



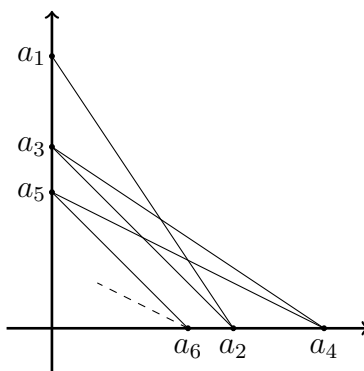
Problem of the Month

Problem 6: March 2022

In this problem, we will explore the following construction: Start with the positive real number $a_1 = 1$ and an infinite sequence m_1, m_2, m_3, \dots of negative slopes that are all distinct. For $n \geq 1$, we define a_{n+1} from a_n as follows.

- For odd n , a_{n+1} is the x -intercept of the line with slope m_n through $(0, a_n)$.
- For even n , a_{n+1} is the y -intercept of the line with slope m_n through $(a_n, 0)$.

The diagram below illustrates this. The line through $(0, a_1)$ and $(a_2, 0)$ has slope m_1 , the line through $(a_2, 0)$ and $(0, a_3)$ has slope m_2 , and so on.



- (a) Suppose that $m_n = -\frac{1}{2^n}$ for all $n \geq 1$.
- Compute a_2 , a_3 , a_4 , and a_5 .
 - Find a general formula for a_n . You will likely need a separate formula for even n and odd n . Describe what happens to a_n as n gets large.
- (b) Suppose that $m_n = -\frac{1}{2^{\frac{1}{2^n}+1}}$ for all n . [The exponent in the denominator is $\frac{1}{2^n} + 1$]
- Find a general formula for a_n .
 - Describe what happens to a_n as n gets large.
- (c) Let u and v be arbitrary positive real numbers with $u \neq 1$. Give a sequence of slopes so that the sequence $a_1, a_3, a_5, a_7, \dots$ approaches u and the sequence $a_2, a_4, a_6, a_8, \dots$ approaches v . Remember that the sequence of slopes should not contain any repetitions.
- (d) Suppose $m_n = -\frac{1}{n}$ for all $n \geq 1$.
- Find an integer n so that $a_n < \frac{1}{100}$.
 - Find an integer n so that $a_n > 100$.
-
-