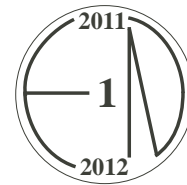


# Emmy Noether - Circle 1 for 2011-2012



## Part I: Problems

### Problem 1

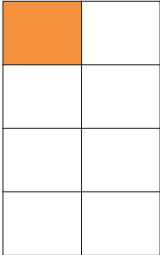
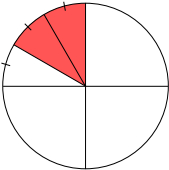
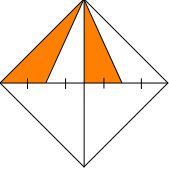
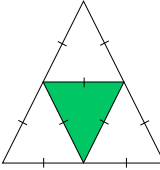
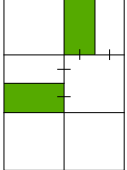
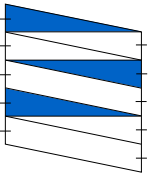
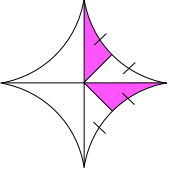
In Wonky-Town, the house numbers on one block follow an unusual pattern. The first house is numbered 1, the next house 3, the third house 7, the fourth 13, the fifth 21, and so on.



- If the house numbers continue to follow this pattern, is there a house number between 60 and 70? Between 80 and 90?
- If the house numbers are less than 100, what is the greatest number of houses there could be?
- Amanda is convinced that an even house number brings good luck. If the house numbers can have any number of digits, and still follow the given pattern, will there ever be an even-numbered house for Amanda? Explain your reasoning.

### Problem 2

Some fraction of the surface of each of the given figures in the chart below is shaded. For which figures is the shaded fraction of the figure the same? Reminder: Fractions are equal parts of the whole.

SHAPE	FRACTION SHADED	SHAPE	FRACTION SHADED	SHAPE	FRACTION SHADED
1. 		3. 		5. 	
2. 		4. 		6. 	
				7. 	

**Problem 3**

Ye Ming has just won a Canadian Falcon points card which gives her points towards free clothes.

- If she spends \$5 in one of their stores, she gets one point.
- A \$10 purchase gives two additional points (i.e., three in total).
- A \$15 purchase gives another four additional points.

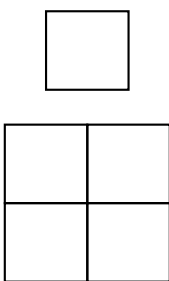
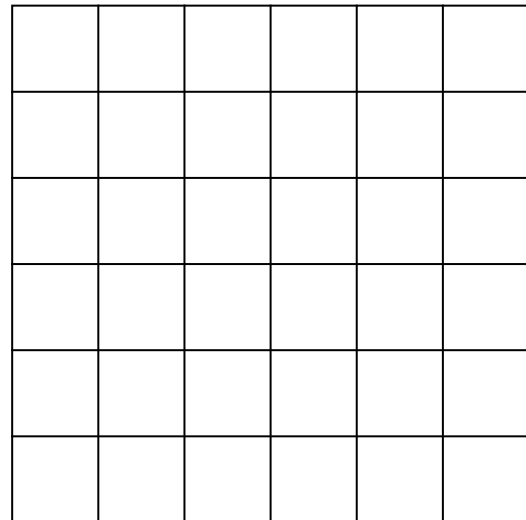


A new pair of jeans requires 350 points. How much money does Ye Ming have to spend in order to receive a total of 350 points? Is it worth it? Justify your answer.

**Problem 4**

Betsy and Josh wish to find out how many squares of any size can be found in a grid that is 6 units by 6 units, as shown below. After trying to count all of the different-sized squares, they realize that it might be a good idea to try solving a simpler problem. (See the chart below.)

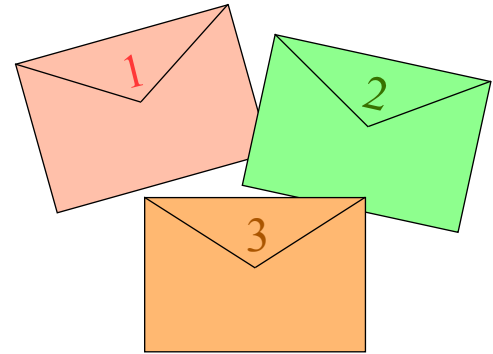
- a) Instead of a  $6 \times 6$  grid, what is the simplest possible grid? How many squares does it contain?
- b) What are the next simplest dimensions for the grid? How many squares does it contain?
- c) Complete the table below and look for a way to help Betsy and Josh solve the original problem. Write a sentence explaining how you found your answers.



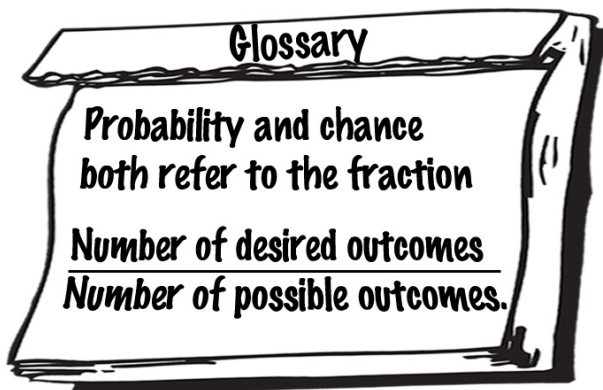
Dimensions of the Grid	Number of "Smallest" Squares	Total Number of Squares
$1 \times 1$	1	1
$2 \times 2$	4	5
$6 \times 6$		

**Problem 5**

Three love-struck boys (Alberto, Butch and Coolroy) each invited Bev to the dance. Bev replied with a special letter to each boy, letting them know whether they were ‘the one’, or not. The first letter was supposed to go to Alberto, the second to Butch, and the third to Coolroy, but Bev forgot to write their names on the outside of the envelopes before sealing them. So now she doesn’t know which is which! She decides to just take a chance and label them and mail them anyhow.



- a) Complete the chart below to show the possible outcomes. For example, if letter 1 went to Alberto (A), then letter 2 could have gone to Butch (B), and letter 3 to Coolroy (C), or vice versa.



Letter	1	2	3
R e c i p i e n t s	A	B	C
	A	C	B
	B		
	B		
	C		
	C		

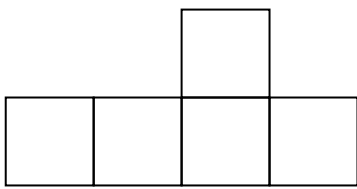
- b) What is the chance that each boy got the letter he was supposed to get?  
 c) What is the chance that at least one boy got the wrong letter?  
 d) What is the probability that the boy she chose as ‘the one’ got the correct letter?

**Extension :**

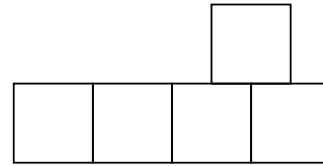
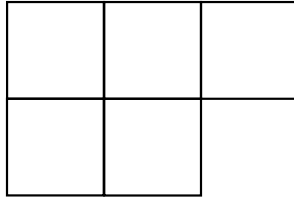
How would your answers change if Bev had 4 admirers instead of 3?

**Problem 6: Pentominoes and Boxes (suitable for pairs or groups of students)**

Pentominoes are formed using 5 square pattern blocks by matching the side(s) of one square exactly to the side(s) of another.

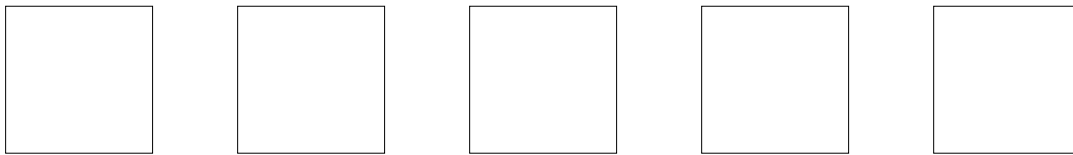


Sample pentominoes

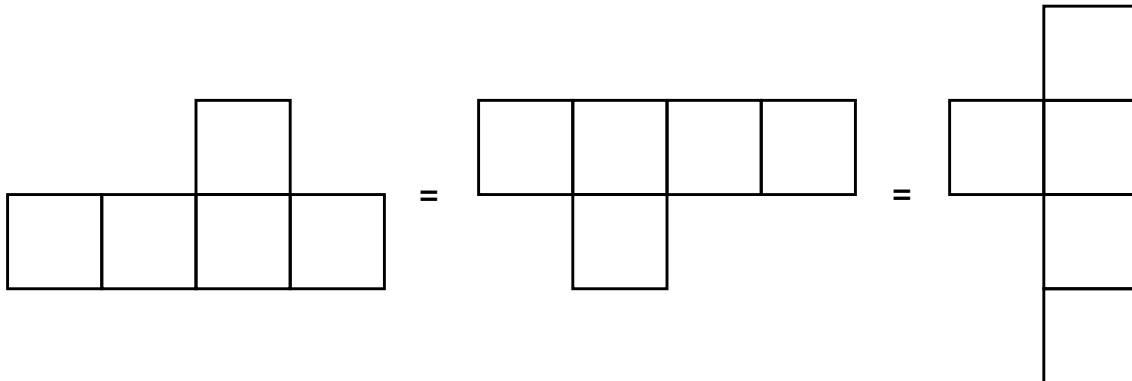


Not a pentomino

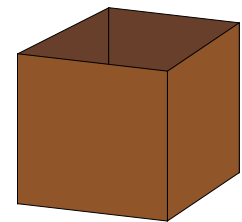
Cut out the squares below and use them to create all possible pentominoes. As you discover each one, sketch it on the grid paper below (as shown for the two samples).

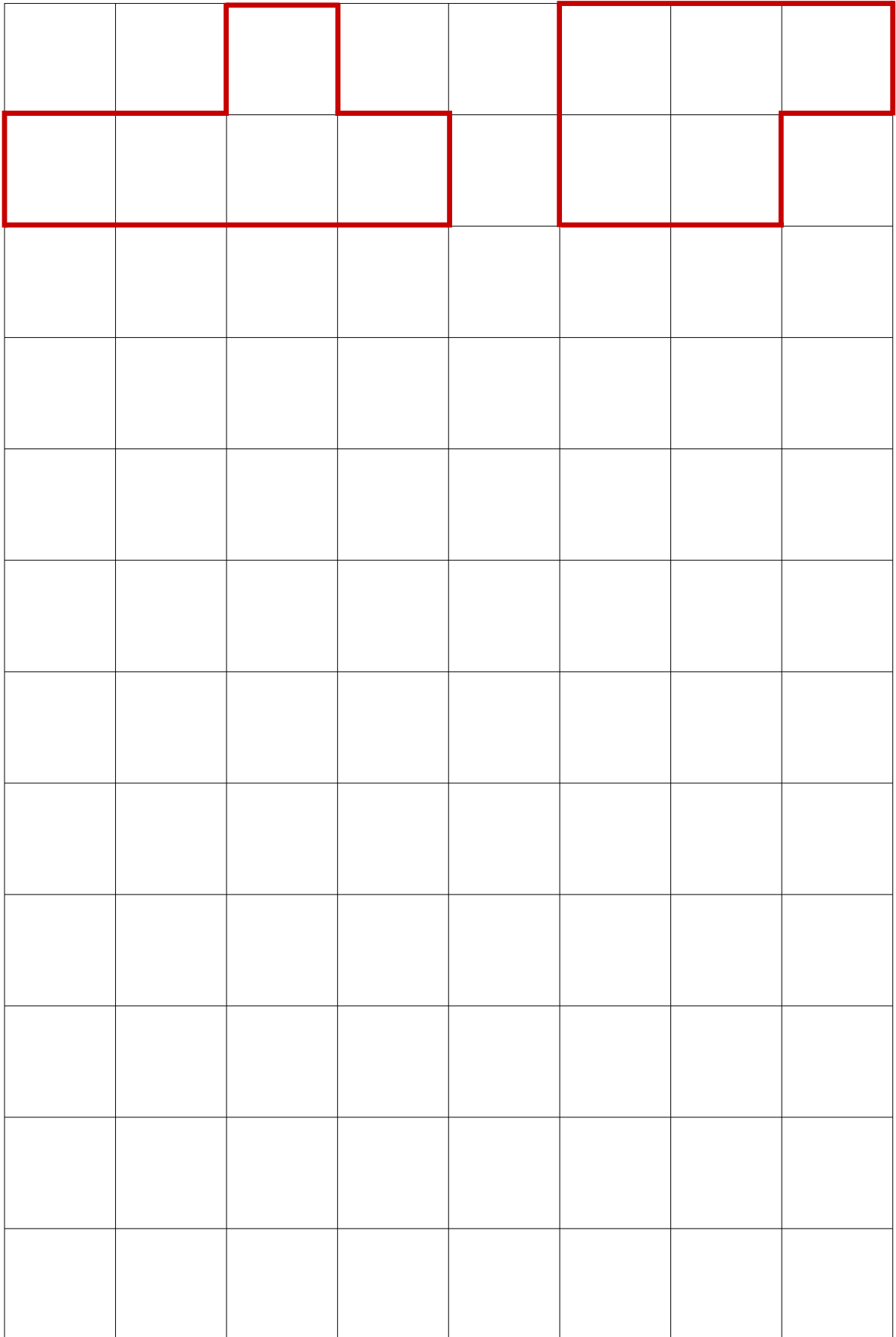


Remember that two shapes are the same pentomino if one can be obtained from flips, reflections or turns (rotations) of the other.



- Which pentomino has the greatest/least area?
- Which pentomino has the greatest/least perimeter?
- Which pentominoes can be made into open boxes (i.e., a cube missing one face?)





***Extension :***

Cut out all the pentominoes you found.

The entire set of pentominoes can be arranged (like a puzzle) to exactly fill the rectangle below. Give it a try!

