Problem of the Month
Problem 0: September 2023

Hint

(a) \( f(r) \) is undefined only when \( r = -2 \). For what value of \( r \) is \( f(r) = -2? \)

(b) The sequence in part (i) is periodic. Can you show that the sequence is periodic for other values of \( r_1? \)

(c) (ii) After substituting the expression for \( f(r) \), multiply the numerator and denominator by \( r + 1 \). Try to find a common factor in the numerator and denominator.

(iii) Use (ii) and the fact that when \( r \) is positive, \( \left| \frac{1 - \sqrt{2}}{r + 1} \right| < \frac{1}{2} \). Try to establish the given inequality for a few small values of \( n \) and observe how knowing the inequality for \( n \) can help you to deduce it for \( n + 1 \).

(d) Three of these sequences are periodic, one of them is constant (after the first term), and one of them always approaches the fixed value \( \frac{3 - \sqrt{13}}{2} \) as long as there are no undefined values in the sequence.