Problem of the Month
Problem 7: April 2023

Hint

(a) Suppose $d(a, b) = k$ for some $k$. Try to construct a path of length $k$ in the natural graph from the vertex labelled $a$ to the vertex labelled $b$.

(b) For fixed $a \in A_n$, how many $b \in A_n$ have the property that $d(a, b) = k$?

(c) Find a function that works for $n = 2$ and use this to build one for $n = 3$. It might be useful to think of the natural graph of $A_2$ as a square and the natural graph of $A_3$ as a cube. As well, a cube can be thought of as two squares on top of each other with vertical edges connecting corresponding vertices in the top and bottom faces.