Problem of the Month
Problem 3: December 2019

Let $a, b, c,$ and $d$ be rational numbers and $f(x) = ax^3 + bx^2 + cx + d$. Suppose $f(n)$ is an integer whenever $n$ is an integer and that

$$\frac{1}{3}n^3 - n - \frac{2}{3} \leq f(n) \leq \frac{1}{3}n^3 + n^2 + 2n + \frac{4}{3}$$

for every integer $n$ with the possible exception of $n = -2$.

(a) Show that $a = \frac{1}{3}$.

(b) Find $f(10^{2019}) - f(10^{2019} - 1)$. 

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