A rectangular array extends up and to the right with infinitely many rows and infinitely many columns. Integers are placed in the four “bottom-left” cells as shown with 4 in the bottom-left corner, 2 in each of the cells sharing a side with the cell containing 4, and 1 in the cell immediately to the right and above the 4.

When referring to rows and columns, we start the enumeration from the bottom and the left. For example, the “third row” refers to the third row from the bottom.

Integers are placed in the remaining cells recursively as follows:

- In the first and second rows, each remaining cell contains the sum of the integer in the cell immediately to its left and twice the integer two cells to its left. For example, the third cell in the first row contains the integer $2 + 2(4) = 10$.

- Cells in or above the third row contain the sum of the integer in the cell immediately below and twice the integer in the cell two below. For example, the second cell in the third row contains the integer $1 + 2(2) = 5$.

We will denote by $f(m, n)$ the integer in the $m^{\text{th}}$ row and the $n^{\text{th}}$ column.

(a) Show that every cell other than those in the first two columns contains the sum of the integer in the cell immediately to its left and twice the integer in the cell two to its left. That is, show that $f(m, n) = f(m, n - 1) + 2f(m, n - 2)$ for all integers $m \geq 1$ and $n \geq 3$.

(b) Prove that $f(m, m)$ is a perfect square for every integer $m \geq 1$. In other words, prove that all of the cells on the diagonal contain perfect squares.

(c) Determine the value of $f(456, 789)$. 