she decided to follow in his footsteps. At the time, it was very difficult for a women to study at a university, but Noether was allowed to audit some courses at the University of Göttingen with the permission of the professor. One of the classes she attended was taught by David Hilbert, who was one of the most influential mathematicians of his time. Hilbert and others encouraged Noether in her academic pursuits. After completing her Ph.D., despite objections by some faculty, Neother taught classes and continued her research at the University of Göttingen. However she was only allowed to teach classes that were officially listed under Hilbert's name. Based on the work she did throughout her career, Emmy Noether is recognized as one of the leading mathematicians of the early 20th century.

During the time that she was teaching at the University of Göttingen, Maria Göppert became a student of mathematics and physics. After marrying her husband, Joseph Edward Mayer, she moved to the United States. Like Noether, Göppert-Mayer had restrictions on her earning potential as a researcher since she was a woman. However, she continued her academic pursuits at institutions such as Johns Hopkins, Columbia University and the University of Chicago. Among her other accomplishments, Göppert-Mayer was a member of the Manhattan Project - the group that developed the first nuclear weapons for the United States. Eventually, Göppert-Mayer became the second woman to receive the Nobel Prize in Physics for research she did after World War II.

As of 2019, only three women have received a Nobel Prize in Physics: Marie Currie, Maria Göppert-Mayer, and Donna Strickland. Strickland received her prize in 2018 (more than 50 years after Göppert-Mayer) for research in the field of lasers. In her Ph.D. thesis, Strickland cited the research of Göppert-Mayer's Ph.D. thesis.

Sources:

- `https://en.wikipedia.org/`
- `https://www.britannica.com/`

Problem of the Week

Problem A

Walking With Walden

Lindsay takes her dog Walden for a walk every day.

- On Monday she took him for a 45 minute walk.
- On Tuesday, they walked for 5 minutes longer than on Monday.
- On Wednesday, they walked for half as much time as Tuesday.
- On Thursday, they walked for 10 minutes less than Monday.
- On Friday, they walked for 15 minutes more than Thursday.
- On Saturday, they walked for twice as long as Monday.

A) How many minutes did Lindsay spend walking Walden between Monday and Saturday?

B) Lindsay wants to make sure that Walden is getting enough exercise. How long must Walden's walk be on Sunday to guarantee he walks for at least 6 hours this week?

Strands Number Sense and Numeration, Measurement

Problem of the Week
Problem A and Solution
Walking With Walden

Problem

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- On Wednesday, they walked for half as much time as Tuesday.
- On Thursday, they walked for 10 minutes less than Monday.
- On Friday, they walked for 15 minutes more than Thursday.
- On Saturday, they walked for twice as long as Monday.

A) How many minutes did Lindsay spend walking Walden between Monday and Saturday?

B) Lindsay wants to make sure that Walden is getting enough exercise. How long must Walden's walk be on Sunday to guarantee he walks for at least 6 hours this week?

Solution

A) First we need to determine how long Lindsay walked each day.

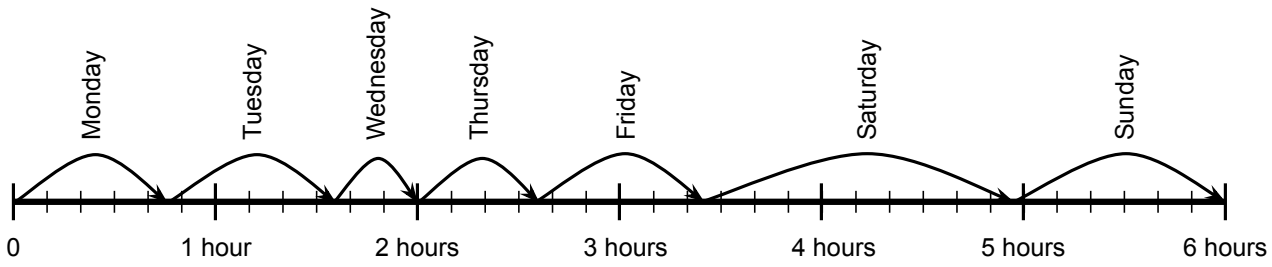
- On Tuesday, they walked for $45 + 5 = 50$ minutes.
- On Wednesday, they walked for $50 \div 2 = 25$ minutes.
- On Thursday, they walked for $45 - 10 = 35$ minutes.
- On Friday, they walked for $35 + 15 = 50$ minutes.
- On Saturday, they walked for $45 \times 2 = 90$ minutes.

The total number of minutes Lindsay walked from Monday to Saturday is:

$45 + 50 + 25 + 35 + 50 + 90 = 295$ minutes.

B) One way to determine how long Lindsay must walk on Sunday is to start by calculating how many minutes there are in 6 hours. Since there are 60 minutes in 1 hour, there are $60 \times 6 = 360$ minutes in 6 hours. Since Lindsay has already walked for 295 minutes, she needs to walk $360 - 295 = 65$ minutes more on Sunday. We could also say that she needs to walk 1 hour and 5 minutes more.

Another way to calculate the required time is to use a timeline that keeps track of how long Walden has been walked during the week:



From the timeline we can see that, by the end of Saturday, Lindsay and Walden have walked 5 minutes short of 5 hours. So, to guarantee a total of 6 hours of exercise, they must walk at least 1 hour and 5 minutes (or 65 minutes) on Sunday.

Teacher's Notes

All of the walking times for the days Tuesday through Sunday can be written as mathematical expressions in terms of the time Lindsay walks on Monday. To do this, we need to interpret the words of the problem into mathematical operations. Suppose we say that m is the number of minutes Lindsay walks on Monday. Then the following expressions describe the other times:

Tuesday: $m + 5$
We determine this by recognizing that *longer* indicates addition.

Wednesday: $(m + 5) \div 2$ or $\frac{1}{2} \cdot (m + 5)$
We determine this by starting with the expression describing Tuesday's walking time, and recognizing that *half* indicates division by 2 or multiplication by $\frac{1}{2}$.

Thursday: $m - 10$
We determine this by recognizing that *less* indicates subtraction.

Friday: $(m - 10) + 15$
We determine this by starting with the expression describing Thursday's walking time, and recognizing that *more* indicates indicates addition.

Saturday: $2 \cdot m$
We determine this by recognizing that *twice* indicates multiplication by 2.

Now the total walking time on Monday through Saturday can be written as:
$m + (m + 5) + (\frac{1}{2} \cdot (m + 5)) + (m - 10) + ((m - 10) + 15) + (2 \cdot m)$

This expression can be simplified. Here is an equivalent expression:
$6 \cdot m + (\frac{1}{2} \cdot (m + 5))$

We can use this expression to calculate the walking time on Sunday.
Sunday: $360 - (6 \cdot m + (\frac{1}{2} \cdot (m + 5))$

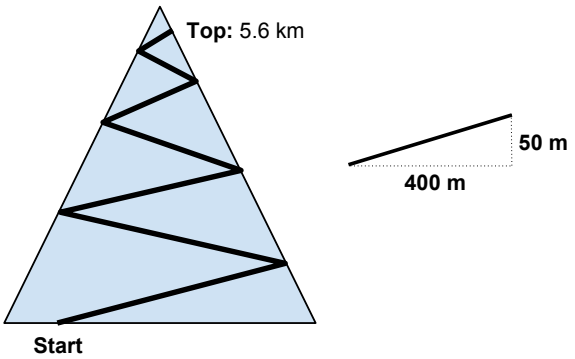
Given any value for the time Walden walks on Monday, we can calculate the times he walks on the other days. With these mathematical expressions, we could use a computer program or a spreadsheet to help calculate these values in general. To solve this particular problem, we would set the variable m to have the value 45.

Problem of the Week

Problem A

Climbing Sulphur Mountain

Climbing a mountain is tough. To make it easier and safer, people build paths that go back and forth across the mountain rather than straight up. This means that after walking for some distance along the path, you will come to a sharp corner, called *switchback*, and start walking in the opposite direction. So when you are hiking on this kind of trail, you will travel much further than than if you climbed straight up from the bottom to the top of the mountain. On average, for every 400 m you walk on the trail to the top of Sulphur Mountain, you are climbing approximately 50 m vertically up the mountain. Here is a diagram representing the trail:



The distance you travel when you walk along the trail from the start to the top is 5.6 km. What is the vertical distance from the bottom of Sulphur Mountain to the top? Justify your answer.

Strands Patterning and Algebra, Measurement, Number Sense and Numeration

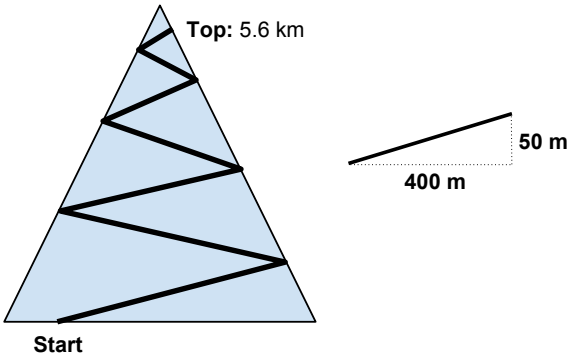
Problem of the Week

Problem A and Solution

Climbing Sulphur Mountain

Problem

Climbing a mountain is tough. To make it easier and safer, people build paths that go back and forth across the mountain rather than straight up. This means that after walking for some distance along the path, you will come to a sharp corner, called *switchback*, and start walking in the opposite direction. So when you are hiking on this kind of trail, you will travel much further than than if you climbed straight up from the bottom to the top of the mountain. On average, for every 400 m you walk on the trail to the top of Sulphur Mountain, you are climbing approximately 50 m vertically up the mountain. Here is a diagram representing the trail:



The distance you travel when you walk along the trail from the start to the top is 5.6 km. What is the vertical distance from the bottom of Sulphur Mountain to the top? Justify your answer.

Solution

One way to solve this problem is to make a table that keeps track of far you have travelled on the trail along with the vertical distance you have climbed.

Each time the trail distance increases by 400 m, the vertical distance increases by 50 m. We want the table to show the pattern until the trail distance is equal to 5.6 km. Since 1 km = 1000 m, the table needs to show the pattern until the trail distance is 5600 m.

Here is a table showing the pattern:

Trail Distance (in metres)	Vertical Distance (in metres)
400	50
800	100
1200	150
1600	200
2000	250
2400	300
2800	350
3200	400
3600	450
4000	500
4400	550
4800	600
5200	650
5600	700

So after walking 5.6 km on the trail, you have travelled approximately 700 m metres vertically up the mountain.

Teacher's Notes

The pattern shown in this problem can be described in the form of an equation of a line. In Cartesian geometry, the equation of a line can be written with this format:

$$y = mx + b$$

where m is defined as the *slope* of the line, and b is defined as the *y-intercept* of the line. The y-intercept describes the point on the line where it crosses the y-axis. In other words, it is the point on the line that satisfies the equation when $x = 0$.

The slope of a line describes a constant relationship between any two points on the line. By definition, if we choose two points that are on the same line: (x_1, y_1) and (x_2, y_2), and compare the ratio of the difference of the y values and the difference of the x values, that ratio will always be the same. We normally work with the rise (the difference in the y values) over the run (the difference in the x values), and call this value the slope. In particular:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

There are two special cases we should consider. Since lines extend infinitely, all lines will cross the y-axis at some point, unless they are parallel to the y-axis. In this case, it is not possible to find a y-intercept. For a line that is parallel to the y-axis, each point on the line will always have the same value of x. We could write the equations of lines like this in the form: $x = c$ where c is a number. The slope of a line that is parallel to the y-axis is undefined since we cannot divide by zero.

Another special case to consider are lines that are parallel to the x-axis. For these lines, each point on the line will always have the same y value. We could write the equations of lines like this in the form: $y = b$. The slope of a line that is parallel to the x-axis is equal to 0.

This problem literally describes a slope. We can think of the vertical distance we travel as being the y value, and the actual distance we walk along the trail as being the x value. The constant ratio in this case is:

$$m = \frac{rise}{run} = \frac{50}{400} = \frac{1}{8}$$

We can think of the start of the trail as being when $x = 0$ and $y = 0$. This means the y-intercept of the line describing this pattern is 0. So an equation describing the relationship between the trail distance and the vertical distance is:

$$y = \frac{1}{8}x$$

To solve this problem, we want to know the value of y when $x = 5600$.

By substitution, $y = \frac{1}{8}(5600)$ or $y = 400$. So the vertical change is 400 m.

Problem of the Week

Problem A

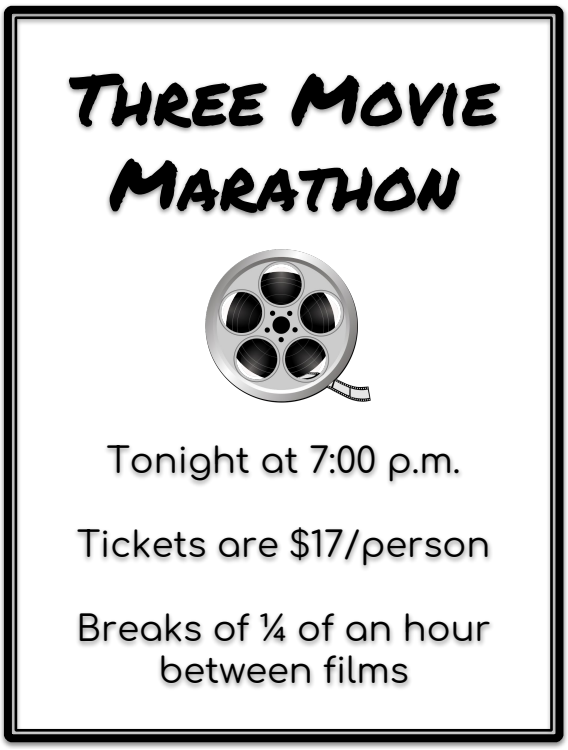
Midnight Movie Madness

Mr. and Mrs. Pretti decided that they wanted to go and watch a special three-movie marathon at the local theatre. They purchased their tickets for the marathon for $17 per person. They called their favourite babysitter to watch their kids.

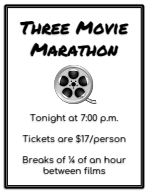
Each movie is 125 minutes long. The movie theatre has promised a break of a quarter of an hour between each movie. The drive to the theatre is 30 minutes, and the marathon starts at 7 p.m. The babysitter arrives at 6:15 p.m.

A) At approximately what time will Mr. and Mrs. Pretti arrive back home?

B) The Prettis pay their babysitter $7.50 per hour. How much will the entire night cost them?



Strands Number Sense and Numeration, Measurement, Patterning and Algebra



Problem of the Week

Problem A and Solution

Midnight Movie Madness

Problem

Mr. and Mrs. Pretti decided that they wanted to go and watch a special three-movie marathon at the local theatre. They purchased their tickets for the marathon for $17 per person. They called their favourite babysitter to watch their kids.

Each movie is 125 minutes long. The movie theatre has promised a break of a quarter of an hour between each movie. The drive to the theatre is 30 minutes, and the marathon starts at 7 p.m. The babysitter arrives at 6:15 p.m.

A) At approximately what time will Mr. and Mrs. Pretti arrive back home?

B) The Prettis pay their babysitter $7.50 per hour. How much will the entire night cost them?

Solution

A) Here is one way to determine when the Prettis will arrive home. Each movie lasts 125 minutes. There are two breaks, one between the first and second movie and one between the second and third movie. Each break is $\frac{1}{4}$ of an hour which is equal to 15 minutes. It will take the Prettis 30 minutes to drive home after the last film is done. The total time between when the first movie starts at 7:00 p.m. and when they arrive home is:

$125 + 15 + 125 + 15 + 125 + 30 = 435$ minutes.

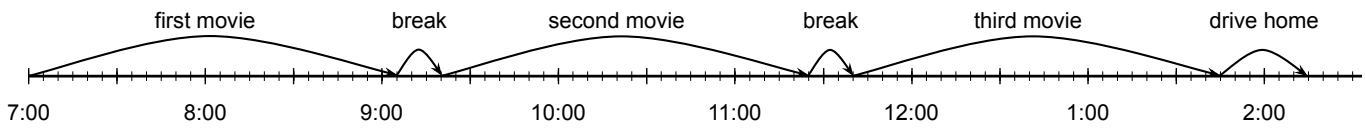
The following pattern shows the relationship between hours and minutes:

Hours	Minutes
1	60
2	120
3	180
4	240
5	300
6	360
7	420

So 435 minutes is $435 - 420 = 15$ minutes more than 7 hours.

There are 5 hours between 7:00 p.m. and midnight. This means Mr. and Mrs. Pretti would get home 2 hours and 15 minutes after midnight or 2:15 a.m.

We could also use a timeline to determine what time the Prettis arrive back home.



B) The babysitter arrived at 6:15 which is 45 minutes before 7:00 p.m. This means the Prettis need to pay for a total of $435 + 45 = 480$ minutes of time. If we continue the pattern from part A), 8 hours = 480 minutes. Now we can find a pattern between the number of hours and the amount the babysitter is paid.

Hours	**Total Paid (in \$)**
1	7.50
2	15.00
3	22.50
4	30.00
5	37.50
6	45.00
7	52.50
8	60.00

Since the Prettis needed to buy two tickets for the movie marathon, this cost $2 \times \$17 = \$34$. The total cost of the evening out is $\$34 + \$60 = \$94$.

Teacher's Notes

The solution for part B) converts minutes to hours to determine how much to pay the babysitter. Here is another way to solve that problem. We know that 480 minutes passed from the time that the babysitter arrived until the time that the Prettis returned home. Rather than converting this time to hours, we could determine how much the babysitter gets paid per minute rather than per hour.

A rate of \$7.50 per hour is equal to 750 cents per 60 minutes. We can determine the rate per minute as $750 \div 60 = 12.5$ cents per minute.

So for 480 minutes, the babysitter should be paid $480 \times 12.5 = 6000$ cents. This is equal to \$60.00.

Problem of the Week

Problem A

Carpet Caper

Bill and Nikki were renovating a basement apartment together. They removed all the carpet to replace it with new flooring. After removing the carpet they took it to the landfill where you pay for the mass of your garbage. At the landfill, they were told that the mass of the carpet was 290 kg. Each square metre of carpet has a mass of 5 kg.

A) How many square metres of carpet did Bill and Nikki remove?

B) They are replacing the carpet with laminate flooring. Each box contains 3 square metres of laminate flooring. How many boxes of flooring do they need?

Strands Measurement, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Carpet Caper

Problem

Bill and Nikki were renovating a basement apartment together. They removed all the carpet to replace it with new flooring. After removing the carpet they took it to the landfill where you pay for the mass of your garbage. At the landfill, they were told that the mass of the carpet was 290 kg. Each square metre of carpet has a mass of 5 kg.

A) How many square metres of carpet did Bill and Nikki remove?

B) They are replacing the carpet with laminate flooring. Each box contains 3 square metres of laminate flooring. How many boxes of flooring do they need?

Solution

A) Here is one way to calculate the area of the carpet that was removed.
Since each square metre of carpet has a mass of 5 kg, then 2 square metres has a mass of $2 \times 5 = 10$ kg. In other words, each 10 kg of the mass comes from 2 square metres of the carpet.
The total mass of the carpet is $29 \times 10 = 290$ kg.
So the total area of the carpet is $29 \times 2 = 58$ square metres.

B) Knowing that Bill and Nikki need at least 58 m^2 of laminate flooring, and that each box contains 3 m^2 of flooring, then we can use division to determine how many boxes they need. We calculate $58 \div 3 = 19$ with a remainder of 2. Since they cannot buy a part of a box, then Bill and Nikki will need to buy 20 boxes of flooring.

Teacher's Notes

In part B) of this question, you were required to determine the number of boxes of laminate flooring. Since you cannot purchase part of a box, in this situation you need to calculate the number of boxes that will cover **at least** 58 square metres. The correct result can be determined by rounding up the result of the division. The action of rounding up can be represented in a mathematical expression using the *ceiling* notation. In this case the actual answer could be calculated this way:

$$\lceil 58 \div 3 \rceil = 20$$

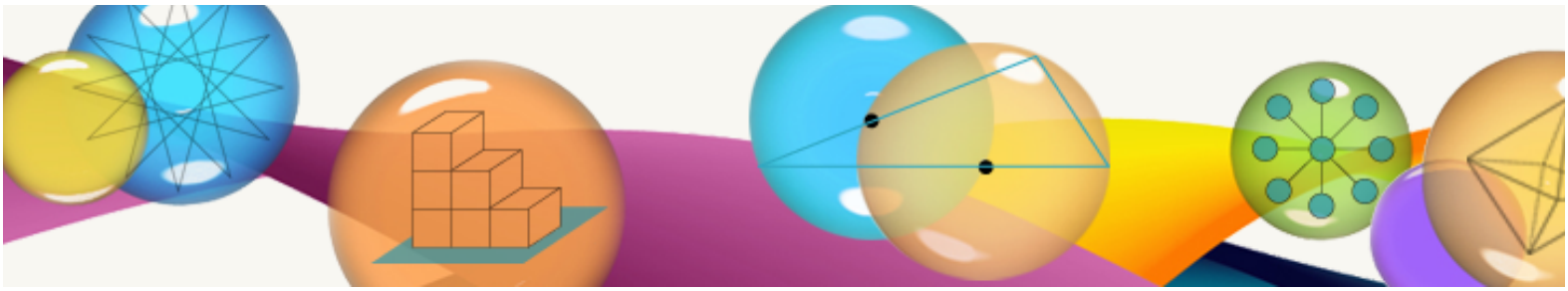
The result of a division calculation could be an integer or a non-integer value. However, the result of calculating the ceiling is always an integer value. In the case where the result of the division is an integer, then the ceiling of the division is equal to the quotient.

For example,

$$30 \div 5 = \lceil 30 \div 5 \rceil = 6$$

However,

$$58 \div 3 \neq \lceil 58 \div 3 \rceil$$



Number Sense & Numeration

TAKE ME TO THE COVER

Problem of the Week

Problem A

Age Action

Marty is ten years old. His grandfather is seven times his age. Marty's mother is half the age of his grandfather.

A) How old is Marty's mother?

B) How many decades old is Marty's grandfather?

Strands Number Sense and Numeration, Measurement

Problem of the Week

Problem A and Solution

Age Action

Problem

Marty is ten years old. His grandfather is seven times his age. Marty's mother is half the age of his grandfather.

A) How old is Marty's mother?

B) How many decades old is Marty's grandfather?

Solution

A) Since Marty's grandfather is seven times his age, his grandfather's age is $7 \times 10 = 70$ years. Since Marty's mother is half his grandfather's age, his mother's age is $70 \div 2 = 35$ years.

B) Since one decade is 10 years, Marty's grandfather is $70 \div 10 = 7$ decades old.

Teacher's Notes

When describing quantities, we have many choices for units of measurement. In this example, the grandfather's age was described in years and decades. However, we could have described Marty's age in weeks, day, hours, or even seconds. When we select a unit of measurement, we want to choose units that fit the value being measured. It is often easier for us to understand relatively small quantitative values. For example, we might describe a new baby as being five weeks old. However, we are unlikely to say something like "Marty is 536 weeks old".

We also choose units based on the precision we require in the measurement. For example, we would not describe the distance between the cities Halifax and Yellowknife in millimetres. But we do use millimetres to describe amounts of rainfall.

It is also important that we choose units that are well known to those with whom we are communicating. We might talk about a fortnight to a tennis fan. North American news reports describe some lengths in terms of a number of football fields. An equestrian would know if a horse that is 12 hands tall is big or small. However there are many people in the world who would not know that a fortnight is 14 days, an American football field is 100 yards, and a hand is 4 inches.

Problem of the Week

Problem A

Population Approximation

Ten years ago Anytown had a population of approximately 8000. Jessie knew the digits 3, 7, 8, 9 were part of the actual population at that time, but could not remember the order of the digits. What might the actual population of Anytown have been if each digit is used once?

Justify your answer.

Strand Number Sense and Numeration

Problem of the Week

Problem A and Solution

Population Approximation

Problem

Ten years ago Anytown had a population of approximately 8000. Jessie knew the digits 3, 7, 8, 9 were part of the actual population at that time, but could not remember the order of the digits. What might the actual population of Anytown have been if each digit is used once?

Justify your answer.

Solution

If we are looking for a number that is close to 8000, the best options would start with either an 8 or a 7. Using the digits $3, 7, 8, 9$ each exactly once we will find the number closest to 8000 but greater, and then find the number closest to 8000 but smaller.

Closest Number to 8000 but greater

For numbers that start with an 8, the smallest value would have the hundreds digit as small as possible, the tens digit would be the next smallest number, and ones digit would be the largest number. This means that the number starting with 8 that is closest to 8000 but greater and uses each of the digits 3, 7, 8, 9 is 8379. This number is almost 400 more than 8000.

Closest Number to 8000 but smaller

For numbers that start with a 7, the largest value would have the hundreds digit as large as possible, the tens digit would be the next largest number and the ones digit would be the smallest number. This means that the number starting with 7 that is closest to 8000 but smaller and uses the digits 3, 7, 8, 9 is 7983. This number is 17 less than 8000.

If we were looking for the number closest to 8000 that uses the digits 3, 7, 8, 9, we would select 7983.

However, we were not asked for the closest number to 8000 but rather a number that could be reasonably approximated by 8000. In one case, we could argue either of 7938 or 7983 since they are both within 100 of 8000.

We could also argue the numbers 7839 or 7893 or 7938 or 7983 or 8379 or 8397 since each is within 400 of 8000 and could be reasonably rounded to 8000.

Each of the above arguments could act as justification for your answer.

Teacher's Notes

The numbers 7983, 7938, and 8379, or even 8397, 7893 and 7839 are all reasonable choices as a population of Anytown, since the result of rounding these numbers to the nearest 1000 gives us 8000 in each case. As quantitative values get bigger, we tend to care less about the precise digits of those values, and are interested mostly in the most *significant digits*.

Calculators are only able to display a limited number of digits. If the calculated values get too big or too small they are usually shown in *scientific notation* or *E-notation*. Numbers written in this format show some number of digits multiplied by a power of 10. For example, the number of atoms in a mole is approximately 6.022×10^{23}, and the mass of an electron is approximately 9.10938×10^{-31} kg. The precision of a number written in scientific notation is determined by the number of significant digits. The mole size has four significant digits and the mass has six significant digits.

In some situations we might not care about precision at all. We may want to have a general sense of the size of some measured value in terms of its *order of magnitude*. In these cases, we are interested in comparing values to the closest power of 10. For example, consider the population of Austria (approximately 8.7 million in 2016) and the population of India (approximately 1.3 billion in 2016). We are unlikely interested in comparing the differences in these numbers by subtracting them. We can say that the order of magnitude difference between the population of India and Austria is 2. This means that the population of India is approximately 10^2 or 100 times that of the population of Austria. However, if we compare the population of Canada (approximately 36 million in 2016) to the population of the United States (approximately 323 million in 2016) we could say that the order of magnitude difference between the population of the United States and Canada is 1. These descriptions give us a broad sense of the relative sizes of the populations.

Problem of the Week

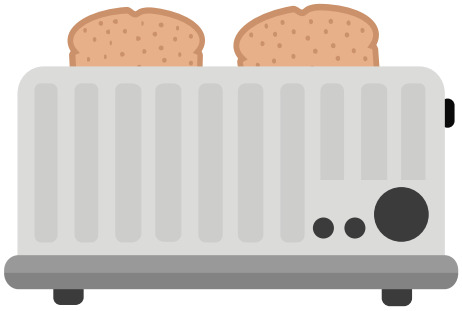
Problem A

Breakfast Food

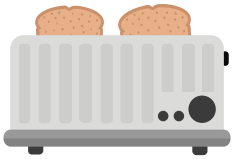
For breakfast, Liz always has a drink, some yogurt, and toast. She likes to drink milk, water, or juice. She likes strawberry, blueberry, raspberry, or vanilla yogurt. She likes whole grain or pumpernickel toast. Liz would like a different combination of a drink, yogurt, and toast every day.

A) List all the different breakfast combinations she could have where she does not drink juice.

B) What is the maximum number of days that will pass before she will have to eat and drink exactly the same combination as a previous breakfast?



Strand Number Sense and Numeration



Problem of the Week
Problem A and Solution
Breakfast Food

Problem

For breakfast, Liz always has a drink, some yogurt, and toast. She likes to drink milk, water, or juice. She likes strawberry, blueberry, raspberry, or vanilla yogurt. She likes whole grain or pumpernickel toast. Liz would like a different combination of a drink, yogurt, and toast every day.

A) List all the different breakfast combinations she could have where she does not drink juice.

B) What is the maximum number of days that will pass before she will have to eat and drink exactly the same combination as a previous breakfast?

Solution

A) We can create a table to determine all the possible combinations.

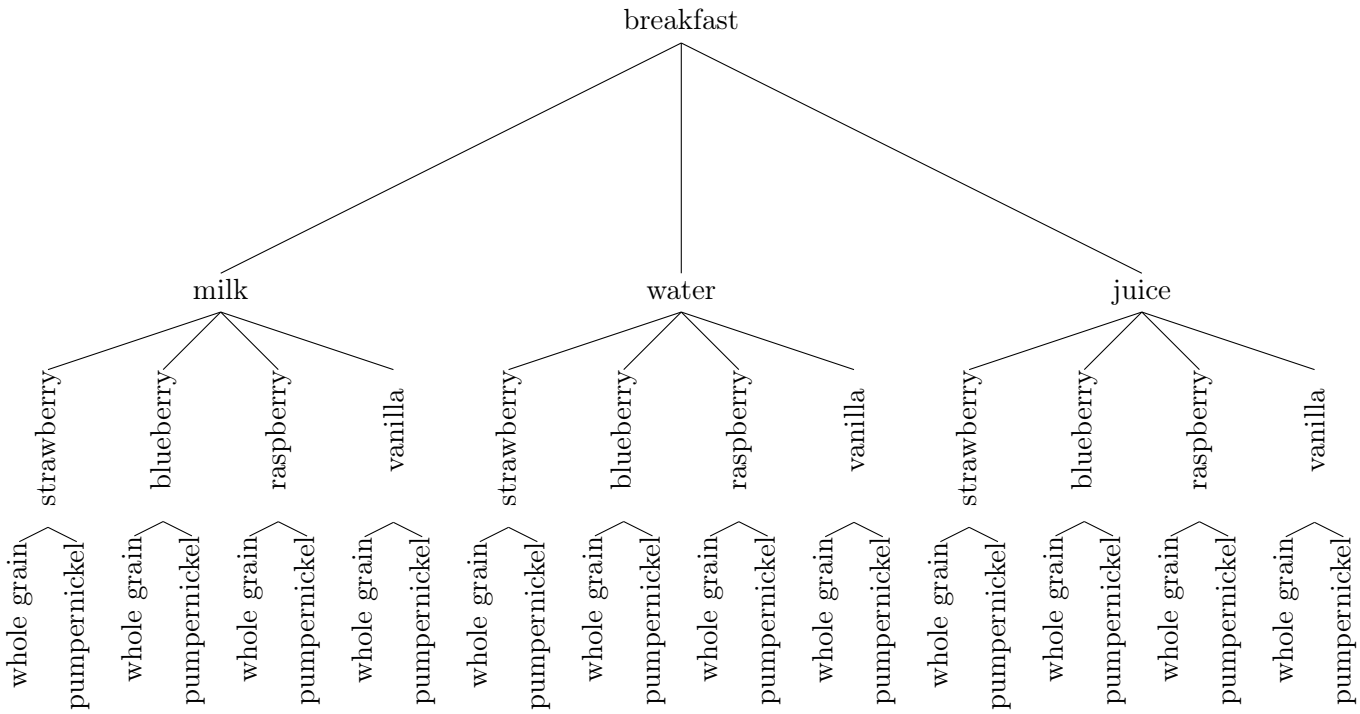
Drink	Yogurt	Toast
milk	strawberry	whole grain
milk	strawberry	pumpernickel
milk	blueberry	whole grain
milk	blueberry	pumpernickel
milk	raspberry	whole grain
milk	raspberry	pumpernickel
milk	vanilla	whole grain
milk	vanilla	pumpernickel
water	strawberry	whole grain
water	strawberry	pumpernickel
water	blueberry	whole grain
water	blueberry	pumpernickel
water	raspberry	whole grain
water	raspberry	pumpernickel
water	vanilla	whole grain
water	vanilla	pumpernickel

Notice the patterns in the table. For example, the eight combinations of yogurt and toast are duplicated, first in combination with milk and then in combination with water.

B) We could extend the table from the previous page to include the juice choice. However, we observe that the eight combinations of yogurt and toast would be repeated for each of the three drink options (milk, water and juice). We then calculate the number of distinct combinations by multiplying: $3 \times 8 = 24$.

We could also look at this calculation by noting that there are 3 choices for a drink. For each of these 3 choices, there are 4 choices of yogurt. So there are $3 \times 4 = 12$ choices of drink and yogurt. And for each of these 12 choices, there are 2 choices of toast. So there are $3 \times 4 \times 2 = 24$ choices of drink, yogurt and toast.

Another way to view the breakfast combinations would be with a tree diagram. This tree shows all of the variations that Liz could have.



Both the calculation and the tree show that there are 24 different breakfast combinations that Liz could have. So at most 24 days could pass before Liz must eat a breakfast combination that she has previously eaten.

Teacher's Notes

The second approach to solving the problem used a *tree* to organize all of the possible breakfast combinations. A tree is a well defined structure used in a field of mathematics known as *graph theory* and is often used in computer science to manage data.

The data in the tree is contained in *nodes* also known as *vertices*. Nodes are connected by *branches* also known as *edges*. This particular tree has a *root*, which is essentially a starting point. In this case, the root is the node that contains the word "breakfast". The nodes at the bottom of the tree are known as *leaf nodes* since they do not have any branches below them. You can follow a *path* from the root node of the tree to any of its nodes. There is only one possible path you can follow to get from the root to any particular node. The paths that we follow from the root of this tree to each of its leaf nodes describe the breakfast combinations that Liz might eat. For example, if you start at the root you could take the following path:

$$\text{breakfast} \rightarrow \text{water} \rightarrow \text{blueberry} \rightarrow \text{pumpernickel}$$

Trees can be used in all sorts of applications. For example, trees can be used to show relationships in a hierarchy or for making logical decisions or for enumerating combinations or for searching efficiently, to name a few possibilities.

Problem of the Week

Problem A

Caring Community

The local community centre is organizing a food drive so that they can provide meals for those in need. Three volunteers have offered to gather food items in bulk. Lance collected crates that have 8 cans of tuna in each. Andre has bins that contain 4 bags of pasta in each. Amal gathered cartons with 6 boxes of oatmeal in each.

The team wants to provide food hampers for 24 families. Each hamper will have three cans of tuna, one bag of pasta, and two boxes of oatmeal. How many crates of canned tuna, bins of pasta, and cartons of oatmeal will each volunteer need to bring?

Jusify your answer.

Strand Number Sense and Numeration

Problem of the Week

Problem A and Solution

Caring Community

Problem

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Jusify your answer.

Solution

The first step is to calculate the number of each item we need to supply the 24 families. We need $24 \times 3 = 72$ cans of tuna, $24 \times 1 = 24$ bags of pasta and $24 \times 2 = 48$ boxes of oatmeal.

We can use division to determine how many of the bulk containers we need to satisfy the needs of the food drive.

- Since each crate has 8 cans of tuna, we will need $72 \div 8 = 9$ crates to provide enough tuna.
- Since each bin contains 4 bags of pasta, and each family will be given one bag, we will need $24 \div 4 = 6$ bins of pasta.
- Since each carton contains 6 boxes of oatmeal, we will need $48 \div 6 = 8$ cartons of oatmeal.

Teacher's Notes

The numbers in this problem were carefully chosen. The number of families (24) is a multiple of all of the other numbers in the problem (1, 2, 3, 4, 6, 8). By choosing a number of families that is a multiple of all of the other numbers, we are guaranteed that there will not be any leftovers in the crates, bins, and cartons that we gather.

The number 24 is actually the smallest positive integer that is a multiple of 1, 2, 3, 4, 6, and 8. We call such a number a *least common multiple* or *LCM*. One way to calculate the LCM of a set of numbers it to start by finding the prime factorization of the numbers in the set. In this case, 2 and 3 are prime numbers themselves. The prime factorizations of the rest of the numbers are:

$$4 = 2 \times 2$$
$$6 = 2 \times 3$$
$$8 = 2 \times 2 \times 2$$

Next we identify the maximum number of times that each prime number appears in any single factorization. The only prime numbers that appear in the factorizations of these numbers are 2 and 3. The number 2 appears a maximum of three times, and the number 3 appears only once. To find the LCM, we multiply the prime factors together, assuring that we include maximum number of each factor appearing in each individual factorization. So in this example, we need to multiply $2 \times 2 \times 2 \times 3 = 24$ to find the LCM.

One of the uses of the LCM is finding a common denominator when adding or subtracting fractions.

Problem of the Week

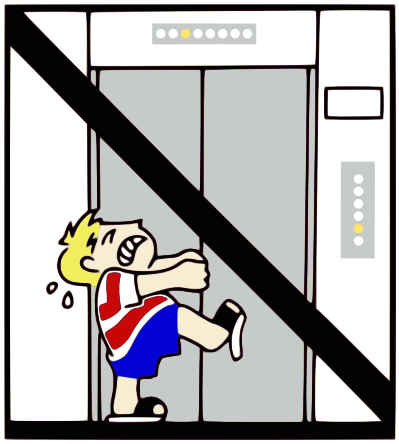
Problem A

Elevator

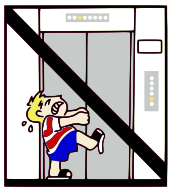
The small school elevator can hold a maximum weight of 227 kg.

A) The average weight of a 9 year old is 28 kg. How many 9 year olds could ride the school elevator safely?

B) The average adult weighs 73 kg. What is the maximum number of adults we expect could ride the school elevator?



Strands Patterning and Algebra, Number Sense and Numeration



Problem of the Week

Problem A and Solution

Elevator

Problem

The small school elevator can hold a maximum weight of 227 kg.

A) The average weight of a 9 year old is 28 kg. How many 9 year olds could ride the school elevator safely?

B) The average adult weighs 73 kg. What is the maximum number of adults we expect could ride the school elevator?

Solution

A) One way to solve this problem is to use a table to keep track of the total weight of the people on the elevator.

Number of Students	Weight (kg)
1	28
2	56
3	84
4	112
5	140
6	168
7	196
8	224
9	252

So the maximum number of 9 year olds that we expect can safely ride the elevator would be 8. Alternatively, we could use division. We can calculate $227 \div 28 = 8$ remainder 3. Since we are looking for a maximum number of riders, and the answer must be a whole number, then the maximum number would be 8.

B) Using a table or division ($227 \div 73 = 3$ remainder 9) we can determine that the maximum number of adults that we expect can safely ride the elevator would be 3. Alternatively, we could estimate the average weight of the adults as 70 kg and the capacity of the elevator as 230 kg. We can see that 3 adults would weigh approximately 210 kg, so we expect that is the maximum number that would safely fit in the elevator.

Teacher's Notes

In part A of this question, when we used division to calculate how many students would be able to safely ride the elevator we got an answer of 8 remainder 3. Since the answer is not a whole number, we need to decide if our final answer needs to be rounded up or rounded down. We need to look for cues in the question to help. In this case the final answer must be a whole number, and we are looking for a maximum capacity. This means we need to round down, since rounding up will exceed the capacity.

In mathematics, we use the operations *floor* and *ceiling* to indicate if our calculated value should be rounded down or rounded up. So for this problem, we would use the floor operation. Symbolically, this would be the calculation:

$$\lfloor 227 \div 28 \rfloor = 8$$

In other cases you will need to round up. For example, suppose you were buying eggs for use at a restaurant. You know that you need 500 eggs each day, and that a carton contains one dozen eggs. If you need to know how many cartons of eggs are required each day, the calculation could be done by using division. The answer will need to be rounded up (i.e. find the ceiling), since you need at least 500 eggs, but you need to purchase a whole number of cartons. Symbolically, this would be the calculation:

$$\lceil 500 \div 12 \rceil = 42$$

Problem of the Week

Problem A

Backpack Limit

Selma is going on a trip with her family. The airline allows each person's carry-on luggage to have a maximum mass of 10 kg. The clothes she wants to bring have a mass of 1.5 kg. She has three books that she wants to read. Two books are the same size, and each has a mass of 2 kg. The third book is 1500 g. She wants to bring her laptop and tablet as well. The laptop's mass is 2.3 kg. Her tablet's mass is 800 g. The shoes she needs for the trip are 1200 g. Her personal toiletries kit is 0.6 kg.

Can Selma bring everything she wants for the trip in her carry-on? Justify your answer.

Strands Measurement, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Backpack Limit

Problem

Selma is going on a trip with her family. The airline allows each person's carry-on luggage to have a maximum mass of 10 kg. The clothes she wants to bring have a mass of 1.5 kg. She has three books that she wants to read. Two books are the same size, and each has a mass of 2 kg. The third book is 1500 g. She wants to bring her laptop and tablet as well. The laptop's mass is 2.3 kg. Her tablet's mass is 800 g. The shoes she needs for the trip are 1200 g. Her personal toiletries kit is 0.6 kg.

Can Selma bring everything she wants for the trip in her carry-on? Justify your answer.

Solution

It would be easier to calculate the total mass of the objects Selma wants to take if the individual masses were all measured in the same units. If we choose grams as the unit, all the masses in kilograms must be multiplied by 1000. Here are the objects' masses all in grams:

clothes: 1500, first book: 2000, second book: 2000, third book: 1500,
laptop: 2300, tablet: 800, shoes: 1200, toiletries kit: 600

The sum of these masses is:
$1500 + 2000 + 2000 + 1500 + 2300 + 800 + 1200 + 600 = 11900$ g

This is equal to 11.9 kg. According to the airline rules, this is too much weight for the carry-on luggage.

Teacher's Notes

Proper handling and conversion of units of measurement is very import in the real world. The metric system makes it easy to convert between units of the same category since you may simply multiply or divide by powers of 10 to do the conversions. However, not everyone in the world uses the metric system. Where different measurement systems are used, there may be problems.

In 1999, NASA sent a satellite to Mars that was intended to orbit the planet and send back climate information. The *Mars Climate Orbiter* was lost when its orbit became unstable and it burned up in the atmosphere surrounding Mars. The cause of the accident was human error. The force calculated in one part of the operating system was assumed to be in *pounds*, but in another part it was assumed to be in *newtons*. This incompatibility ended up with an incorrect amount of power given to the thrusters which led to the eventual destruction of the satellite. *(Source: wired.com)*.

In 1983, an Air Canada flight ran out of fuel on its way from Toronto to Edmonton. The flight specifications had indicated that the aircraft required an amount of fuel in kilograms. However, the ground crew pumped the amount given in pounds. Since one pound is less than one-half a kilogram, the flight had much less fuel than it required. Luckily the pilots landed the aircraft safely, with no engine power, at a closed air force base in Manitoba. *(Source: wikipedia.org)*

Problem of the Week

Problem A

Fall Fair Fare

Avneet is at the Fall Fair with her friends. They plan to go on 10 rides each. They are trying to figure out which is the best price to pay for admission. The fair has two options:

Option #1
- Admission: \$15
- Single ride ticket: \$3

Option #2
- Admission including unlimited ride wristband: \$35

A) Which option should Avneet choose? Justify your answer.

B) What is the most number of rides Avneet could plan to go on that would make Option #1 the better choice? Justify your answer.

Strands Number Sense and Numeration, Patterning and Algebra

Problem of the Week
Problem A and Solution
Fall Fair Fare

Problem

Avneet is at the Fall Fair with her friends. They plan to go on 10 rides each. They are trying to figure out which is the best price to pay for admission. The fair has two options:

Option #1
- Admission: \$15
- Single ride ticket: \$3

Option #2
- Admission including unlimited ride wristband: \$35

A) Which option should Avneet choose? Justify your answer.

B) What is the most number of rides Avneet could plan to go on that would make Option #1 the better choice? Justify your answer.

Solution

A) If Avneet chooses Option #1 and she goes on 10 rides, the ride tickets will cost $10 \times 3 = \$30$. Then the total cost for the fair with this option would be $15 + 30 = \$45$. Since Option #2 costs \$35, this would be the better choice.

B) One way to figure out this would be to make a table showing the number of rides and the cost of Option #1.

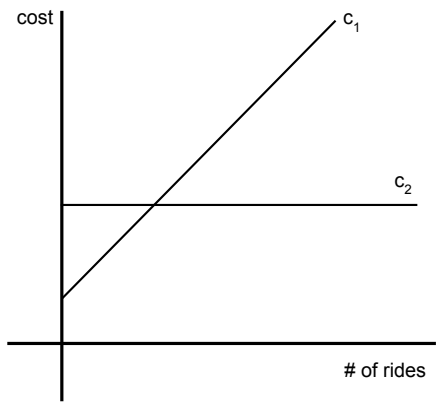
# of Rides	Total Cost (\$)
0	15
1	18
2	21
3	24
4	27
5	30
6	33
7	36

So the most number of rides you can take to make Option #1 the better choice would be 6. Alternatively, since we must pay \$15 for Option #1 no matter how many rides we take, and we must pay \$35 for Option #2 no matter how many rides we take, then the difference between these amounts describes how much of the admission price for Option #2 is covering the cost of the individual rides. This difference is $35 - 15 = \$20$. If we had \$20 for rides that cost \$3 each, we can figure out how many rides we can take by using division: $20 \div 3 = 6$ remainder 2. Since there would be \$2 unspent with Option #2 when we take 6 rides, then Option #1 would be a better choice.

Teacher's Notes

In this problem, we could describe these two options as mathematical functions.
Option #1 could be written as $c_1 = 3 \cdot r + 15$, where r is the number of rides a person takes.
Option #2 could be written as $c_2 = 35$.

If can describe options using mathematical functions, then we have the ability to analyze and compare those functions. In this case, the function describing Option #1 is a **linear** function. This means that the value of c_1 grows at approximately the same rate as the number of rides that Avneet takes. The function describing Option #2 is a **constant** function. This means that the value of c_2 is unaffected by the number of rides that Avneet takes. We can draw a graph showing the difference:



At some point, the linear function crosses the constant function. After that point, the cost of the linear function is always greater than the cost of the constant function.

In computer science, we often use broad categories to compare different functions. Generally, we expect a constant function to be more efficient than a linear function, although, as we see in this problem, a constant function is not necessarily better for **all** values. However, since we normally care about efficiency when dealing with large values, we usually ignore the small cases where the linear function performs better.

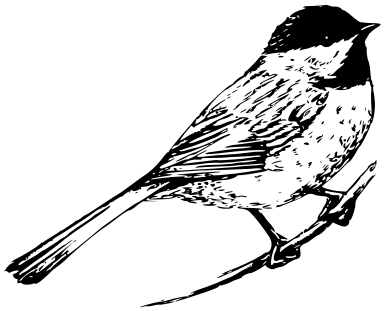
Problem of the Week

Problem A

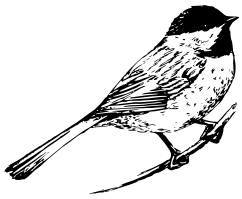
Watching Carefully

Steve is a birdwatcher. On one of his walks he saw a total of 28 birds. He saw twice as many chickadees as blue jays. He saw three fewer blue jays than woodpeckers. He saw one bald eagle and 10 chickadees. He also saw some herons.

How many herons, blue jays and woodpeckers did he see?



Strand Number Sense and Numeration



Problem of the Week

Problem A and Solution

Watching Carefully

Problem

Steve is a birdwatcher. On one of his walks he saw a total of 28 birds. He saw twice as many chickadees as blue jays. He saw three fewer blue jays than woodpeckers. He saw one bald eagle and 10 chickadees. He also saw some herons.

How many herons, blue jays and woodpeckers did he see?

Solution

- Since Steve saw twice as many chickadees as blue jays, he must have seen $10 \div 2 = 5$ blue jays.
- Since he saw three fewer blue jays than woodpeckers, he must have seen $5 + 3 = 8$ woodpeckers.
- The total number of chickadees, blue jays, woodpeckers, and bald eagles is $10 + 5 + 8 + 1 = 24$.
- Since Steve saw a total of 28 birds, he must have seen $28 - 24 = 4$ herons.

Teacher's Notes

This problem could be solved algebraically. We could use c to represent the number of chickadees, h to represent the number of herons, b to represent the number of blue jays, w to represent the number of woodpeckers, and e to represent the number of eagles. Then we could write the following equations:

$$\begin{aligned} 2 \times b &= c && (eqn\ 1) \\ w - 3 &= b && (eqn\ 2) \\ e &= 1 && (eqn\ 3) \\ c &= 10 && (eqn\ 4) \\ c + h + b + w + e &= 28 && (eqn\ 5) \end{aligned}$$

This is known as a system of equations. To be sure we can solve the system, we must have at least as many *linearly independent* equations as we have variables. Two equations are *linearly dependent* if one is a multiple of the other. In this case, we have 5 linearly independent equations with 5 variables. We are guaranteed to have enough information to find the values of the unknown variables. Now we can solve the system. We already know the values for c and e.

We can substitute $c = 10$ into $(eqn\ 1)$ to get $2 \times b = 10$ so $b = 5$.

Now we can substitute $b = 5$ into $(eqn\ 2)$ to get $w - 3 = 5$ so $w = 8$.

Now we can substitute the values for c, b, w, and e into $(eqn\ 5)$ to get

$$\begin{aligned} 10 + h + 5 + 8 + 1 &= 28 \\ h + 24 &= 28 \\ h &= 4 \end{aligned}$$

Problem of the Week

Problem A

My Beating Heart

A) My resting heart rate is 21 beats in 15 seconds. Predict my number of heart beats in 1 minute.

B) After skipping rope for 1 minute, my heart rate is now 30 beats in 15 seconds. Now, predict my number of heart beats in 1 minute.

C) If I skip for 10 minutes and my heart rate continues to be 30 beats in 15 seconds, predict how many times my heart will beat in 10 minutes.

Strands Number Sense and Numeration, Measurement

Problem of the Week

Problem A and Solution

My Beating Heart

Problem

A) My resting heart rate is 21 beats in 15 seconds. Predict my number of heart beats in 1 minute.

B) After skipping rope for 1 minute, my heart rate is now 30 beats in 15 seconds. Now, predict my number of heart beats in 1 minute.

C) If I skip for 10 minutes and my heart rate continues to be 30 beats in 15 seconds, predict how many times my heart will beat in 10 minutes.

Solution

A) One way to solve this problem is by making a table keeping track of how times your heart beats every 15 seconds.

Time Elapsed in seconds	Heart Beats
15	21
30	42
45	63
60	84

Since 60 seconds equals one minute, we can expect that you will have had 84 heart beats over that time.

B) Since $4 \times 15 = 60$ seconds, if we know your heart rate for 15 seconds we can calculate your heart rate for 1 minute by multiplying by 4. So, we can predict that your heart beats $4 \times 30 = 120$ times in 1 minute after skipping.

C) We expect that the number of times your heart beats in 10 minutes will be 10 times the number of beats you have in 1 minute. So after 10 minutes of skipping we expect your heart beats $10 \times 120 = 1200$ times.

Teacher's Notes

To solve this problem, we used a concept from probability theory known as *expected value*. We assumed that the actual measurement of our heart beat over a short period of time would allow us to predict the number of heart beats over a longer period of time. Our predictions in this case are reasonable if the conditions remain stable.

Although expected value is useful for predicting the future, there are other factors that affect predictions. Statisticians would also consider the *variance* of a particular situation. Variance describes the range of possible values that could actually occur. When the variance is low, then our predictions are more likely to be correct or at least close to correct. When the variance is high, then our prediction may be correct, but the actual outcome may be much different than the expected value.

Problem of the Week

Problem A

Secret Message

Andrea wants to send secret messages to her friends. She creates a grid and fills it with the letters of the alphabet in order. She uses letters to identify the columns of the grid and numbers to identify the rows of the grid. Then she sends a message by using the positions of the letters and spaces in the grid.

To make it harder for people to decode her messages, she uses different sized grids for different messages, and sometimes she leaves some blank spaces at the beginning of the grid. For example, this is a grid with 8 columns and 3 blank spaces at the beginning:

	A	B	C	D	E	F	G	H
1				a	b	c	d	e
2	f	g	h	i	j	k	l	m
3	n	o	p	q	r	s	t	u
4	v	w	x	y	z			

A) Using the grid above, decode the message: `H2D1G3C2C1D2F3F4A2H3A3`

B) Without seeing the particular grid Andrea uses, she can still send her friends secret messages that they can decode if they know how many columns are in the grid and how many blank spaces are at the beginning of the grid. We call this kind of information the key. So for the grid above, the key is 83.

If the key is 62, the grid would have six columns and two blank spaces at the beginning. How do you encode the message: **this is secret** using the key 62?

C) Decode the message `D1D4A6B4A5A4C2D4B1B4D2A6` using the key 51.

Strands Geometry and Spatial Sense, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Secret Message

Problem

Andrea wants to send secret messages to her friends. She creates a grid and fills it with the letters of the alphabet in order. She uses letters to identify the columns of the grid and numbers to identify the rows of the grid. Then she sends a message by using the positions of the letters and spaces in the grid.

To make it harder for people to decode her messages, she uses different sized grids for different messages, and sometimes she leaves some blank spaces at the beginning of the grid. For example, this is a grid with 8 columns and 3 blank spaces at the beginning:

	A	B	C	D	E	F	G	H
1				a	b	c	d	e
2	f	g	h	i	j	k	l	m
3	n	o	p	q	r	s	t	u
4	v	w	x	y	z			

A) Using the grid above, decode the message: `H2D1G3C2C1D2F3F4A2H3A3`

B) Without seeing the particular grid Andrea uses, she can still send her friends secret messages that they can decode if they know how many columns are in the grid and how many blank spaces are at the beginning of the grid. We call this kind of information the key. So for the grid above, the key is 83.

If the key is 62, the grid would have six columns and two blank spaces at the beginning. How do you encode the message: **this is secret** using the key 62?

C) Decode the message `D1D4A6B4A5A4C2D4B1B4D2A6` using the key 51.

Solution

A) The decoded message is: **math is fun**

B) Here is the grid that matches the key 62:

	A	B	C	D	E	F
1			a	b	c	d
2	e	f	g	h	i	j
3	k	l	m	n	o	p
4	q	r	s	t	u	v
5	w	x	y	z		

We could use any of `A1`, `B1`, `E5`, or `F5` for the spaces in the message. The letters can only be encoded in one way. So here is one solution for the encoding of **this is secret**: `D4D2E2C4A1E2C4B1C4A2E1B4A2D4`.

C) Here is the grid that matches the key 51:

	A	B	C	D	E
1		a	b	c	d
2	e	f	g	h	i
3	j	k	l	m	n
4	o	p	q	r	s
5	t	u	v	w	x
6	y	z			

So the decoded message is: **cryptography**

Teacher's Notes

Cryptography is the study of encoding private data. It is a practice that goes back thousands of years. In most cryptographic schemes, encoding and decoding a message requires a *key*. One of the trickiest parts of sending secret messages is that both the sender and the receiver need to know a key in order to encode and decode the message. The question becomes how do you transmit the key without someone finding out what it is.

Today, most data online is secured using *public key encryption*. In this case, the receiver has two keys: a public key and a private key. The receiver provides the sender with a public key that the sender uses to encode the message. However, without the private key, it would be almost impossible to decode the message - even for the sender.

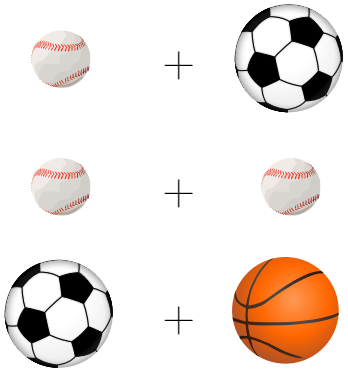
Here is one way to think of public key cryptography. If I want you to be able to send me a secret message, I can send you a box with an open (public) lock. You can put the message in the box and lock it. After it is locked, nobody except the person with a (private) key can read the message. Now you can safely send me a secret message.

Problem of the Week

Problem A

Sporting Sleuth

A) A physical education teacher weighed some of the equipment in the gym. She recorded the following measurements:



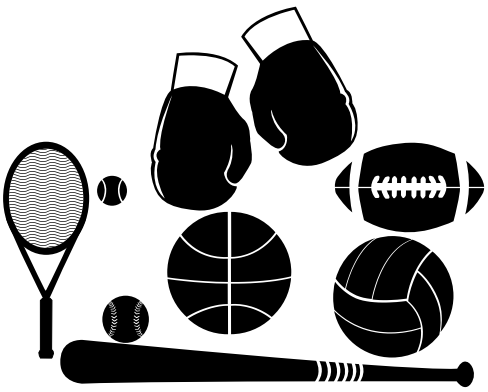
530 grams

280 grams

890 grams

Determine the mass of each type of ball: baseball soccer ball basketball

B) A gym bag contains three baseballs, a soccer ball and two basketballs. What is the approximate total mass of the gym equipment?



Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Sporting Sleuth

Problem

A) A physical education teacher weighed some of the equipment in the gym. She recorded the following measurements:

 + = **530 grams**

 + = **280 grams**

 + = **890 grams**

Determine the mass of each type of ball: baseball soccer ball basketball

B) A gym bag contains three baseballs, a soccer ball and two basketballs. What is the approximate total mass of the gym equipment?

Solution

A) From the second equation we know that two baseballs weigh 280 grams. So each baseball is $280 \div 2 = 140$ grams. We can use that information and the first equation to determine that a soccer ball weighs 140 grams less than 530 grams. So a soccer ball weighs $530 - 140 = 390$ grams. Looking at the last equation, now we know that a basketball weighs 390 grams less than 890 grams. So a basketball weighs $890 - 390 = 500$ grams.

B) Using the results from the first part, we can calculate that three baseballs will weigh $3 \times 140 = 420$ grams. We can also calculate the mass of two basketballs as $2 \times 500 = 1000$ grams. So the total mass of the gym equipment would be $420 + 1000 + 390 = 1810$ grams.

Teacher's Notes

The images of baseballs, soccer balls and basketballs are acting like variables in mathematical equations. We could rewrite the statements algebraically with more traditional symbols for variables such as x, y, and z.

Let x be the weight of a baseball.
Let y be the the weight of a soccer ball.
Let z be the weight of a basketball.

Therefore:

$$x + y = 530 \tag{1}$$
$$2x = 280 \tag{2}$$
$$y + z = 890 \tag{3}$$

Now, if we wanted, we can use standard mathematical methods to solve the equations and find the mass of each piece of sports equipment.
From equation (2), we can divide by 2 on both sides, giving us the following:

$$x = 140 \tag{4}$$

Knowing the value of x, we can use *substitution* to solve the other two equations:
Substituting $x = 140$ into equation (1) we get:

$$(140) + y = 530 \tag{5}$$
$$140 - 140 + y = 530 - 140 \tag{6}$$
$$y = 390 \tag{7}$$

Substituting $y = 390$ into equation (3) we get:

$$(390) + z = 890 \tag{8}$$
$$390 - 390 + z = 890 - 390 \tag{9}$$
$$z = 500 \tag{10}$$

Using images or symbols in equations is an example of *abstraction*. The idea of abstraction is to take a real life situation and create a mathematical model. Once the model has been created, then it is possible to use known mathematical techniques to solve the problem. When solving the problem, the actual symbols used to represent the real life situation do not affect the outcome.

Problem of the Week

Problem A

Chasing the Great One

Auston McCrosby (a professional hockey player) has earned 1000 points (goals and assists) in his career so far. Wayne Gretzky, the highest point getter of all time in the National Hockey League, scored 2875 points in his career. If Auston averages 2 points per game in an 82 game season, approximately how many more seasons will he need to complete in order to reach Wayne Gretzky's record.

Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Chasing the Great One

Problem

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Solution

If Auston averages 2 points per game and there are 82 games in a season then he is expected to earn $2 \times 82 = 164$ points in each season. With this information we can make a table that estimates how many points he has accumulated after each season.

Year	Points in Season	Accumulated Points
Start		1000
1	164	1164
2	164	1328
3	164	1492
4	164	1656
5	164	1820
6	164	1984
7	164	2148
8	164	2312
9	164	2476
10	164	2640
11	164	2804
12	164	2968

So after 12 more seasons we expect that Auston will have accumulated more points than Wayne Gretzky.

Alternatively, we can calculate the number of points that Auston needs to earn to match Wayne Gretzky as $2875 - 1000 = 1875$. Then we can divide by 164 to determine how many years we expect it will take to accumulate 1875 more points. So we calculate $1875 \div 164 = 11$ remainder 71. Since there is a remainder with this division, we expect that it will take 12 seasons for Auston to surpass Wayne Gretzky's record.

It is also possible to use estimation to predict how many more seasons it will take Auston to reach Gretzky's record. We expect that Auston will score approximately 160 points per season. After 10 seasons, we expect him to score approximately 1600 points, so he would have accumulated approximately 2600 points. Since he needs to accumulate approximately 2900 points, he will need at least two more years where he would be expected to accumulate approximately 320 points. This means we expect it would take a total of 12 years for Auston to surpass the record.

Teacher's Notes

Another way to solve this problem would be to use a spreadsheet or to write a program to calculate how many goals Auston is predicted to have scored at the end of each year. Here is a program written in Python that would compute the prediction:

```
CURRENT_GOALS = 1000
GOALS_PER_GAME = 2
GAMES_PER_SEASON = 82
GOALS_PER_SEASON = GOALS_PER_GAME * GAMES_PER_SEASON
RECORD_GOALS = 2875

year = 0
accumulated_goals = CURRENT_GOALS
while accumulated_goals <= RECORD_GOALS:
    year = year + 1
    accumulated_goals = accumulated_goals + GOALS_PER_SEASON

print(year)
```

The values at the beginning of the program such as CURRENT_GOALS and GOALS_PER_GAME are called constants. These are values that can be easily changed to do the same calculation for other players with other numbers. It would also be easy to make the program interactive so you can ask someone to enter the starting number of goals and goals per game for a particular player and then calculate how many seasons it would take to surpass the record.

The main calculations happen starting at the line year = 0. The program initializes the values of two variables that are keeping track of the **year** and the **accumulated_goals**.

The line:

while accumulated_goals <= RECORD_GOALS:

tells the computer to repeat the next two instructions as long as the number of accumulated goals is less than the record number of goals. The repeated instructions are to increase the year by 1, and to increase the accumulated goals by the number of goals expected to be scored in a season. When the number of accumulated goals is greater than the record, the repetition will stop. At that point the program will continue to the last line which is and instruction to print the year.

Since the year has been increasing by 1 as long as the number of accumulated goals is less than the record, the value printed at the end will be the year in which the total number of goals surpasses the record.

Problem of the Week

Problem A

Patterned Savings

Rebecca wants to save money starting on January 1. She decided to collect money in a jar in the following way. Every day she puts a quarter in the jar. Every third day, starting on January 3, she puts a loonie in the jar. Every fifth day, starting on January 5, she puts a toonie in the jar. So some days she puts one coin in the jar, some days she puts two coins in the jar, and some days she puts three coins in the jar.

In Canada, a quarter is worth 25 cents, a loonie is worth one dollar, and a toonie is worth two dollars. There are 100 cents in one dollar.

A) How many coins does she put in the jar on January 12?

B) How many coins does she put in the jar on January 26?

C) How many coins does she put in the jar on January 30?

D) How many coins in total does she have in the jar by the end of January?

E) If she keeps saving this way throughout the year, how much money will she have after 90 days? Try to figure this out without counting the money she puts in the jar every day for all 90 days.

Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Patterned Savings

Problem

Rebecca wants to save money starting on January 1. She decided to collect money in a jar in the following way. Every day she puts a quarter in the jar. Every third day, starting on January 3, she puts a loonie in the jar. Every fifth day, starting on January 5, she puts a toonie in the jar. So some days she puts one coin in the jar, some days she puts two coins in the jar, and some days she puts three coins in the jar.

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B) How many coins does she put in the jar on January 26?

C) How many coins does she put in the jar on January 30?

D) How many coins in total does she have in the jar by the end of January?

E) If she keeps saving this way throughout the year, how much money will she have after 90 days? Try to figure this out without counting the money she puts in the jar every day for all 90 days.

Solution

One way to answer most of these questions is to make a table that keeps track of how many coins are deposited each day and how much money is accumulated each day.

Day	Quarters Added	Loonies Added	Toonies Added	Money Added ($)	Total Coins	Total Money Money ($)
1	1	0	0	0.25	1	0.25
2	1	0	0	0.25	2	0.50
3	1	1	0	1.25	4	1.75
4	1	0	0	0.25	5	2.00
5	1	0	1	2.25	7	4.25
6	1	1	0	1.25	9	5.50
7	1	0	0	0.25	10	5.75
8	1	0	0	0.25	11	6.00
9	1	1	0	1.25	13	7.25
10	1	0	1	2.25	15	9.50
11	1	0	0	0.25	16	9.75
12	1	1	0	1.25	18	11.00
13	1	0	0	0.25	19	11.25
14	1	0	0	0.25	20	11.50
15	1	1	1	3.25	23	14.75

We could continue the table, and we would see that the amounts of coins and money added would be repeated exactly every 15 days. From this table, we observe that on every day that is a multiple of 3 (except on day 15) Rebecca adds two coins to the jar, and on every day that is a multiple of five (except on day 15) Rebecca adds two coins to the jar. On day 15, which is a multiple of both 3 and 5, Rebecca adds three coins to the jar. On all other days, Rebecca adds just one coin to the jar.

A) From the table, we see that, on January 12, Rebecca added 2 coins to the jar.

B) Since 26 is neither a multiple of 3 nor a multiple of 5, Rebecca will only add 1 coin to the jar.

C) Since 30 is a multiple of both 3 and 5, Rebecca will add 3 coins to the jar.

D) From the table, we see that after 15 days, Rebecca will have saved 23 coins. This pattern will be repeated in the next 15 days. On January 31, she will add one more quarter. So, by January 31 Rebecca will have $23 + 23 + 1 = 47$ coins in the jar.

E) Every 15 days Rebecca will have saved a total of \$14.75. This pattern will be repeated 6 times over a 90 day period. So after 90 days Rebecca will have $14.75 + 14.75 + 14.75 + 14.75 + 14.75 + 14.75 = \$88.50$. Alternatively we could calculate the savings as $6 \times 14.75 = \$88.50$.

Teacher's Notes

This problem shows a repeating pattern every 15 days. In mathematics, we could refer to this kind of repetition as a *periodic function*. The length of the interval between repeated elements is known as the *period* of the function. In this case, the period of the savings function is 15, which is the *least common multiple* or *LCM* of the values 1, 3, and 5. The LCM of a set of numbers is the smallest number that is a multiple of each element of the set. In this case, we are looking for the smallest multiple of each of the individual periods of savings.

Periodic functions appear in mathematics and in the real world. Trigonometric functions such as *sin*, *cos*, and *tan* are periodic functions. Sound waves, phases of the moon, and your blood pressure, are all examples of periodic functions in nature.

Problem of the Week

Problem A

Managing Muffins

The family studies class baked two dozen muffins. The class plans to share one third of the muffins with the school office staff, and one quarter of the muffins with Mr. Ahmed's chess club.

A) Which group will receive more muffins? Justify your answer.

B) How many muffins will be left over after they give the office staff and chess club their shares?

Strand Number Sense and Numeration

Problem of the Week
Problem A and Solution
Managing Muffins

Problem

The family studies class baked two dozen muffins. The class plans to share one third of the muffins with the school office staff, and one quarter of the muffins with Mr. Ahmed's chess club.

A) Which group will receive more muffins? Justify your answer.

B) How many muffins will be left over after they give the office staff and chess club their shares?

Solution

One way to solve this problem is to create a diagram showing the batch of two dozen muffins. Then we can group the diagram into four equal parts (quarters) using solid black lines and three equal parts (thirds) using different background shading. We can arrange the 24 muffins this way:

A) From the diagram, we can see that one quarter of the batch is equal to 6 muffins, and that one third of the batch is equal to 8 muffins. Since the school office staff received one third of the muffins, they will get more muffins.

B) The class started with $2 \times 12 = 24$ muffins. They gave away a total of $6 + 8 = 14$ muffins. That means they will have a total of $24 - 14 = 10$ muffins left over.

Teacher's Notes

Consider the relationship between a positive integer n and the fraction $\frac{1}{n}$. We can think of $\frac{1}{n}$ as being one part of a whole that has n parts in total. So the fewer parts we have, the bigger each individual part must be. This means that if we compare two positive fractions, $\frac{1}{a}$ and $\frac{1}{b}$, if $a < b$ we know that $\frac{1}{a} > \frac{1}{b}$. For example, since $3 < 4$ we know that $\frac{1}{3} > \frac{1}{4}$.

As n increases in size, then $\frac{1}{n}$ decreases in size proportionally. In other words, if n is a very large number, then $\frac{1}{n}$ is a very small number. Since there are an infinite number of integers, there are also an infinite number of fractions in the form $\frac{1}{n}$. As n grows bigger the value of $\frac{1}{n}$ gets closer to 0, although it never actually reaches 0. We can say that the limit of $\frac{1}{n}$, as n approaches infinity, is 0. This can be written mathematically as:

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

The study of limits is a fundamental part of calculus.

Problem of the Week

Problem A

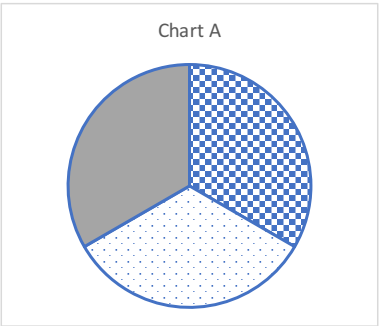
Surveys and Siblings

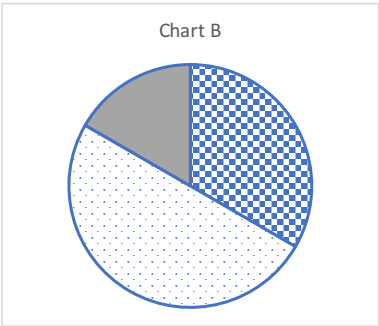
Zheng surveys his friends at school and gathers the following information:

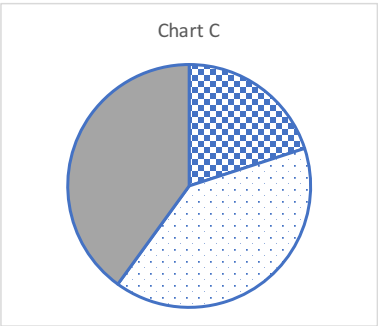
- 13 grade 3 students have no siblings
- 16 grade 3 students have exactly one sibling
- 8 grade 3 students have more than one sibling
- 7 grade 4 students have no siblings
- 14 grade 4 students have exactly one sibling
- 2 grade 4 students have more than one sibling

A) Choose the pie chart that properly shows the proportions of students in the school who have no siblings, one sibling, and more than one sibling.

Justify your answer.







B) Create a legend for the correct pie chart that shows which section represents no siblings, which section represents one sibling, and which section represents more than one sibling.

Problem of the Week

Problem A and Solution

Surveys and Siblings

Problem

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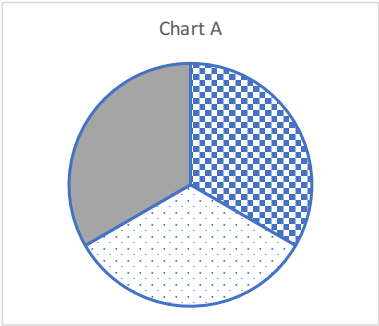
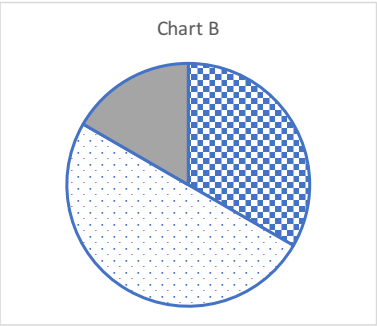
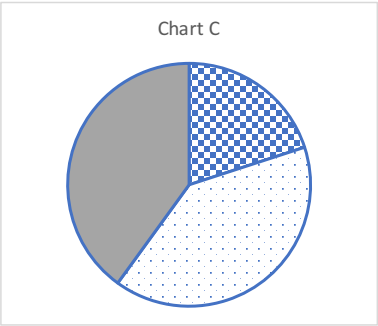
Justify your answer.







B) Create a legend for the correct pie chart that shows which section represents no siblings, which section represents one sibling, and which section represents more than one sibling.

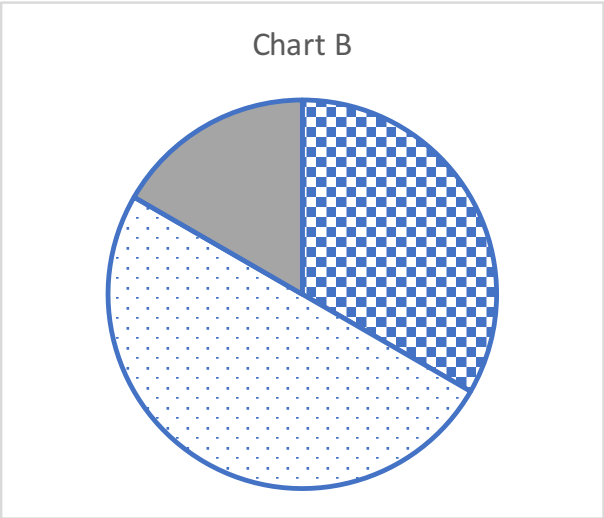
Solution

A) One way to determine which chart is correct is to collate and analyze the original data. Zheng surveyed a total of $13 + 16 + 8 + 14 + 2 = 60$ students.

We can also calculate the following data:

- A total of $13 + 7 = 20$ students have no siblings.
- A total of $16 + 14 = 30$ students have one sibling.
- A total of $8 + 2 = 10$ students have more than one sibling.

Examining the charts, we can see that the sections of Chart A are all approximately the same size. However, in our data analysis, there are clearly many more students who have one sibling than have more than one sibling. So Chart A cannot be correct. We also notice that half of the students (30 out of 60) have exactly one sibling. In Chart C, none of the sections fill half of the pie. So Chart C cannot be correct. Chart B has one section that fills half of the pie. The other two sections of Chart B are clearly not the same size. Since the number of students who have no siblings and the number of students who have more than one sibling are not equal, this means that Chart B is correct.



B) Since half of the students in the survey said they had one sibling, and since the dotted section of the chart fills half the pie, then that section must be representing the students with one sibling. Since the fewest number of students in the survey indicated that they had more than one sibling, then the smallest section of the chart which has a solid fill, is representing that group. This means that the checker-board filled section of the chart must be representing the students with no siblings.

Teacher's Notes

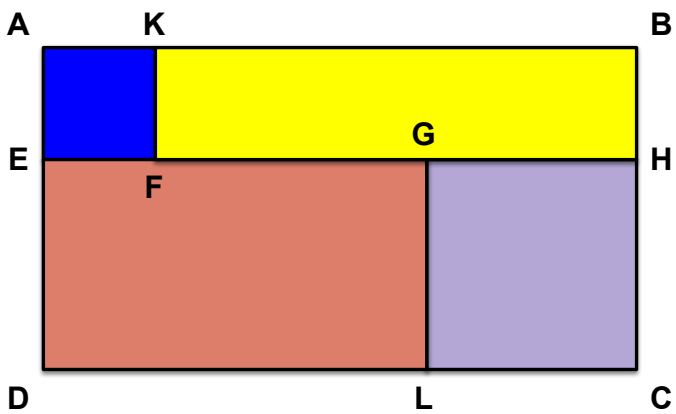
For this problem, we are given possible answers and expected to determine which one is correct. In mathematics and computer science, producing an answer is only one part of a good solution. Complete solutions to problems usually involve being able to explain how you got the solution and how you know that your solution is correct. Producing a logical argument to support your answer is an important skill in many situations.

Problem of the Week

Problem A

Area Issues

The employees of Sew Inspired need to make a quilt for a special puzzle project. They draw a pattern for the four pieces of cloth they will cut for each section. The pattern includes two square pieces (**AKFE** and **GHCL**), and two rectangular pieces (**KBHF** and **EGLD**).



The area of the piece **AKFE** is 4 square units. The area of the piece **GHCL** is 9 square units. The points **E**, **F**, **G**, and **H** are on the same straight line, and the line segment **FG** is 5 units long.

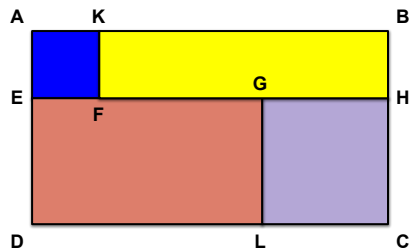
What is the area of the rectangular section **ABCD**?

Strands Measurement, Geometry and Spatial Sense, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Area Issues

Problem

The employees of Sew Inspired need to make a quilt for a special puzzle project. They draw a pattern for the four pieces of cloth they will cut for each section. The pattern includes two square pieces (**AKFE** and **GHCL**), and two rectangular pieces (**KBHF** and **EGLD**).



The area of the piece **AKFE** is 4 square units. The area of the piece **GHCL** is 9 square units. The points **E**, **F**, **G**, and **H** are on the same straight line, and the line segment **FG** is 5 units long.

What is the area of the rectangular section **ABCD**?

Solution

One way to calculate the area of rectangle **ABCD**, is to determine the lengths of its sides. We know the area of the square **AKFE** is 4 square units, and we know that the lengths of the sides of a square must be the same. So if the length of one side of a square is n, then the area of the square must be $n \times n$. By trial and error we can see that $2 \times 2 = 4$, so each side of the square **AKFE** must be 2 units. Similarly, since the area of the square **GHCL** is 9 square units, we can see that $3 \times 3 = 9$. So each side of the square **GHCL** must be 3 units.

Another way to determine the lengths of the sides of square **AKFE**, would be to start with 4 unit squares (using blocks or cut out of paper for example) and determine how to arrange them into a larger square. The only possible arrangement is a 2×2 square.

The opposite sides of a rectangle must be the same length, and the bottom of the square **AKFE** is on the same line as the top of square **GHCL**. This means the length side **AD** is equal to the sum of the lengths of the sides of the two squares. So the length of side **AD** is $2 + 3 = 5$ units. Also, since the length of line segment **FG** is 5 units, the length of line **EH** must be $2 + 5 + 3 = 10$ units. The length of this line is the same as the length of the side **AB** or the side **DC**.

Now we can calculate the area of rectangle **ABCD**. The area of this rectangle is the product of the length of side **AD** and the length of side **AB**. So the area of rectangle **ABCD** is $5 \times 10 = 50$ square units.

Teacher's Notes

Exploring the areas of rectangles or squares is a good way to practice multiplication of positive numbers. The result of calculating $a \times b$ is equivalent to calculating the area of a rectangle with side lengths a and b.

Similarly, exploring the relationship between the area of a square and the lengths of its sides is a good way to learn about *square roots*. However, this exploration only produces half of the answer.

A square root of a number is defined as follows:

$$\text{If } y \text{ is a square root of } x\text{, then } y \cdot y = x.$$

Since the product of two negative numbers is positive, the value of y could be positive or negative. So the square root of 9 is either 3 or -3. When we use the radical symbol, $\sqrt{}$, we are looking for the principal (or positive) square root. Although -3 and 3 are both square roots of 9, the $\sqrt{9} = 3$.

If x is a *perfect square* (i.e. it is the the result of multiplying a rational number by itself), then the result of calculating $\sqrt{x}$ is also a rational number. A *rational number* is any number that can be represented as a fraction $\frac{a}{b}$, where a and b are integers, and b is not 0.

For example, $\sqrt{144}$ is exactly 12. There are also non-integers values that have rational square roots. For example, $\sqrt{\frac{9}{16}}$ is exactly $\frac{3}{4}$.

There are many numbers, however, that are not perfect squares. Square roots of these numbers are known as *irrational numbers*. An irrational number cannot be represented precisely as a fraction. If we try to write an irrational number in decimal form, we end up with a result where the digits after the decimal place never end, and never repeat a pattern. So, we cannot compute the square root of an irrational number on a calculator exactly. The result will appear as a limited number of digits after the decimal place. The calculator rounds the answer to a fixed number of decimal places.

Problem of the Week

Problem A

A Ball of a Birthday Party

Jaylen, Shelby, Kara and Logan went to a birthday party for their friend Zabrina. Each of them brought a gift. The gifts were a skipping rope, marbles, a book, and a soccer ball. After the party each child went home with a different colour balloon: red, blue, green, or orange. Using the clues, figure out which child brought what gift, and what colour balloon they took home.

- Jaylen did not bring the marbles as a gift, but he did go home with a blue balloon.
- Shelby brought the soccer ball as a gift, but did not go home with a red or green balloon.
- The friend who brought the skipping rope as a gift went home with a red balloon.
- Kara brought the marbles as a gift.

The table below may be helpful to keep track of information.

	Skipping Rope	Marbles	Book	Soccer Ball	Red Balloon	Blue Balloon	Green Balloon	Orange Balloon
Jaylen								
Shelby								
Kara								
Logan								

Strand Logic

Problem of the Week
Problem A and Solution
A Ball of a Birthday Party

Problem

Jaylen, Shelby, Kara and Logan went to a birthday party for their friend Zabrina. Each of them brought a gift. The gifts were a skipping rope, marbles, a book, and a soccer ball. After the party each child went home with a different colour balloon: red, blue, green, or orange. Using the clues, figure out which child brought what gift, and what colour balloon they took home.

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The table below may be helpful to keep track of information.

	Skipping Rope	Marbles	Book	Soccer Ball	Red Balloon	Blue Balloon	Green Balloon	Orange Balloon
Jaylen								
Shelby								
Kara								
Logan								

Solution

One way to determine the connections between the people attending the party, the presents, and the balloons is as follows:

We start by examining the clues. We can check the boxes that we **know** are connected, and fill in the boxes that we know are **not** connected. For example, based on the first clue, we can check the box that connects Jaylen with a blue balloon, and fill in the box that connects Jaylen with marbles. The result after evaluating all the clues is this:

	Skipping Rope	Marbles	Book	Soccer Ball	Red Balloon	Blue Balloon	Green Balloon	Orange Balloon
Jaylen						✓		
Shelby				✓				
Kara		✓						
Logan								

Next, wherever there is a ✓, we can eliminate all of the other choices in the same category. For example, since we know that Kara brought the marbles, then we know that none of the other guests brought the marbles, and we know that Kara did not bring any of the other gifts. Updating the table with this information results in the following table:

	Skipping Rope	Marbles	Book	Soccer Ball	Red Balloon	Blue Balloon	Green Balloon	Orange Balloon
Jaylen						✓		
Shelby				✓				
Kara		✓						
Logan								

Now we can make some logical deductions. At this point we know that Shelby did not take home the red, blue, or green balloon, so she must have received the orange balloon. Since Shelby received the orange balloon, neither Kara nor Logan received it. Since we know that Jaylen did not take home the red balloon, based on the third clue he did not bring the skipping rope. The updated table is now:

	Skipping Rope	Marbles	Book	Soccer Ball	Red Balloon	Blue Balloon	Green Balloon	Orange Balloon
Jaylen						✓		
Shelby				✓				✓
Kara		✓						
Logan								

Again, we can make some logical deductions. The only choice left for the gift Jaylen brought is the book. This means that Logan did not bring the book, so he must have brought the skipping rope. From the third clue, this also means that Logan took home the red balloon. This leaves only the green balloon unassigned, so Kara must have received that balloon. In summary:

	Skipping Rope	Marbles	Book	Soccer Ball	Red Balloon	Blue Balloon	Green Balloon	Orange Balloon
Jaylen			✓			✓		
Shelby				✓				✓
Kara		✓					✓	
Logan	✓				✓			

Jaylen brought the book and took home the blue balloon.

Shelby brought the soccer ball and took home the orange balloon.

Kara brought the marbles and took home the green balloon.

Logan brought the skipping rope and took home the red balloon.

Teacher's Notes

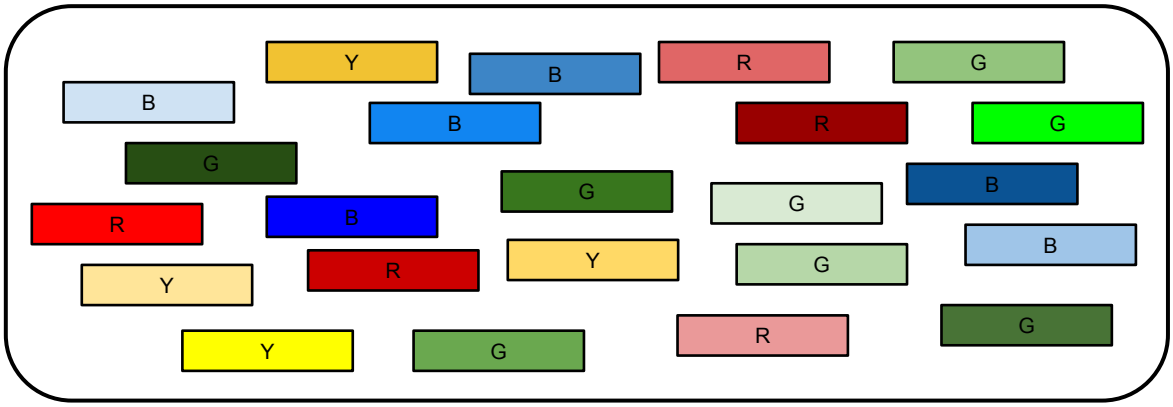
Logic is an essential part of Mathematics and Computer Science. Many post-secondary institutions offer entire courses dedicated to understanding and using logic.

Formal logic allows people to make convincing arguments and come to logical conclusions based on known facts. People make logical arguments all the time, but they normally use natural language to make the case. Spoken or written natural language can be imprecise and ambiguous.

Mathematicians and computer scientists use tools such as *propositional logic*, to make clearly structured arguments. These arguments start with statements that are known to be either true or false (known as *propositions*), and then use standard rules to prove some other conclusions. The propositions and arguments are written using mathematical notation, which in turn can be translated to statements that a computer can interpret. Formal logic methods are used for everything from the design of computer hardware, to proving that systems designed by software engineers are reliable, to the creation of expert systems by people studying artificial intelligence.

Problem of the Week
Problem A
Colourful Conundrum

Ian has a pencil case full of coloured pencils. He has many shades of four main colours: red (R), blue (B), green (G), and yellow (Y). The diagram below represents the contents of the pencil case:



A) Use the chart below to tally the coloured pencils and then create a pictograph using the key provided:

Key:  = 2 pencils

Main Colour	Tally	Pictograph
red		
blue		
green		
yellow		

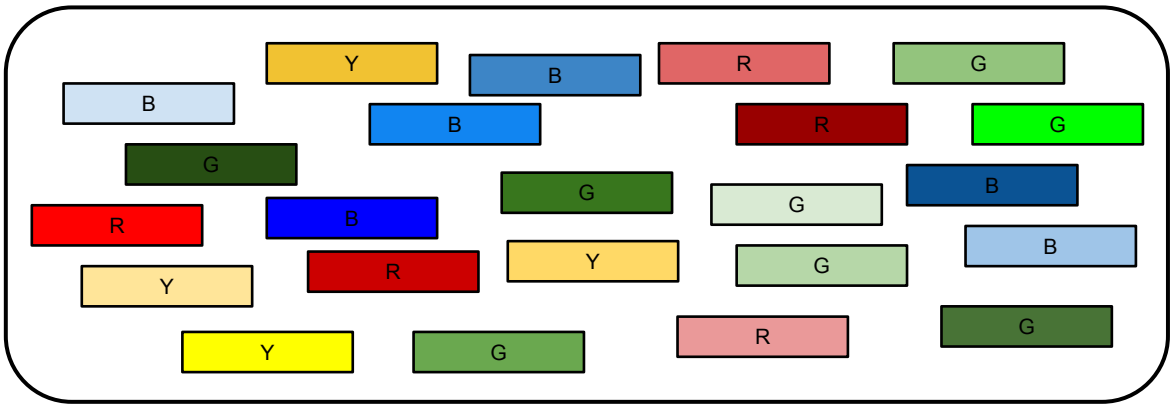
B) How many pencils will you have to draw from the pencil case to guarantee you will get two pencils of the same main colour?

Strands Data Management and Probability, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Colourful Conundrum

Problem

Ian has a pencil case full of coloured pencils. He has many shades of four main colours: red (R), blue (B), green (G), and yellow (Y). The diagram below represents the contents of the pencil case:



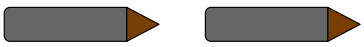
A) Use the chart below to tally the coloured pencils and then create a pictograph using the key provided:

B) How many pencils will you have to draw from the pencil case to guarantee you will get two pencils of the same main colour?

Solution

A) Here is the completed tally and pictograph for the pencil case data:

Key:  = 2 pencils

Main Colour	Tally	Pictograph
red	𝍸	
blue	𝍸 |	
green	𝍸 |||	
yellow	||||	

B) Since there are four main colours, then it is possible to pick four pencils from the case that all have different main colours. So to guarantee you will get two pencils of the same main colour, you will need to pick more than four from the case. Since there are not five main colours, if you pick five pencils, at least two of the pencils will be the same colour. So five is the smallest number of pencils you could draw that would guarantee two of the same main colour.

Teacher's Notes

The solution for part **B** is an example of the *Pigeonhole Principle* at work. The following analogy is often used to describe the Pigeonhole Principle.

> Imagine that you have n pigeons and that you have k pigeonholes where the birds roost. All of the pigeons come home to roost at night. If $n > k$ (i. e. the number of pigeons is greater than the number of pigeonholes), then at least one of the pigeonholes contains more than one pigeon.
>
> We can show that this is true by trying to prove the opposite. Let's assume that no pigeonhole contains more than one pigeon. As each pigeon flies home to roost, it enters an empty pigeon hole. However, after k pigeons have come home all of the pigeonholes will contain a bird. If $n > k$, then there is at least one more pigeon that needs somewhere to roost. It will need to share one of the pigeonholes.

Note that the Pigeonhole Principle does not guarantee that each pigeonhole contains a pigeon. In theory (although maybe not in reality), all of the pigeons could be roosting in one pigeonhole and there would be many empty spots. The only guarantee is that there will be at least one pigeonhole that contains more than one pigeon.

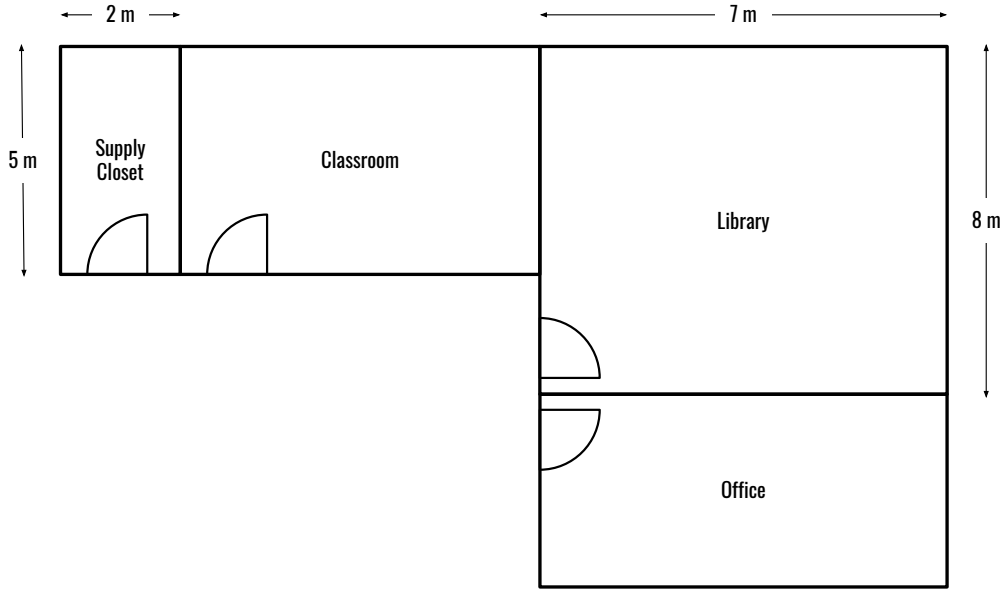
The key to using the Pigeonhole Principle in a mathematics problem is to determine what are the *pigeons* and what are the *pigeonholes*. In part **B** of this problem we can imagine that we have one container, each of which is labelled with one of the four main colours. As we draw pencils out of the case, we will put them in the container labelled with a matching main colour. The question becomes, how many pencils do we need to draw from the pencil case before at least one of the containers has two pencils? In this case, the containers are like the pigeonholes and the coloured pencils are like the pigeons. Since there are four containers, then if we have more than four coloured pencils, one of the containers must have at least two pencils. In particular, when we draw five pencils from the case we must have two that are the same main colour. Since five is the smallest integer that is greater than four, then five is the fewest number of pencils we must draw to guarantee two have the same main colour. However, we do not know which two are the same main colour, and it is possible we have three, four, or even all five pencils that are the same main colour.

Problem of the Week

Problem A

School Expansion

Emmy Noether Public School needs to add a new wing to its main building. The contractor has the plans for the addition, but some measurements were missing. The new rooms are rectangular and are connected as shown.



The good news is that the contractor remembered some extra details.

- When describing the dimensions of a rectangle, the contractor uses *length* to describe the longer side, and *width* to describe the shorter side.
- The width of the classroom is the same as the length of the supply closet.
- The length of the classroom is three times longer than the width of the supply closet.
- The width of the library and the length of the office are the same.
- The width of the office is half the length of the library.

The contractor needs to order tiles for the new rooms.

A) What are the missing dimensions of the classroom and the office?

B) What is the total area needed to be covered by the tiles?

Strands Measurement, Number Sense and Numeration

Problem of the Week
Problem A and Solution
School Expansion

Problem

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- The width of the library and the length of the office are the same.
- The width of the office is half the length of the library.

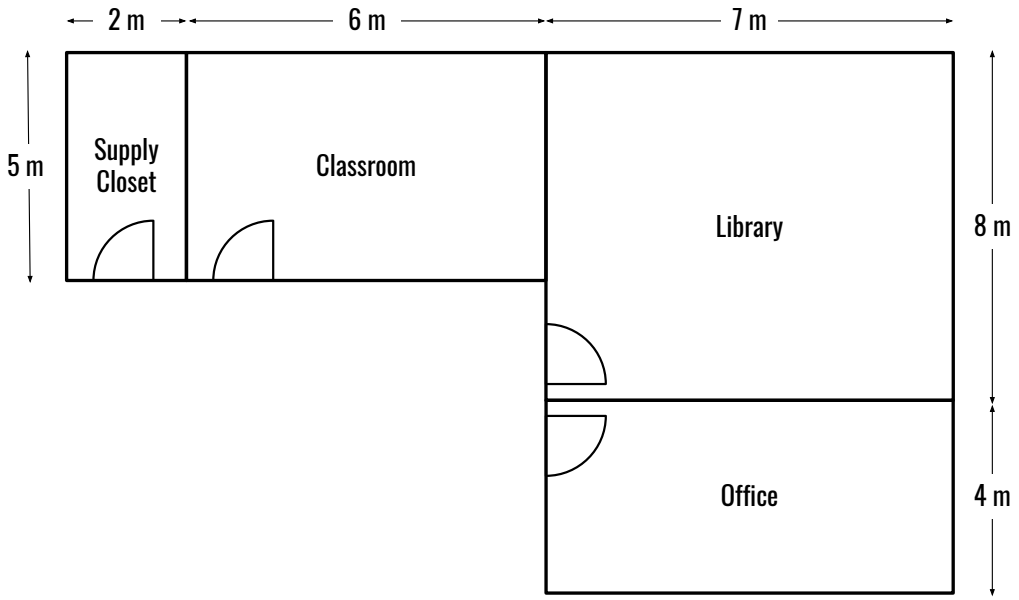
The contractor needs to order tiles for the new rooms.

A) What are the missing dimensions of the classroom and the office?

B) What is the total area needed to be covered by the tiles?

Solution

A) Since the classroom length is three times longer than the supply closet width, its length is $2 \times 3 = 6$ m. Since the office width is half the length of the library, its width is $8 \div 2 = 4$ m. Here is a diagram with all of the dimensions:



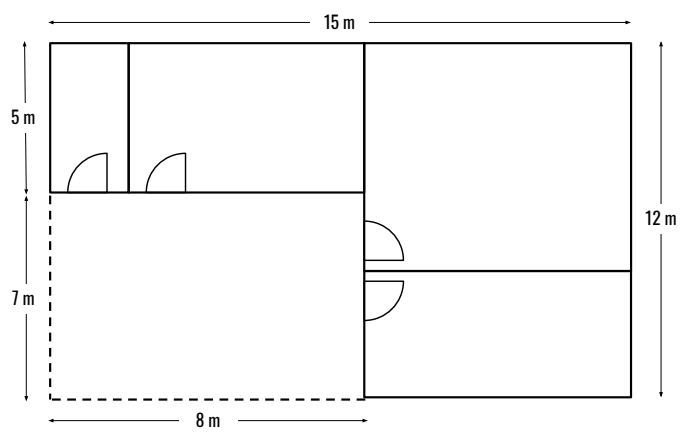
B) There are many ways to calculate the area needed to be covered. One way is to calculate the area of each room and add those numbers together.

- The area of the supply closet is $5 \times 2 = 10$ m^2.
- The area of the classroom is $5 \times 6 = 30$ m^2.
- The area of the library is $7 \times 8 = 56$ m^2.
- The area of the office is $7 \times 4 = 28$ m^2.

So the total area the contractor needs to tile is $10 + 30 + 56 + 28 = 124$ m^2.

Another way to find the total area is to calculate the area classroom based on the area of the supply closet and to calculate the office based on the area of the library. Since the length of the classroom is three times longer than the supply closet width, but the other dimensions are the same, the area of the classroom is three times the area of the supply closet. We can imagine that we could put three closets side-by-side and they would fill in the same area as the classroom. This means that the area of the classroom is $10 \times 3 = 30$ m^2. Similarly we can imagine that the area of the office would fill half of the area of the library. This means that the area of the office is $56 \div 2 = 28$ m^2.

A third way we could calculate the area that needs to be covered in tiles is to square off the diagram, as shown below.



The length across the top is $2 + 6 + 7 = 15$ m. The width along the side is $8 + 4 = 12$ m. The total area of that rectangle is $15 \times 12 = 180$ m^2.

However, there is a section of that rectangle that is not part of the building and hence needs no tiles. That section has a length equal to $2 + 6 = 8$ m and a width equal to $12 - 5 = 7$ m. The section of the diagram with no tile has an area of $8 \times 7 = 56$ m^2.

So the tiled area is the difference between these the areas of these two rectangles: $180 - 56 = 124$ m^2.

Teacher's Notes

Throughout most of history women were not actively encouraged to pursue academia, especially in the fields of mathematics and science. Emmy Noether (the namesake of the school in this problem) was born in Germany in 1882. Her father was a mathematician and she decided to follow in his footsteps. At the time, it was very difficult for a women to study at a university, but Noether was allowed to audit some courses at the University of Göttingen with the permission of the professor. One of the classes she attended was taught by David Hilbert, who was one of the most influential mathematicians of his time. Hilbert and others encouraged Noether in her academic pursuits. After completing her Ph.D., despite objections by some faculty, Neother taught classes and continued her research at the University of Göttingen. However she was only allowed to teach classes that were officially listed under Hilbert's name. Based on the work she did throughout her career, Emmy Noether is recognized as one of the leading mathematicians of the early 20th century.

During the time that she was teaching at the University of Göttingen, Maria Göppert became a student of mathematics and physics. After marrying her husband, Joseph Edward Mayer, she moved to the United States. Like Noether, Göppert-Mayer had restrictions on her earning potential as a researcher since she was a woman. However, she continued her academic pursuits at institutions such as Johns Hopkins, Columbia University and the University of Chicago. Among her other accomplishments, Göppert-Mayer was a member of the Manhattan Project - the group that developed the first nuclear weapons for the United States. Eventually, Göppert-Mayer became the second woman to receive the Nobel Prize in Physics for research she did after World War II.

As of 2019, only three women have received a Nobel Prize in Physics: Marie Currie, Maria Göppert-Mayer, and Donna Strickland. Strickland received her prize in 2018 (more than 50 years after Göppert-Mayer) for research in the field of lasers. In her Ph.D. thesis, Strickland cited the research of Göppert-Mayer's Ph.D. thesis.

Sources:

- `https://en.wikipedia.org/`
- `https://www.britannica.com/`

Problem of the Week

Problem A

Walking With Walden

Lindsay takes her dog Walden for a walk every day.

- On Monday she took him for a 45 minute walk.
- On Tuesday, they walked for 5 minutes longer than on Monday.
- On Wednesday, they walked for half as much time as Tuesday.
- On Thursday, they walked for 10 minutes less than Monday.
- On Friday, they walked for 15 minutes more than Thursday.
- On Saturday, they walked for twice as long as Monday.

A) How many minutes did Lindsay spend walking Walden between Monday and Saturday?

B) Lindsay wants to make sure that Walden is getting enough exercise. How long must Walden's walk be on Sunday to guarantee he walks for at least 6 hours this week?

Strands Number Sense and Numeration, Measurement

Problem of the Week
Problem A and Solution
Walking With Walden

Problem

Lindsay takes her dog Walden for a walk every day.

- On Monday she took him for a 45 minute walk.
- On Tuesday, they walked for 5 minutes longer than on Monday.
- On Wednesday, they walked for half as much time as Tuesday.
- On Thursday, they walked for 10 minutes less than Monday.
- On Friday, they walked for 15 minutes more than Thursday.
- On Saturday, they walked for twice as long as Monday.

A) How many minutes did Lindsay spend walking Walden between Monday and Saturday?

B) Lindsay wants to make sure that Walden is getting enough exercise. How long must Walden's walk be on Sunday to guarantee he walks for at least 6 hours this week?

Solution

A) First we need to determine how long Lindsay walked each day.

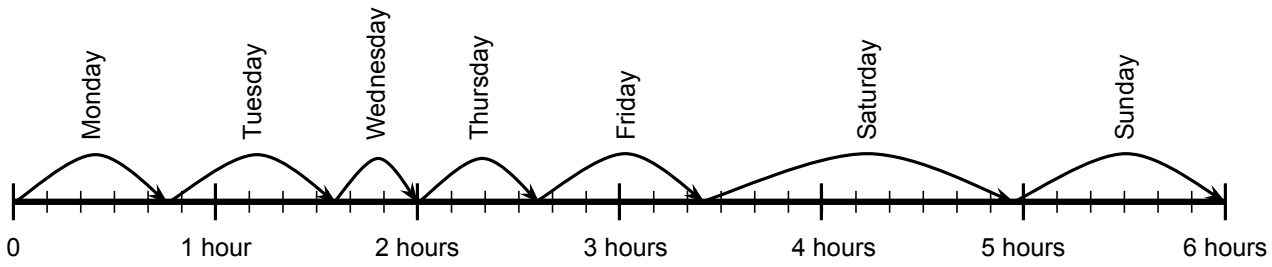
- On Tuesday, they walked for $45 + 5 = 50$ minutes.
- On Wednesday, they walked for $50 \div 2 = 25$ minutes.
- On Thursday, they walked for $45 - 10 = 35$ minutes.
- On Friday, they walked for $35 + 15 = 50$ minutes.
- On Saturday, they walked for $45 \times 2 = 90$ minutes.

The total number of minutes Lindsay walked from Monday to Saturday is:

$45 + 50 + 25 + 35 + 50 + 90 = 295$ minutes.

B) One way to determine how long Lindsay must walk on Sunday is to start by calculating how many minutes there are in 6 hours. Since there are 60 minutes in 1 hour, there are $60 \times 6 = 360$ minutes in 6 hours. Since Lindsay has already walked for 295 minutes, she needs to walk $360 - 295 = 65$ minutes more on Sunday. We could also say that she needs to walk 1 hour and 5 minutes more.

Another way to calculate the required time is to use a timeline that keeps track of how long Walden has been walked during the week:



From the timeline we can see that, by the end of Saturday, Lindsay and Walden have walked 5 minutes short of 5 hours. So, to guarantee a total of 6 hours of exercise, they must walk at least 1 hour and 5 minutes (or 65 minutes) on Sunday.

Teacher's Notes

All of the walking times for the days Tuesday through Sunday can be written as mathematical expressions in terms of the time Lindsay walks on Monday. To do this, we need to interpret the words of the problem into mathematical operations. Suppose we say that m is the number of minutes Lindsay walks on Monday. Then the following expressions describe the other times:

Tuesday: $m + 5$
We determine this by recognizing that *longer* indicates addition.

Wednesday: $(m + 5) \div 2$ or $\frac{1}{2} \cdot (m + 5)$
We determine this by starting with the expression describing Tuesday's walking time, and recognizing that *half* indicates division by 2 or multiplication by $\frac{1}{2}$.

Thursday: $m - 10$
We determine this by recognizing that *less* indicates subtraction.

Friday: $(m - 10) + 15$
We determine this by starting with the expression describing Thursday's walking time, and recognizing that *more* indicates indicates addition.

Saturday: $2 \cdot m$
We determine this by recognizing that *twice* indicates multiplication by 2.

Now the total walking time on Monday through Saturday can be written as:
$m + (m + 5) + (\frac{1}{2} \cdot (m + 5)) + (m - 10) + ((m - 10) + 15) + (2 \cdot m)$

This expression can be simplified. Here is an equivalent expression:
$6 \cdot m + (\frac{1}{2} \cdot (m + 5))$

We can use this expression to calculate the walking time on Sunday.
Sunday: $360 - (6 \cdot m + (\frac{1}{2} \cdot (m + 5))$

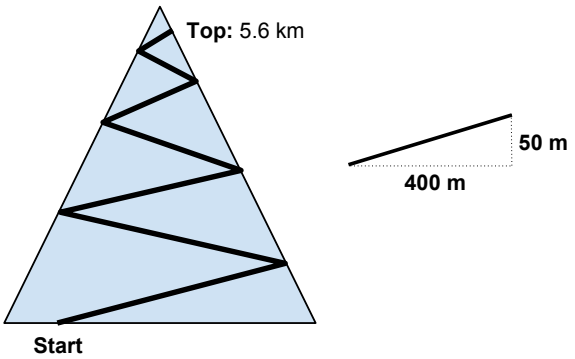
Given any value for the time Walden walks on Monday, we can calculate the times he walks on the other days. With these mathematical expressions, we could use a computer program or a spreadsheet to help calculate these values in general. To solve this particular problem, we would set the variable m to have the value 45.

Problem of the Week

Problem A

Climbing Sulphur Mountain

Climbing a mountain is tough. To make it easier and safer, people build paths that go back and forth across the mountain rather than straight up. This means that after walking for some distance along the path, you will come to a sharp corner, called *switchback*, and start walking in the opposite direction. So when you are hiking on this kind of trail, you will travel much further than than if you climbed straight up from the bottom to the top of the mountain. On average, for every 400 m you walk on the trail to the top of Sulphur Mountain, you are climbing approximately 50 m vertically up the mountain. Here is a diagram representing the trail:



The distance you travel when you walk along the trail from the start to the top is 5.6 km. What is the vertical distance from the bottom of Sulphur Mountain to the top? Justify your answer.

Strands Patterning and Algebra, Measurement, Number Sense and Numeration

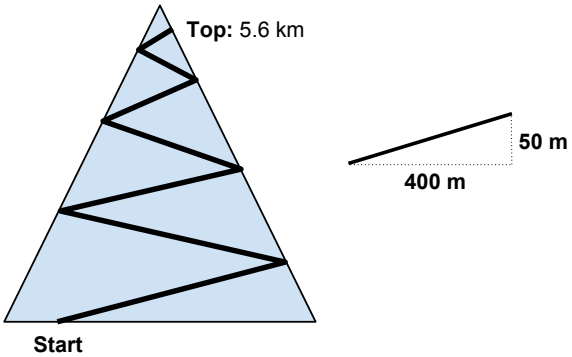
Problem of the Week

Problem A and Solution

Climbing Sulphur Mountain

Problem

Climbing a mountain is tough. To make it easier and safer, people build paths that go back and forth across the mountain rather than straight up. This means that after walking for some distance along the path, you will come to a sharp corner, called *switchback*, and start walking in the opposite direction. So when you are hiking on this kind of trail, you will travel much further than than if you climbed straight up from the bottom to the top of the mountain. On average, for every 400 m you walk on the trail to the top of Sulphur Mountain, you are climbing approximately 50 m vertically up the mountain. Here is a diagram representing the trail:



The distance you travel when you walk along the trail from the start to the top is 5.6 km. What is the vertical distance from the bottom of Sulphur Mountain to the top? Justify your answer.

Solution

One way to solve this problem is to make a table that keeps track of far you have travelled on the trail along with the vertical distance you have climbed.

Each time the trail distance increases by 400 m, the vertical distance increases by 50 m. We want the table to show the pattern until the trail distance is equal to 5.6 km. Since 1 km = 1000 m, the table needs to show the pattern until the trail distance is 5600 m.

Here is a table showing the pattern:

Trail Distance (in metres)	Vertical Distance (in metres)
400	50
800	100
1200	150
1600	200
2000	250
2400	300
2800	350
3200	400
3600	450
4000	500
4400	550
4800	600
5200	650
5600	700

So after walking 5.6 km on the trail, you have travelled approximately 700 m metres vertically up the mountain.

Teacher's Notes

The pattern shown in this problem can be described in the form of an equation of a line. In Cartesian geometry, the equation of a line can be written with this format:

$$y = mx + b$$

where m is defined as the *slope* of the line, and b is defined as the *y-intercept* of the line. The y-intercept describes the point on the line where it crosses the y-axis. In other words, it is the point on the line that satisfies the equation when $x = 0$.

The slope of a line describes a constant relationship between any two points on the line. By definition, if we choose two points that are on the same line: (x_1, y_1) and (x_2, y_2), and compare the ratio of the difference of the y values and the difference of the x values, that ratio will always be the same. We normally work with the rise (the difference in the y values) over the run (the difference in the x values), and call this value the slope. In particular:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

There are two special cases we should consider. Since lines extend infinitely, all lines will cross the y-axis at some point, unless they are parallel to the y-axis. In this case, it is not possible to find a y-intercept. For a line that is parallel to the y-axis, each point on the line will always have the same value of x. We could write the equations of lines like this in the form: $x = c$ where c is a number. The slope of a line that is parallel to the y-axis is undefined since we cannot divide by zero.

Another special case to consider are lines that are parallel to the x-axis. For these lines, each point on the line will always have the same y value. We could write the equations of lines like this in the form: $y = b$. The slope of a line that is parallel to the x-axis is equal to 0.

This problem literally describes a slope. We can think of the vertical distance we travel as being the y value, and the actual distance we walk along the trail as being the x value. The constant ratio in this case is:

$$m = \frac{rise}{run} = \frac{50}{400} = \frac{1}{8}$$

We can think of the start of the trail as being when $x = 0$ and $y = 0$. This means the y-intercept of the line describing this pattern is 0. So an equation describing the relationship between the trail distance and the vertical distance is:

$$y = \frac{1}{8}x$$

To solve this problem, we want to know the value of y when $x = 5600$.

By substitution, $y = \frac{1}{8}(5600)$ or $y = 400$. So the vertical change is 400 m.

Problem of the Week
Problem A
Scrabble Scoring

Scrabble^{TM} is a board game where players make words using tiles containing individual letters. Each of the letters have a point value which are shown in the following table:

A	B	C	D	E	F	G	H	I	J	K	L	M
1	3	3	2	1	4	2	4	1	8	5	1	3
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	1	3	10	1	1	1	1	4	4	8	4	10

A player places tiles on the board to form a word. The board is divided up into squares. Some of the squares have special values. The words are placed in a straight line horizontally (across, reading from left to right) or vertically (reading from the top down).

- Blue squares are double letter squares, which means if you put a tile on that square the letter is worth twice as many points.
- Green squares are triple letter squares, which means if you put a tile on that square the letter is worth three times as many points.
- Red squares are double word squares, which means if you put a tile on that square, the whole word is worth twice as many points.
- Orange squares are triple word squares, which means if you put a tile on that square, the whole word is worth three times as many points.
- Double and triple letter scores are calculated before the double and triple word scores.

Here is a diagram of part of the Scrabble^{TM} board.

	A	B	C	D	E	F
1	G				G	
2		R				
3			R			B
4				R		
5	G				R	
6			B			O

A) How many points would the word MATH be worth if you started the word in square B1 and placed it horizontally across the board?

B) How many points would the word ZEBRA be worth if you started the word in square D2 and placed it vertically down the board?

C) How many points would the word HAPPY be worth if you started the word in square F2 and placed it vertically down the board?

Strands Number Sense and Numeration, Geometry and Spatial Sense

Problem of the Week
Problem A and Solution
Scrabble Scoring

Problem

Scrabble^{TM} is a board game where players make words using tiles containing individual letters. Each of the letters have a point value which are shown in the following table:

A	B	C	D	E	F	G	H	I	J	K	L	M
1	3	3	2	1	4	2	4	1	8	5	1	3

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	1	3	10	1	1	1	1	4	4	8	4	10

A player places tiles on the board to form a word. The board is divided up into squares. Some of the squares have special values. The words are placed in a straight line horizontally (across, reading from left to right) or vertically (reading from the top down).

- Blue squares, marked **B** on the board below, are double letter squares, which means if you put a tile on that square the letter is worth twice as many points.
- Green squares, marked **G** on the board below, are triple letter squares, which means if you put a tile on that square the letter is worth three times as many points.
- Red squares, marked **R** on the board below, are double word squares, which means if you put a tile on that square, the whole word is worth twice as many points.
- Orange squares, marked **O** on the board below, are triple word squares, which means if you put a tile on that square, the whole word is worth three times as many points.
- Double and triple letter scores are calculated before the double and triple word scores.

Here is a diagram of part of the Scrabble^{TM} board.

	A	B	C	D	E	F
1	**G**				**G**	
2		**R**				
3			**R**			**B**
4				**R**		
5	**G**				**R**	
6			**B**			**O**

A) How many points would the word MATH be worth if you started the word in square B1 and placed it horizontally across the board?

B) How many points would the word ZEBRA be worth if you started the word in square D2 and placed it vertically down the board?

C) How many points would the word HAPPY be worth if you started the word in square F2 and placed it vertically down the board?

Solution

A) If we place the letters for **MATH** horizontally on the board starting at square B1 then the **H** will be on a green square. This means that letter is worth three times its normal value. That is, the **H** is worth $3 \times 4 = \mathbf{12}$ points when placed on the green square located in this placement of the word. So the score for this word is: $3 + 1 + 1 + \mathbf{12} = 17$ points.

B) If we place the letters for **ZEBRA** vertically down the board starting at square D2 then the word covers a red square. This means the word is worth twice its normal value. The normal score for the word would be $10 + 1 + 3 + 1 + 1 = 16$. The red square means that the word would be worth twice as much, so the score for the word at this location on the board is $2 \times 16 = 32$ points.

C) If we place the letters for **HAPPY** vertically down the board starting at square F2 then the **A** will be on a blue square. This means that letter is worth twice its normal value. That is, in this placement of the word, the letter **A** is worth $2 \times 1 = \mathbf{2}$ points. The word also covers an orange square, which means the word is worth three times its normal value. We calculate the score including the double value of the letter **A** before applying the triple word score. The score for the word without considering the triple word square, but including its double letter score, is $4 + \mathbf{2} + 3 + 3 + 4 = 16$. When we consider the triple word square, the result is $3 \times 16 = 48$ points.

Teacher's Notes

When we apply the rule that double and triple letter scores are calculated before the double and triple word scores, we are enforcing an *order of operations* to our calculations.

We often use the mnemonic **BEDMAS** to describe the standard order of operations for mathematical operators. This stands for **B**rackets, **E**xponents, **D**ivision and **M**ultiplication in the order that they appear, and **A**ddition and **S**ubtraction in the order that they appear.

We use brackets in places where we want some operation to take precedence over another operation that would otherwise happen first according to BEDMAS. For example, in part B) of this problem, we want to add the letter values together before we apply the double word score. We could calculate the result like this:

$$(10 + 1 + 3 + 1 + 1) \times 2 = (16) \times 2 = 32$$

Without the brackets the calculation

$$10 + 1 + 3 + 1 + 1 \times 2$$

would equal

$$10 + 1 + 3 + 1 + 2 = 17$$

since according to BEDMAS the multiplication is done before the addition.

In part C), we need to multiply the point value of the letter **A** by 2 since it is on a double letter square. Then we add the rest of the letter values together before multiplying that sum by 3. So we could calculate the result for part C) like this:

$$(4 + 2 \times 1 + 3 + 3 + 4) \times 3 = (4 + 2 + 3 + 3 + 4) \times 3 = (16) \times 3 = 48$$

Note that we do not need brackets around 2×1 since the multiplication will be done automatically before the addition in the calculation. However, sometimes we use brackets (even when they are not required) to make calculations clearer. It may be easier to understand the calculation for part C) if we used this expression:

$$(4 + (2 \times 1) + 3 + 3 + 4) \times 3$$

just to emphasize that the double letter score must be calculated before the sum of the letters is calculated.

Problem of the Week

Problem A

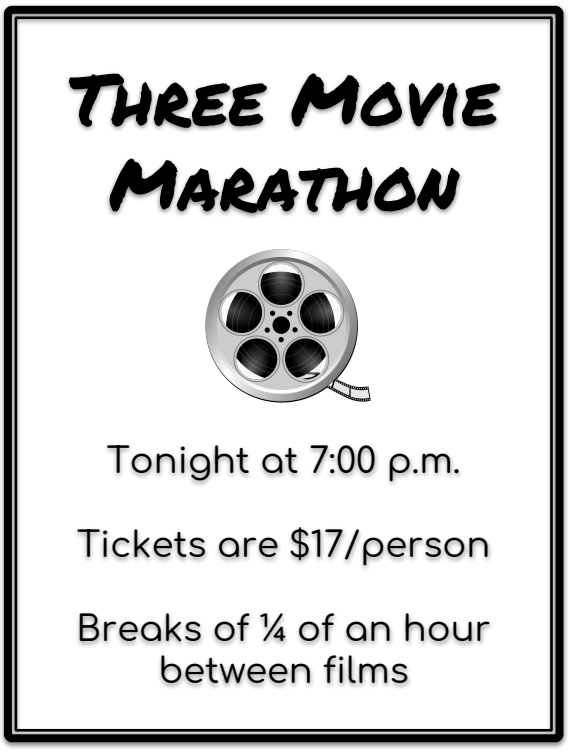
Midnight Movie Madness

Mr. and Mrs. Pretti decided that they wanted to go and watch a special three-movie marathon at the local theatre. They purchased their tickets for the marathon for $17 per person. They called their favourite babysitter to watch their kids.

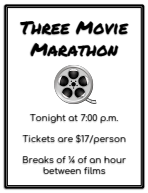
Each movie is 125 minutes long. The movie theatre has promised a break of a quarter of an hour between each movie. The drive to the theatre is 30 minutes, and the marathon starts at 7 p.m. The babysitter arrives at 6:15 p.m.

A) At approximately what time will Mr. and Mrs. Pretti arrive back home?

B) The Prettis pay their babysitter $7.50 per hour. How much will the entire night cost them?



Strands Number Sense and Numeration, Measurement, Patterning and Algebra



Problem of the Week

Problem A and Solution

Midnight Movie Madness

Problem

Mr. and Mrs. Pretti decided that they wanted to go and watch a special three-movie marathon at the local theatre. They purchased their tickets for the marathon for $17 per person. They called their favourite babysitter to watch their kids.

Each movie is 125 minutes long. The movie theatre has promised a break of a quarter of an hour between each movie. The drive to the theatre is 30 minutes, and the marathon starts at 7 p.m. The babysitter arrives at 6:15 p.m.

A) At approximately what time will Mr. and Mrs. Pretti arrive back home?

B) The Prettis pay their babysitter $7.50 per hour. How much will the entire night cost them?

Solution

A) Here is one way to determine when the Prettis will arrive home. Each movie lasts 125 minutes. There are two breaks, one between the first and second movie and one between the second and third movie. Each break is $\frac{1}{4}$ of an hour which is equal to 15 minutes. It will take the Prettis 30 minutes to drive home after the last film is done. The total time between when the first movie starts at 7:00 p.m. and when they arrive home is:

$125 + 15 + 125 + 15 + 125 + 30 = 435$ minutes.

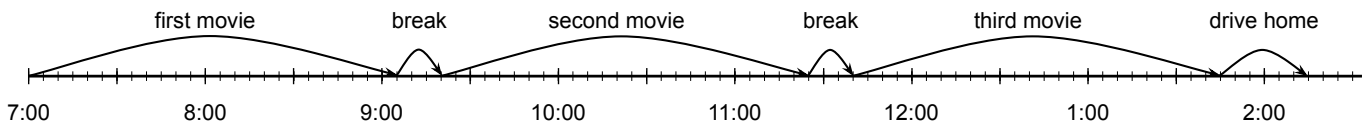
The following pattern shows the relationship between hours and minutes:

Hours	Minutes
1	60
2	120
3	180
4	240
5	300
6	360
7	420

So 435 minutes is $435 - 420 = 15$ minutes more than 7 hours.

There are 5 hours between 7:00 p.m. and midnight. This means Mr. and Mrs. Pretti would get home 2 hours and 15 minutes after midnight or 2:15 a.m.

We could also use a timeline to determine what time the Prettis arrive back home.



B) The babysitter arrived at 6:15 which is 45 minutes before 7:00 p.m. This means the Prettis need to pay for a total of $435 + 45 = 480$ minutes of time. If we continue the pattern from part A), 8 hours = 480 minutes. Now we can find a pattern between the number of hours and the amount the babysitter is paid.

Hours	**Total Paid (in \$)**
1	7.50
2	15.00
3	22.50
4	30.00
5	37.50
6	45.00
7	52.50
8	60.00

Since the Prettis needed to buy two tickets for the movie marathon, this cost $2 \times \$17 = \$34$. The total cost of the evening out is $\$34 + \$60 = \$94$.

Teacher's Notes

The solution for part B) converts minutes to hours to determine how much to pay the babysitter. Here is another way to solve that problem. We know that 480 minutes passed from the time that the babysitter arrived until the time that the Prettis returned home. Rather than converting this time to hours, we could determine how much the babysitter gets paid per minute rather than per hour.

A rate of \$7.50 per hour is equal to 750 cents per 60 minutes. We can determine the rate per minute as $750 \div 60 = 12.5$ cents per minute.

So for 480 minutes, the babysitter should be paid $480 \times 12.5 = 6000$ cents. This is equal to \$60.00.

Problem of the Week

Problem A

Carpet Caper

Bill and Nikki were renovating a basement apartment together. They removed all the carpet to replace it with new flooring. After removing the carpet they took it to the landfill where you pay for the mass of your garbage. At the landfill, they were told that the mass of the carpet was 290 kg. Each square metre of carpet has a mass of 5 kg.

A) How many square metres of carpet did Bill and Nikki remove?

B) They are replacing the carpet with laminate flooring. Each box contains 3 square metres of laminate flooring. How many boxes of flooring do they need?

Strands Measurement, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Carpet Caper

Problem

Bill and Nikki were renovating a basement apartment together. They removed all the carpet to replace it with new flooring. After removing the carpet they took it to the landfill where you pay for the mass of your garbage. At the landfill, they were told that the mass of the carpet was 290 kg. Each square metre of carpet has a mass of 5 kg.

A) How many square metres of carpet did Bill and Nikki remove?

B) They are replacing the carpet with laminate flooring. Each box contains 3 square metres of laminate flooring. How many boxes of flooring do they need?

Solution

A) Here is one way to calculate the area of the carpet that was removed.
Since each square metre of carpet has a mass of 5 kg, then 2 square metres has a mass of $2 \times 5 = 10$ kg. In other words, each 10 kg of the mass comes from 2 square metres of the carpet.
The total mass of the carpet is $29 \times 10 = 290$ kg.
So the total area of the carpet is $29 \times 2 = 58$ square metres.

B) Knowing that Bill and Nikki need at least 58 m^2 of laminate flooring, and that each box contains 3 m^2 of flooring, then we can use division to determine how many boxes they need. We calculate $58 \div 3 = 19$ with a remainder of 2. Since they cannot buy a part of a box, then Bill and Nikki will need to buy 20 boxes of flooring.

Teacher's Notes

In part B) of this question, you were required to determine the number of boxes of laminate flooring. Since you cannot purchase part of a box, in this situation you need to calculate the number of boxes that will cover **at least** 58 square metres. The correct result can be determined by rounding up the result of the division. The action of rounding up can be represented in a mathematical expression using the *ceiling* notation. In this case the actual answer could be calculated this way:

$$\lceil 58 \div 3 \rceil = 20$$

The result of a division calculation could be an integer or a non-integer value. However, the result of calculating the ceiling is always an integer value. In the case where the result of the division is an integer, then the ceiling of the division is equal to the quotient.

For example,

$$30 \div 5 = \lceil 30 \div 5 \rceil = 6$$

However,

$$58 \div 3 \neq \lceil 58 \div 3 \rceil$$

Problem of the Week

Problem A

Circular Calculations

Robbie was practising his math and discovered some interesting sequences.

A) Robbie picked a random number to start. Then he wrote out that number in words and counted its number of characters. Then he wrote the resulting number, and counted its number of characters. He continued this sequence for as long as he could. For example when he started with seventy-eight, this was the result:

seventy-eight $\rightarrow$ (13) thirteen $\rightarrow$ (8) eight $\rightarrow$ (5) five $\rightarrow$ (4) four

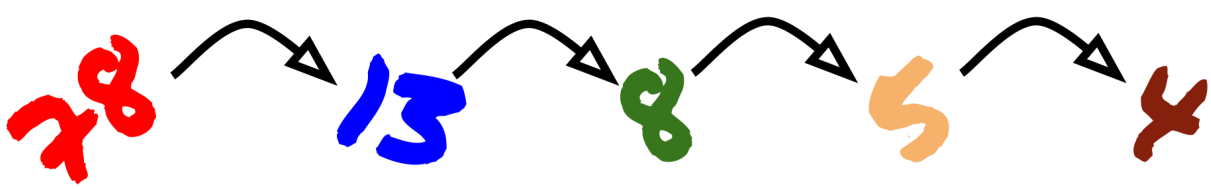
Try following this procedure with other numbers to start. Do you see any pattern? Do you expect to see the same result for any starting number?

B) Robbie discovered another interesting result. Follow this procedure:

- Start with any positive integer.
- If the number is even, divide by 2. For example if the number is 26, the next number in the sequence is 13.
- If the number is odd, multiply by 3 and add 1. For example, if the number is 13, the next number in the sequence is $(13 \times 3) + 1 = 40$.
- Continue the sequence as long as you can.

Try this procedure with the following numbers to start: 5, 26, 15, 50. Do you see a pattern? Do you expect to see the same result for any starting number?

Challenge: Try this procedure starting with 27.



Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Circular Calculations

Problem

Robbie was practising his math and discovered some interesting sequences.

A) Robbie picked a random number to start. Then he wrote out that number in words and counted its number of characters. Then he wrote the resulting number, and counted its number of characters. He continued this sequence for as long as he could. For example when he started with seventy-eight, this was the result:

$$\text{seventy-eight} \to (13)\ \text{thirteen} \to (8)\ \text{eight} \to (5)\ \text{five} \to (4)\ \text{four}$$

Try following this procedure with other numbers to start. Do you see any pattern? Do you expect to see the same result for any starting number?

B) Robbie discovered another interesting result. Follow this procedure:

- Start with any positive integer.
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- Continue the sequence as long as you can.

Try this procedure with the following numbers to start: 5, 26, 15, 50. Do you see a pattern? Do you expect to see the same result for any starting number?

Challenge: Try this procedure starting with 27.

Solution

A) All of the sequences will converge to *four*. Interestingly, this will be true even with spelling mistakes. For example, the sequence will be different if you start with *fourty-three* instead of *forty-three*, but both sequences will converge to *four*.

Credit to Lorna Morrow

B) All of the sequences will converge to 1.

- 5, 16, 8, 4, 2, 1
- 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
- 15, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1
- 50, 25, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

Teacher's Notes

We could make a logical argument that the sequence in part A) will always converge to *four*.

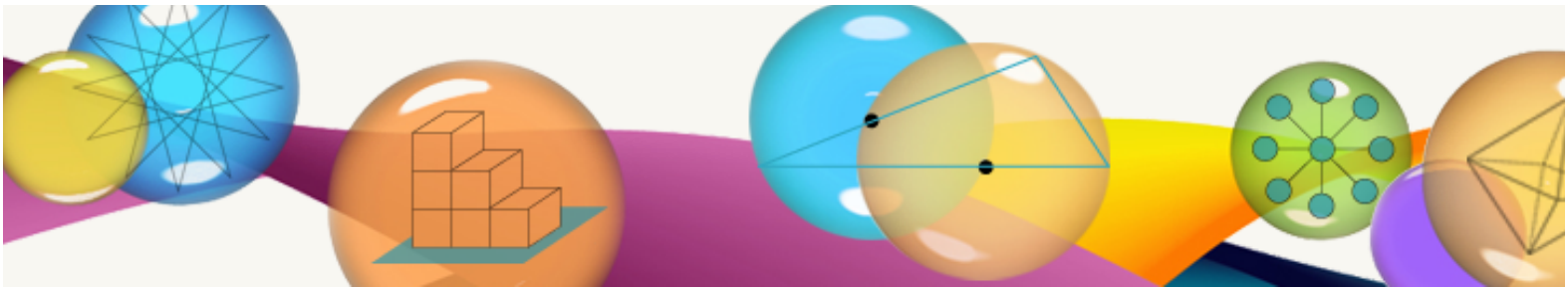
- The number *four* has 4 characters. So the sequence will stop at this point.
- The numbers *five* and *nine* each have 4 characters. So they are one step away from the end of the sequence.
- The numbers *three*, *seven*, and *eight* each have 5 characters. So they are two steps away from the end of the sequence.
- The numbers *one*, *two*, *six*, and *ten* each have 3 characters. So they are three steps away from the end of the sequence.

This covers all cases of the numbers from 1 to 10. We can argue that all numbers that are greater than 10 have fewer characters in their written versions than their individual numerical values. For example, *seventeen* is the number between 11 and 20 that has the most characters in its name (9 characters). So, the next step in the sequence for any number that is greater than 10 must be a smaller number. This means eventually, the sequence will lead to a number that is less than or equal to 10. Once we get to a point in the sequence where we have a number between 1 and 10, we are at most three steps away from the end of the sequence. So this sequence will always converge to *four*.

Although it takes a long time when you start with the number 27, using the procedure in part B), the sequence will converge to 1. However it takes 111 steps.

> 27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1

Mathematicians believe that starting with any positive integer, the steps described in part B) will always lead to a sequence that converges to 1. This is known as the *Collatz Conjecture*. However, proving this is true for all positive integers is an open problem. There is experimental evidence that shows this is true for very large numbers; however, there is no formal proof that the conjecture holds for all positive integers.



Patterning & Algebra

TAKE ME TO THE COVER

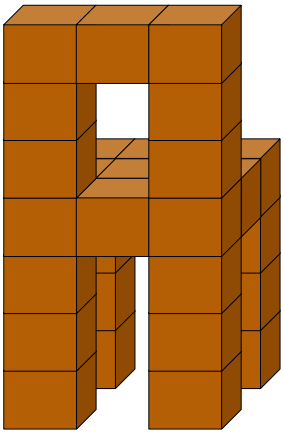
Problem of the Week

Problem A

Block Furniture

The Suite Deal Furniture company is famous for their best selling block furniture that is built out of interlocking wooden cubes.

A) Use the diagram below, to determine the number of blocks required to build the chair.



B) Suite Deal wants to make a larger chair, by increasing the leg height by 1 and the seat width and depth by 1. How many blocks are required to build the larger version?

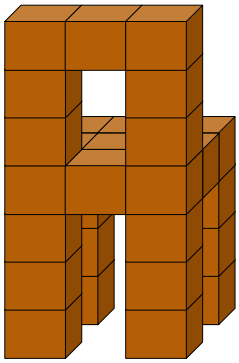
Strands Geometry and Spatial Sense, Patterning and Algebra

Problem of the Week
Problem A and Solution
Block Furniture

Problem

The Suite Deal Furniture company is famous for their best selling block furniture that is built out of interlocking wooden cubes.

A) Use the diagram below, to determine the number of blocks required to build the chair.



B) Suite Deal wants to make a larger chair, by increasing the leg height by 1 and the seat width and depth by 1. How many blocks are required to build the larger version?

Solution

A) From what we can see in the diagram, the seat of the chair is 3×3 blocks. This is a total of 9 blocks for the seat. Each of the four legs of the chair are 3 blocks tall, so this is a total of $4 \times 3 = 12$ blocks for the legs. We can count the blocks that form the back of the chair and see that there are a total of 7 blocks for the back. This means it takes a total of $9 + 12 + 7 = 28$ blocks to build the chair in the diagram.

B) If the seat width and depth of the chair are increased by 1, then the seat is formed using $4 \times 4 = 16$ blocks. If the leg height of the chair is increased by 1, then each leg would be 4 blocks tall, so you would have $4 \times 4 = 16$ blocks for the legs. The height of the back of the chair is unchanged, but when the seat width increases by 1, you will need 4 blocks across the top of the back. This is one more block than the original for a total of 8 blocks for the back. So, the total number of blocks required is $16 + 16 + 8 = 40$ blocks.

Alternatively we can count the extra blocks we would need to build the bigger chair. We would need to add 1 block to each leg for a total of 4 blocks. We would need to add 7 blocks to the seat, since a 4×4 square has 7 more units than a 3×3 square. And, we would need to add one more block to the back of the chair to handle the new seat width. This means we would need a total of $28 + 4 + 7 + 1 = 40$ blocks for the bigger chair.

Teacher's Notes

This problem illustrates the literal relationship between *linear* and *square* functions. To calculate the square of a number, we could build a square out of blocks. For example, if we want to know the value of 3^2 we can build a square like the seat of the chair. The seat has dimensions 3×3 and there are 9 blocks that form the seat. Thus $3^2 = 9$. Similarly, we can calculate the cube of a number by building a cube out of blocks. For example, the value of 4^3 is equal to the total number of blocks required to build a $4 \times 4 \times 4$ cube. If we count the number of blocks in that cube we would see that we needed 64 in total. Thus $4^3 = 64$.

Another thing we can notice from this problem is that a small change in a linear value leads to a much bigger change in the square value. When we increased the width of the seat from 3 to 4, the number of blocks required to build the square seat increased from 9 to 16. More generally, if we consider any number x that is greater than or equal to 1, and know that another number y is double the size of x, then we can prove that y^2 is always four times as big as x^2. We can check this result with a few examples:

If $x = 5$, then $y = 10$
Therefore, $x^2 = 25$ and $y^2 = 100$
Since $4 \times 25 = 100$ we can say $4 \times x^2 = y^2$

or

If $x = 30$, then $y = 60$
Therefore $x^2 = 900$ and $y^2 = 3600$
Since $4 \times 900 = 3600$ we can say $4 \times x^2 = y^2$

We can prove the general case with algebra:

If $y = 2x$, then $y^2 = (2x)^2$
$(2x)^2 = (2x) \times (2x) = 4x^2$
Therefore, y^2 is always 4 times the size of x^2.

Problem of the Week

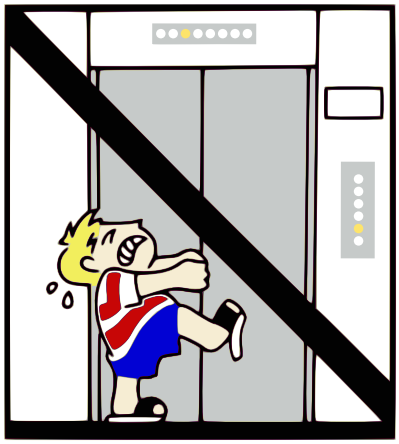
Problem A

Elevator

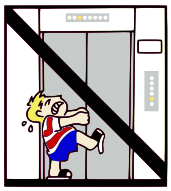
The small school elevator can hold a maximum weight of 227 kg.

A) The average weight of a 9 year old is 28 kg. How many 9 year olds could ride the school elevator safely?

B) The average adult weighs 73 kg. What is the maximum number of adults we expect could ride the school elevator?



Strands Patterning and Algebra, Number Sense and Numeration



Problem of the Week

Problem A and Solution

Elevator

Problem

The small school elevator can hold a maximum weight of 227 kg.

A) The average weight of a 9 year old is 28 kg. How many 9 year olds could ride the school elevator safely?

B) The average adult weighs 73 kg. What is the maximum number of adults we expect could ride the school elevator?

Solution

A) One way to solve this problem is to use a table to keep track of the total weight of the people on the elevator.

Number of Students	Weight (kg)
1	28
2	56
3	84
4	112
5	140
6	168
7	196
8	224
9	252

So the maximum number of 9 year olds that we expect can safely ride the elevator would be 8. Alternatively, we could use division. We can calculate $227 \div 28 = 8$ remainder 3. Since we are looking for a maximum number of riders, and the answer must be a whole number, then the maximum number would be 8.

B) Using a table or division ($227 \div 73 = 3$ remainder 9) we can determine that the maximum number of adults that we expect can safely ride the elevator would be 3. Alternatively, we could estimate the average weight of the adults as 70 kg and the capacity of the elevator as 230 kg. We can see that 3 adults would weigh approximately 210 kg, so we expect that is the maximum number that would safely fit in the elevator.

Teacher's Notes

In part A of this question, when we used division to calculate how many students would be able to safely ride the elevator we got an answer of 8 remainder 3. Since the answer is not a whole number, we need to decide if our final answer needs to be rounded up or rounded down. We need to look for cues in the question to help. In this case the final answer must be a whole number, and we are looking for a maximum capacity. This means we need to round down, since rounding up will exceed the capacity.

In mathematics, we use the operations *floor* and *ceiling* to indicate if our calculated value should be rounded down or rounded up. So for this problem, we would use the floor operation. Symbolically, this would be the calculation:

$$\lfloor 227 \div 28 \rfloor = 8$$

In other cases you will need to round up. For example, suppose you were buying eggs for use at a restaurant. You know that you need 500 eggs each day, and that a carton contains one dozen eggs. If you need to know how many cartons of eggs are required each day, the calculation could be done by using division. The answer will need to be rounded up (i.e. find the ceiling), since you need at least 500 eggs, but you need to purchase a whole number of cartons. Symbolically, this would be the calculation:

$$\lceil 500 \div 12 \rceil = 42$$

Problem of the Week

Problem A

Fall Fair Fare

Avneet is at the Fall Fair with her friends. They plan to go on 10 rides each. They are trying to figure out which is the best price to pay for admission. The fair has two options:

Option #1
- Admission: \$15
- Single ride ticket: \$3

Option #2
- Admission including unlimited ride wristband: \$35

A) Which option should Avneet choose? Justify your answer.

B) What is the most number of rides Avneet could plan to go on that would make Option #1 the better choice? Justify your answer.

Strands Number Sense and Numeration, Patterning and Algebra

Problem of the Week
Problem A and Solution
Fall Fair Fare

Problem

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Option #1
- Admission: \$15
- Single ride ticket: \$3

Option #2
- Admission including unlimited ride wristband: \$35

A) Which option should Avneet choose? Justify your answer.

B) What is the most number of rides Avneet could plan to go on that would make Option #1 the better choice? Justify your answer.

Solution

A) If Avneet chooses Option #1 and she goes on 10 rides, the ride tickets will cost $10 \times 3 = \$30$. Then the total cost for the fair with this option would be $15 + 30 = \$45$. Since Option #2 costs \$35, this would be the better choice.

B) One way to figure out this would be to make a table showing the number of rides and the cost of Option #1.

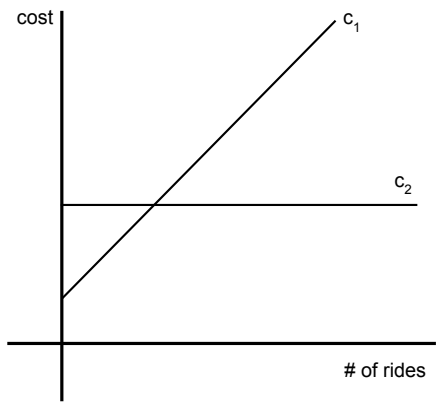
# of Rides	Total Cost (\$)
0	15
1	18
2	21
3	24
4	27
5	30
6	33
7	36

So the most number of rides you can take to make Option #1 the better choice would be 6. Alternatively, since we must pay \$15 for Option #1 no matter how many rides we take, and we must pay \$35 for Option #2 no matter how many rides we take, then the difference between these amounts describes how much of the admission price for Option #2 is covering the cost of the individual rides. This difference is $35 - 15 = \$20$. If we had \$20 for rides that cost \$3 each, we can figure out how many rides we can take by using division: $20 \div 3 = 6$ remainder 2. Since there would be \$2 unspent with Option #2 when we take 6 rides, then Option #1 would be a better choice.

Teacher's Notes

In this problem, we could describe these two options as mathematical functions.
Option #1 could be written as $c_1 = 3 \cdot r + 15$, where r is the number of rides a person takes.
Option #2 could be written as $c_2 = 35$.

If can describe options using mathematical functions, then we have the ability to analyze and compare those functions. In this case, the function describing Option #1 is a **linear** function. This means that the value of c_1 grows at approximately the same rate as the number of rides that Avneet takes. The function describing Option #2 is a **constant** function. This means that the value of c_2 is unaffected by the number of rides that Avneet takes. We can draw a graph showing the difference:



At some point, the linear function crosses the constant function. After that point, the cost of the linear function is always greater than the cost of the constant function.

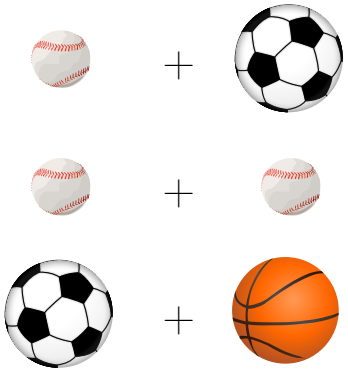
In computer science, we often use broad categories to compare different functions. Generally, we expect a constant function to be more efficient than a linear function, although, as we see in this problem, a constant function is not necessarily better for **all** values. However, since we normally care about efficiency when dealing with large values, we usually ignore the small cases where the linear function performs better.

Problem of the Week

Problem A

Sporting Sleuth

A) A physical education teacher weighed some of the equipment in the gym. She recorded the following measurements:



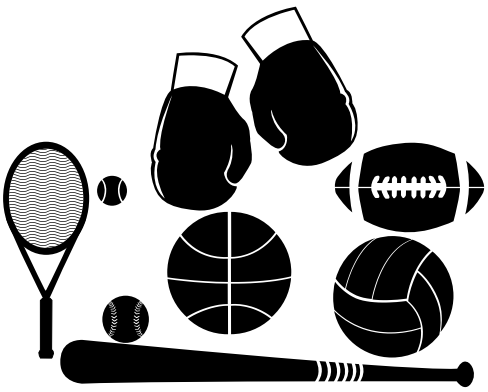
530 grams

280 grams

890 grams

Determine the mass of each type of ball: baseball soccer ball basketball

B) A gym bag contains three baseballs, a soccer ball and two basketballs. What is the approximate total mass of the gym equipment?



Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Sporting Sleuth

Problem

A) A physical education teacher weighed some of the equipment in the gym. She recorded the following measurements:

 + = **530 grams**

 + = **280 grams**

 + = **890 grams**

Determine the mass of each type of ball: baseball soccer ball basketball

B) A gym bag contains three baseballs, a soccer ball and two basketballs. What is the approximate total mass of the gym equipment?

Solution

A) From the second equation we know that two baseballs weigh 280 grams. So each baseball is $280 \div 2 = 140$ grams. We can use that information and the first equation to determine that a soccer ball weighs 140 grams less than 530 grams. So a soccer ball weighs $530 - 140 = 390$ grams. Looking at the last equation, now we know that a basketball weighs 390 grams less than 890 grams. So a basketball weighs $890 - 390 = 500$ grams.

B) Using the results from the first part, we can calculate that three baseballs will weigh $3 \times 140 = 420$ grams. We can also calculate the mass of two basketballs as $2 \times 500 = 1000$ grams. So the total mass of the gym equipment would be $420 + 1000 + 390 = 1810$ grams.

Teacher's Notes

The images of baseballs, soccer balls and basketballs are acting like variables in mathematical equations. We could rewrite the statements algebraically with more traditional symbols for variables such as x, y, and z.

Let x be the weight of a baseball.
Let y be the the weight of a soccer ball.
Let z be the weight of a basketball.

Therefore:

$$x + y = 530 \tag{1}$$
$$2x = 280 \tag{2}$$
$$y + z = 890 \tag{3}$$

Now, if we wanted, we can use standard mathematical methods to solve the equations and find the mass of each piece of sports equipment.
From equation (2), we can divide by 2 on both sides, giving us the following:

$$x = 140 \tag{4}$$

Knowing the value of x, we can use *substitution* to solve the other two equations:
Substituting $x = 140$ into equation (1) we get:

$$(140) + y = 530 \tag{5}$$
$$140 - 140 + y = 530 - 140 \tag{6}$$
$$y = 390 \tag{7}$$

Substituting $y = 390$ into equation (3) we get:

$$(390) + z = 890 \tag{8}$$
$$390 - 390 + z = 890 - 390 \tag{9}$$
$$z = 500 \tag{10}$$

Using images or symbols in equations is an example of *abstraction*. The idea of abstraction is to take a real life situation and create a mathematical model. Once the model has been created, then it is possible to use known mathematical techniques to solve the problem. When solving the problem, the actual symbols used to represent the real life situation do not affect the outcome.

Problem of the Week

Problem A

Chasing the Great One

Auston McCrosby (a professional hockey player) has earned 1000 points (goals and assists) in his career so far. Wayne Gretzky, the highest point getter of all time in the National Hockey League, scored 2875 points in his career. If Auston averages 2 points per game in an 82 game season, approximately how many more seasons will he need to complete in order to reach Wayne Gretzky's record.

Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Chasing the Great One

Problem

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Solution

If Auston averages 2 points per game and there are 82 games in a season then he is expected to earn $2 \times 82 = 164$ points in each season. With this information we can make a table that estimates how many points he has accumulated after each season.

Year	Points in Season	Accumulated Points
Start		1000
1	164	1164
2	164	1328
3	164	1492
4	164	1656
5	164	1820
6	164	1984
7	164	2148
8	164	2312
9	164	2476
10	164	2640
11	164	2804
12	164	2968

So after 12 more seasons we expect that Auston will have accumulated more points than Wayne Gretzky.

Alternatively, we can calculate the number of points that Auston needs to earn to match Wayne Gretzky as $2875 - 1000 = 1875$. Then we can divide by 164 to determine how many years we expect it will take to accumulate 1875 more points. So we calculate $1875 \div 164 = 11$ remainder 71. Since there is a remainder with this division, we expect that it will take 12 seasons for Auston to surpass Wayne Gretzky's record.

It is also possible to use estimation to predict how many more seasons it will take Auston to reach Gretzky's record. We expect that Auston will score approximately 160 points per season. After 10 seasons, we expect him to score approximately 1600 points, so he would have accumulated approximately 2600 points. Since he needs to accumulate approximately 2900 points, he will need at least two more years where he would be expected to accumulate approximately 320 points. This means we expect it would take a total of 12 years for Auston to surpass the record.

Teacher's Notes

Another way to solve this problem would be to use a spreadsheet or to write a program to calculate how many goals Auston is predicted to have scored at the end of each year. Here is a program written in Python that would compute the prediction:

```
CURRENT_GOALS = 1000
GOALS_PER_GAME = 2
GAMES_PER_SEASON = 82
GOALS_PER_SEASON = GOALS_PER_GAME * GAMES_PER_SEASON
RECORD_GOALS = 2875

year = 0
accumulated_goals = CURRENT_GOALS
while accumulated_goals <= RECORD_GOALS:
    year = year + 1
    accumulated_goals = accumulated_goals + GOALS_PER_SEASON

print(year)
```

The values at the beginning of the program such as CURRENT_GOALS and GOALS_PER_GAME are called constants. These are values that can be easily changed to do the same calculation for other players with other numbers. It would also be easy to make the program interactive so you can ask someone to enter the starting number of goals and goals per game for a particular player and then calculate how many seasons it would take to surpass the record.

The main calculations happen starting at the line year = 0. The program initializes the values of two variables that are keeping track of the **year** and the **accumulated_goals**.

The line:

while accumulated_goals <= RECORD_GOALS:

tells the computer to repeat the next two instructions as long as the number of accumulated goals is less than the record number of goals. The repeated instructions are to increase the year by 1, and to increase the accumulated goals by the number of goals expected to be scored in a season. When the number of accumulated goals is greater than the record, the repetition will stop. At that point the program will continue to the last line which is and instruction to print the year.

Since the year has been increasing by 1 as long as the number of accumulated goals is less than the record, the value printed at the end will be the year in which the total number of goals surpasses the record.

Problem of the Week

Problem A

Patterned Savings

Rebecca wants to save money starting on January 1. She decided to collect money in a jar in the following way. Every day she puts a quarter in the jar. Every third day, starting on January 3, she puts a loonie in the jar. Every fifth day, starting on January 5, she puts a toonie in the jar. So some days she puts one coin in the jar, some days she puts two coins in the jar, and some days she puts three coins in the jar.

In Canada, a quarter is worth 25 cents, a loonie is worth one dollar, and a toonie is worth two dollars. There are 100 cents in one dollar.

A) How many coins does she put in the jar on January 12?

B) How many coins does she put in the jar on January 26?

C) How many coins does she put in the jar on January 30?

D) How many coins in total does she have in the jar by the end of January?

E) If she keeps saving this way throughout the year, how much money will she have after 90 days? Try to figure this out without counting the money she puts in the jar every day for all 90 days.

Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week

Problem A and Solution

Patterned Savings

Problem

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D) How many coins in total does she have in the jar by the end of January?

E) If she keeps saving this way throughout the year, how much money will she have after 90 days? Try to figure this out without counting the money she puts in the jar every day for all 90 days.

Solution

One way to answer most of these questions is to make a table that keeps track of how many coins are deposited each day and how much money is accumulated each day.

Day	Quarters Added	Loonies Added	Toonies Added	Money Added ($)	Total Coins	Total Money Money ($)
1	1	0	0	0.25	1	0.25
2	1	0	0	0.25	2	0.50
3	1	1	0	1.25	4	1.75
4	1	0	0	0.25	5	2.00
5	1	0	1	2.25	7	4.25
6	1	1	0	1.25	9	5.50
7	1	0	0	0.25	10	5.75
8	1	0	0	0.25	11	6.00
9	1	1	0	1.25	13	7.25
10	1	0	1	2.25	15	9.50
11	1	0	0	0.25	16	9.75
12	1	1	0	1.25	18	11.00
13	1	0	0	0.25	19	11.25
14	1	0	0	0.25	20	11.50
15	1	1	1	3.25	23	14.75

We could continue the table, and we would see that the amounts of coins and money added would be repeated exactly every 15 days. From this table, we observe that on every day that is a multiple of 3 (except on day 15) Rebecca adds two coins to the jar, and on every day that is a multiple of five (except on day 15) Rebecca adds two coins to the jar. On day 15, which is a multiple of both 3 and 5, Rebecca adds three coins to the jar. On all other days, Rebecca adds just one coin to the jar.

A) From the table, we see that, on January 12, Rebecca added 2 coins to the jar.

B) Since 26 is neither a multiple of 3 nor a multiple of 5, Rebecca will only add 1 coin to the jar.

C) Since 30 is a multiple of both 3 and 5, Rebecca will add 3 coins to the jar.

D) From the table, we see that after 15 days, Rebecca will have saved 23 coins. This pattern will be repeated in the next 15 days. On January 31, she will add one more quarter. So, by January 31 Rebecca will have $23 + 23 + 1 = 47$ coins in the jar.

E) Every 15 days Rebecca will have saved a total of \$14.75. This pattern will be repeated 6 times over a 90 day period. So after 90 days Rebecca will have $14.75 + 14.75 + 14.75 + 14.75 + 14.75 + 14.75 = \$88.50$. Alternatively we could calculate the savings as $6 \times 14.75 = \$88.50$.

Teacher's Notes

This problem shows a repeating pattern every 15 days. In mathematics, we could refer to this kind of repetition as a *periodic function*. The length of the interval between repeated elements is known as the *period* of the function. In this case, the period of the savings function is 15, which is the *least common multiple* or *LCM* of the values 1, 3, and 5. The LCM of a set of numbers is the smallest number that is a multiple of each element of the set. In this case, we are looking for the smallest multiple of each of the individual periods of savings.

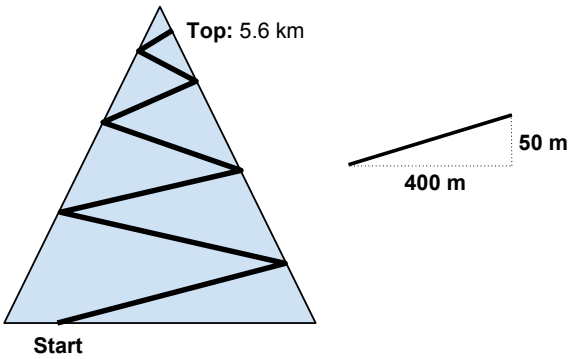
Periodic functions appear in mathematics and in the real world. Trigonometric functions such as *sin*, *cos*, and *tan* are periodic functions. Sound waves, phases of the moon, and your blood pressure, are all examples of periodic functions in nature.

Problem of the Week

Problem A

Climbing Sulphur Mountain

Climbing a mountain is tough. To make it easier and safer, people build paths that go back and forth across the mountain rather than straight up. This means that after walking for some distance along the path, you will come to a sharp corner, called *switchback*, and start walking in the opposite direction. So when you are hiking on this kind of trail, you will travel much further than than if you climbed straight up from the bottom to the top of the mountain. On average, for every 400 m you walk on the trail to the top of Sulphur Mountain, you are climbing approximately 50 m vertically up the mountain. Here is a diagram representing the trail:



The distance you travel when you walk along the trail from the start to the top is 5.6 km. What is the vertical distance from the bottom of Sulphur Mountain to the top? Justify your answer.

Strands Patterning and Algebra, Measurement, Number Sense and Numeration

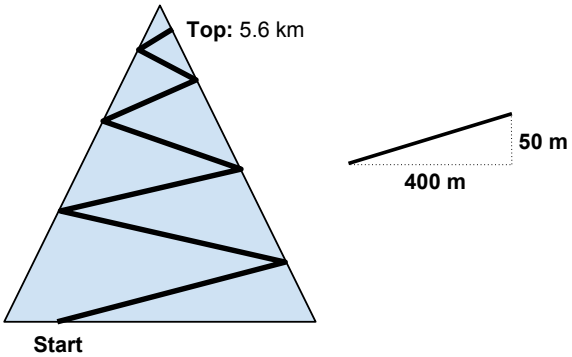
Problem of the Week

Problem A and Solution

Climbing Sulphur Mountain

Problem

Climbing a mountain is tough. To make it easier and safer, people build paths that go back and forth across the mountain rather than straight up. This means that after walking for some distance along the path, you will come to a sharp corner, called *switchback*, and start walking in the opposite direction. So when you are hiking on this kind of trail, you will travel much further than than if you climbed straight up from the bottom to the top of the mountain. On average, for every 400 m you walk on the trail to the top of Sulphur Mountain, you are climbing approximately 50 m vertically up the mountain. Here is a diagram representing the trail:



The distance you travel when you walk along the trail from the start to the top is 5.6 km. What is the vertical distance from the bottom of Sulphur Mountain to the top? Justify your answer.

Solution

One way to solve this problem is to make a table that keeps track of far you have travelled on the trail along with the vertical distance you have climbed.

Each time the trail distance increases by 400 m, the vertical distance increases by 50 m. We want the table to show the pattern until the trail distance is equal to 5.6 km. Since 1 km = 1000 m, the table needs to show the pattern until the trail distance is 5600 m.

Here is a table showing the pattern:

Trail Distance (in metres)	Vertical Distance (in metres)
400	50
800	100
1200	150
1600	200
2000	250
2400	300
2800	350
3200	400
3600	450
4000	500
4400	550
4800	600
5200	650
5600	700

So after walking 5.6 km on the trail, you have travelled approximately 700 m metres vertically up the mountain.

Teacher's Notes

The pattern shown in this problem can be described in the form of an equation of a line. In Cartesian geometry, the equation of a line can be written with this format:

$$y = mx + b$$

where m is defined as the *slope* of the line, and b is defined as the *y-intercept* of the line. The y-intercept describes the point on the line where it crosses the y-axis. In other words, it is the point on the line that satisfies the equation when $x = 0$.

The slope of a line describes a constant relationship between any two points on the line. By definition, if we choose two points that are on the same line: (x_1, y_1) and (x_2, y_2), and compare the ratio of the difference of the y values and the difference of the x values, that ratio will always be the same. We normally work with the rise (the difference in the y values) over the run (the difference in the x values), and call this value the slope. In particular:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

There are two special cases we should consider. Since lines extend infinitely, all lines will cross the y-axis at some point, unless they are parallel to the y-axis. In this case, it is not possible to find a y-intercept. For a line that is parallel to the y-axis, each point on the line will always have the same value of x. We could write the equations of lines like this in the form: $x = c$ where c is a number. The slope of a line that is parallel to the y-axis is undefined since we cannot divide by zero.

Another special case to consider are lines that are parallel to the x-axis. For these lines, each point on the line will always have the same y value. We could write the equations of lines like this in the form: $y = b$. The slope of a line that is parallel to the x-axis is equal to 0.

This problem literally describes a slope. We can think of the vertical distance we travel as being the y value, and the actual distance we walk along the trail as being the x value. The constant ratio in this case is:

$$m = \frac{rise}{run} = \frac{50}{400} = \frac{1}{8}$$

We can think of the start of the trail as being when $x = 0$ and $y = 0$. This means the y-intercept of the line describing this pattern is 0. So an equation describing the relationship between the trail distance and the vertical distance is:

$$y = \frac{1}{8}x$$

To solve this problem, we want to know the value of y when $x = 5600$.

By substitution, $y = \frac{1}{8}(5600)$ or $y = 400$. So the vertical change is 400 m.

Problem of the Week

Problem A

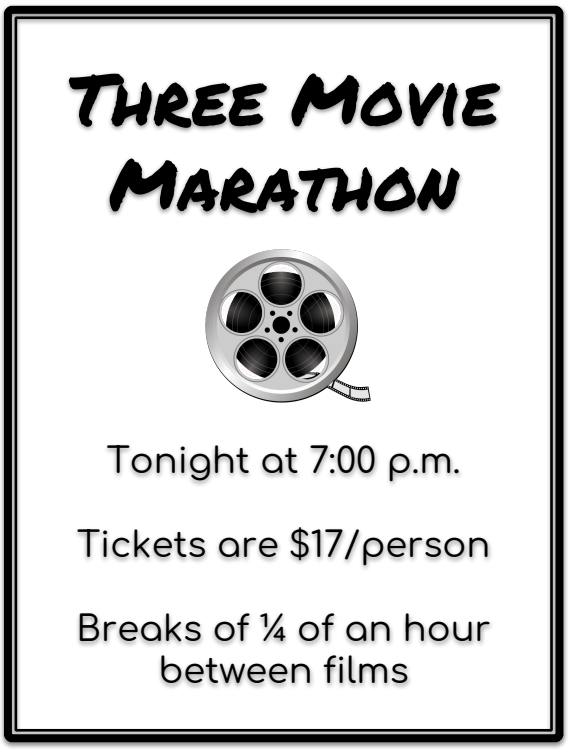
Midnight Movie Madness

Mr. and Mrs. Pretti decided that they wanted to go and watch a special three-movie marathon at the local theatre. They purchased their tickets for the marathon for $17 per person. They called their favourite babysitter to watch their kids.

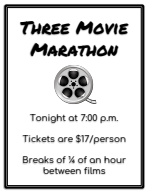
Each movie is 125 minutes long. The movie theatre has promised a break of a quarter of an hour between each movie. The drive to the theatre is 30 minutes, and the marathon starts at 7 p.m. The babysitter arrives at 6:15 p.m.

A) At approximately what time will Mr. and Mrs. Pretti arrive back home?

B) The Prettis pay their babysitter $7.50 per hour. How much will the entire night cost them?



Strands Number Sense and Numeration, Measurement, Patterning and Algebra



Problem of the Week

Problem A and Solution

Midnight Movie Madness

Problem

Mr. and Mrs. Pretti decided that they wanted to go and watch a special three-movie marathon at the local theatre. They purchased their tickets for the marathon for $17 per person. They called their favourite babysitter to watch their kids.

Each movie is 125 minutes long. The movie theatre has promised a break of a quarter of an hour between each movie. The drive to the theatre is 30 minutes, and the marathon starts at 7 p.m. The babysitter arrives at 6:15 p.m.

A) At approximately what time will Mr. and Mrs. Pretti arrive back home?

B) The Prettis pay their babysitter $7.50 per hour. How much will the entire night cost them?

Solution

A) Here is one way to determine when the Prettis will arrive home. Each movie lasts 125 minutes. There are two breaks, one between the first and second movie and one between the second and third movie. Each break is $\frac{1}{4}$ of an hour which is equal to 15 minutes. It will take the Prettis 30 minutes to drive home after the last film is done. The total time between when the first movie starts at 7:00 p.m. and when they arrive home is:

$125 + 15 + 125 + 15 + 125 + 30 = 435$ minutes.

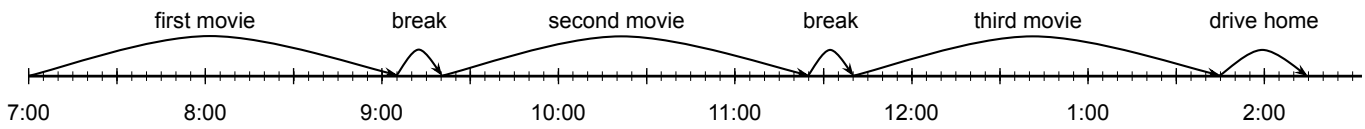
The following pattern shows the relationship between hours and minutes:

Hours	Minutes
1	60
2	120
3	180
4	240
5	300
6	360
7	420

So 435 minutes is $435 - 420 = 15$ minutes more than 7 hours.

There are 5 hours between 7:00 p.m. and midnight. This means Mr. and Mrs. Pretti would get home 2 hours and 15 minutes after midnight or 2:15 a.m.

We could also use a timeline to determine what time the Prettis arrive back home.



B) The babysitter arrived at 6:15 which is 45 minutes before 7:00 p.m. This means the Prettis need to pay for a total of $435 + 45 = 480$ minutes of time. If we continue the pattern from part A), 8 hours = 480 minutes. Now we can find a pattern between the number of hours and the amount the babysitter is paid.

Hours	**Total Paid (in \$)**
1	7.50
2	15.00
3	22.50
4	30.00
5	37.50
6	45.00
7	52.50
8	60.00

Since the Prettis needed to buy two tickets for the movie marathon, this cost $2 \times \$17 = \$34$. The total cost of the evening out is $\$34 + \$60 = \$94$.

Teacher's Notes

The solution for part B) converts minutes to hours to determine how much to pay the babysitter. Here is another way to solve that problem. We know that 480 minutes passed from the time that the babysitter arrived until the time that the Prettis returned home. Rather than converting this time to hours, we could determine how much the babysitter gets paid per minute rather than per hour.

A rate of \$7.50 per hour is equal to 750 cents per 60 minutes. We can determine the rate per minute as $750 \div 60 = 12.5$ cents per minute.

So for 480 minutes, the babysitter should be paid $480 \times 12.5 = 6000$ cents. This is equal to \$60.00.

Problem of the Week

Problem A

Circular Calculations

Robbie was practising his math and discovered some interesting sequences.

A) Robbie picked a random number to start. Then he wrote out that number in words and counted its number of characters. Then he wrote the resulting number, and counted its number of characters. He continued this sequence for as long as he could. For example when he started with seventy-eight, this was the result:

seventy-eight $\rightarrow$ (13) thirteen $\rightarrow$ (8) eight $\rightarrow$ (5) five $\rightarrow$ (4) four

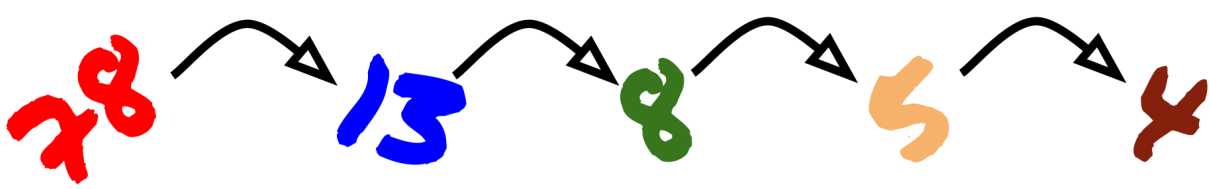
Try following this procedure with other numbers to start. Do you see any pattern? Do you expect to see the same result for any starting number?

B) Robbie discovered another interesting result. Follow this procedure:

- Start with any positive integer.
- If the number is even, divide by 2. For example if the number is 26, the next number in the sequence is 13.
- If the number is odd, multiply by 3 and add 1. For example, if the number is 13, the next number in the sequence is $(13 \times 3) + 1 = 40$.
- Continue the sequence as long as you can.

Try this procedure with the following numbers to start: 5, 26, 15, 50. Do you see a pattern? Do you expect to see the same result for any starting number?

Challenge: Try this procedure starting with 27.



Strands Patterning and Algebra, Number Sense and Numeration

Problem of the Week
Problem A and Solution
Circular Calculations

Problem

Robbie was practising his math and discovered some interesting sequences.

A) Robbie picked a random number to start. Then he wrote out that number in words and counted its number of characters. Then he wrote the resulting number, and counted its number of characters. He continued this sequence for as long as he could. For example when he started with seventy-eight, this was the result:

$$\text{seventy-eight} \to (13)\ \text{thirteen} \to (8)\ \text{eight} \to (5)\ \text{five} \to (4)\ \text{four}$$

Try following this procedure with other numbers to start. Do you see any pattern? Do you expect to see the same result for any starting number?

B) Robbie discovered another interesting result. Follow this procedure:

- Start with any positive integer.
- If the number is even, divide by 2. For example if the number is 26, the next number in the sequence is 13.
- If the number is odd, multiply by 3 and add 1. For example, if the number is 13, the next number in the sequence is $(13 \times 3) + 1 = 40$.
- Continue the sequence as long as you can.

Try this procedure with the following numbers to start: 5, 26, 15, 50. Do you see a pattern? Do you expect to see the same result for any starting number?

Challenge: Try this procedure starting with 27.

Solution

A) All of the sequences will converge to *four*. Interestingly, this will be true even with spelling mistakes. For example, the sequence will be different if you start with *fourty-three* instead of *forty-three*, but both sequences will converge to *four*.

Credit to Lorna Morrow

B) All of the sequences will converge to 1.

- 5, 16, 8, 4, 2, 1
- 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
- 15, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1
- 50, 25, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

Teacher's Notes

We could make a logical argument that the sequence in part A) will always converge to *four*.

- The number *four* has 4 characters. So the sequence will stop at this point.
- The numbers *five* and *nine* each have 4 characters. So they are one step away from the end of the sequence.
- The numbers *three*, *seven*, and *eight* each have 5 characters. So they are two steps away from the end of the sequence.
- The numbers *one*, *two*, *six*, and *ten* each have 3 characters. So they are three steps away from the end of the sequence.

This covers all cases of the numbers from 1 to 10. We can argue that all numbers that are greater than 10 have fewer characters in their written versions than their individual numerical values. For example, *seventeen* is the number between 11 and 20 that has the most characters in its name (9 characters). So, the next step in the sequence for any number that is greater than 10 must be a smaller number. This means eventually, the sequence will lead to a number that is less than or equal to 10. Once we get to a point in the sequence where we have a number between 1 and 10, we are at most three steps away from the end of the sequence. So this sequence will always converge to *four*.

Although it takes a long time when you start with the number 27, using the procedure in part B), the sequence will converge to 1. However it takes 111 steps.

> 27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1

Mathematicians believe that starting with any positive integer, the steps described in part B) will always lead to a sequence that converges to 1. This is known as the *Collatz Conjecture*. However, proving this is true for all positive integers is an open problem. There is experimental evidence that shows this is true for very large numbers; however, there is no formal proof that the conjecture holds for all positive integers.