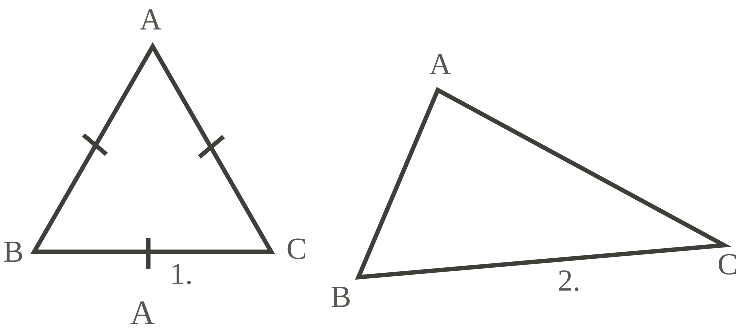
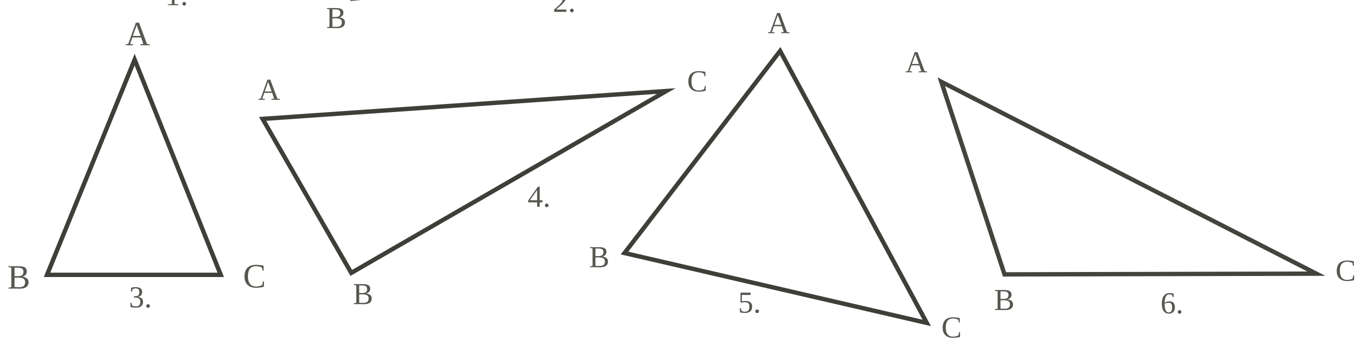
Problem

Polygons: How Many Degrees per Vertex? (For pairs or groups of students)

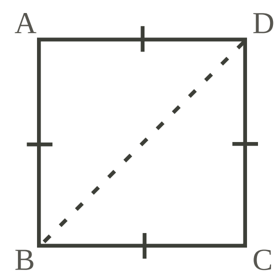
a) Below are several triangles. For each triangle, measure the angles at each vertex A, B, C, find their sum, and enter your results in the table at right. What conclusion do your observations seem to show about the sum of the angles in a triangle?

Triangle	Angles			Sum of Angles
	A	B	C	
1	60°	60°	60°	
2				
3				
4				
5				
6				

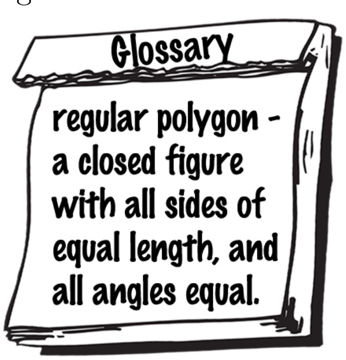


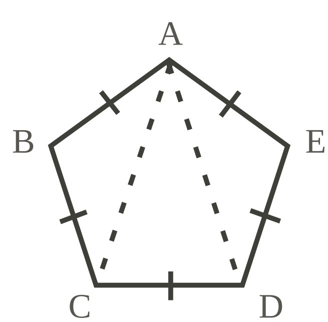


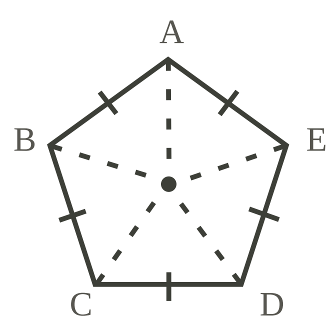
b) For the square at right, use your result from part a) to show that the sum of the angles at the vertices A, B, C, D equals 360°.



c) (i) The diagrams below suggest two ways you could find the sum of the angles in a regular pentagon. Show that each method leads to the same result. (Recall that a complete rotation, (i.e., a 'round' angle whose two sides coincide), measures 360°, so the sum of the angles at the centre of the diagram on the right is 360°.)



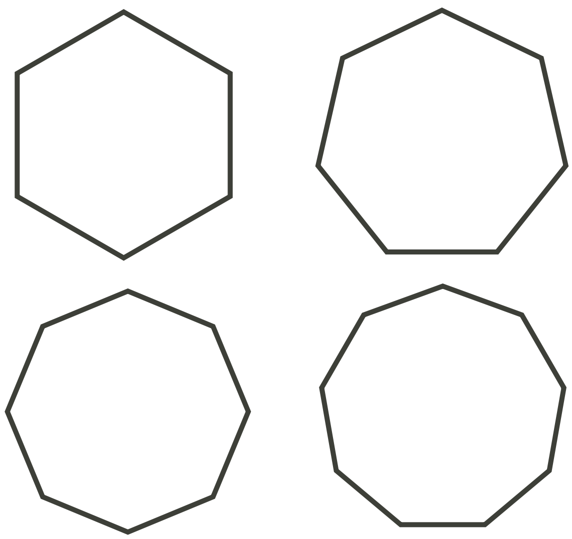




(ii) What is the number of degrees in the angle at each vertex? How do you know?

d) (i) Below are several more regular polygons. Find the sum of the angles at all vertices, and the size of the angle at each vertex. Write your results in the chart at right for 6, 7, 8, and 9 sides.

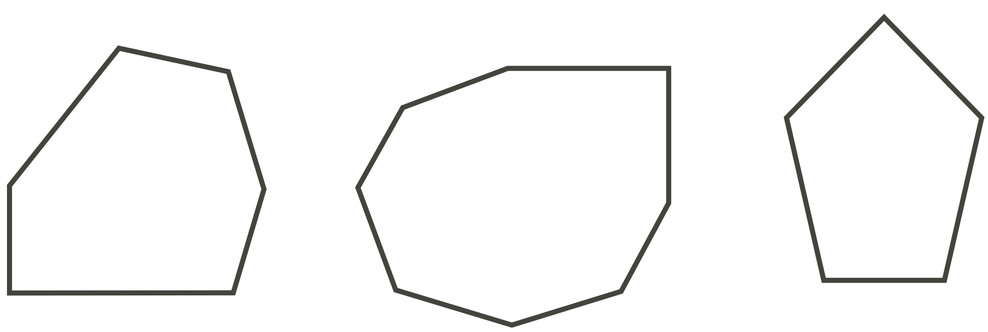
No. of Sides	Sum of all angles at vertices	Size of angle at each vertex
3	180°	60°
4	360°	90°
5		
6		
7		
8		
9		
10		
11		
12		



(ii) Look for a pattern in your chart. Then use it to predict the results for a 10-sided, 11-sided, and 12-sided regular polygon.

***Extension* :**

1. Suppose the polygons in a) to e) above were not regular, i.e., they could have unequal sides. Which of your two columns of the results would remain unchanged? Experiment with the three non-regular polygons below.



Hints

Suggestions:
This activity works best if students work in small groups with some direction from the teacher. Here are some suggestions.

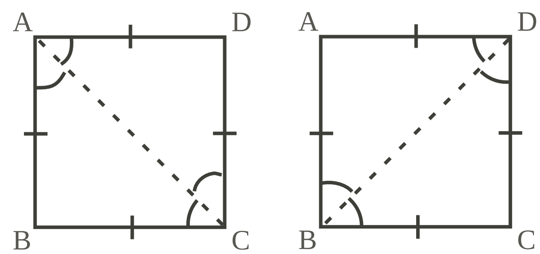
1. For part a), divide students into six small groups, and have each group do the measurements for one triangle. Then collect the data for the whole class to verify that every triangle's angles sum to 180°. If you have already covered this topic in geometry, you may wish to do just one triangle to remind them of this fact.
2. For part b), ask "If you draw a diagonal on the square, what sort of figures appear?"
3. For part c), have half the groups do one method, and half the other, and report to the class.
4. For part d)(i), assign one figure to each group of students, and then have each group report its results to complete the table up to 9 sides. Discuss the pattern and the predictions for the 10-, 11-, and 12-sided polygons.

Solution

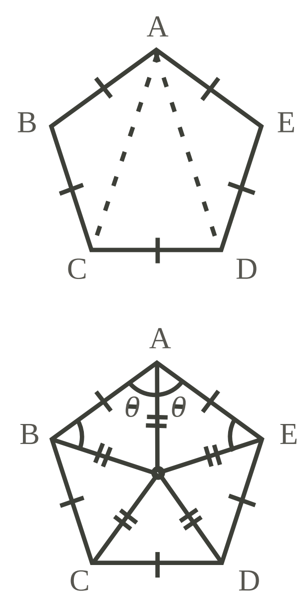
Triangle	Angles A	Angles B	Angles C	Sum of Angles
1	60°	60°	60°	180°
2	83°	63°	34°	180°
3	42°	69°	69°	180°
4	64°	90°	26°	180°
5	65°	65°	50°	180°
6	45°	108°	27°	180°

a) The sum of the angles in each triangle will be 180°, give or take errors in measuring the angles. At right is a table which should roughly coincide with the students' measurements.

b) Divide the square into two congruent triangles, using either diagonal. Then each triangle has angles that sum to 180°, as in part a), giving a total of 360°. *Note:* Since the square is symmetric about both diagonals, and by definition has one right angle, the triangles are congruent. Thus the angles at the vertices are all equal, each being $360° \div 4 = 90°$.

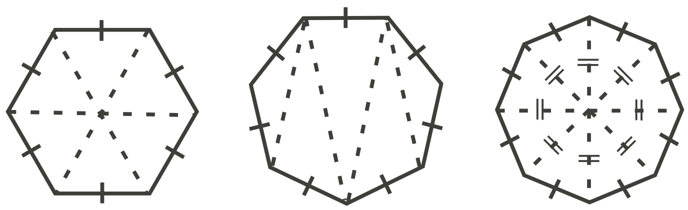


c)(i) In the upper diagram, the pentagon is divided into three triangles, each having angles that sum to 180°. Since $\angle C = \angle BCA + \angle ACD$, $\angle D = \angle CDA + \angle ADE$, and $\angle A = \angle BAC + \angle CAD + \angle DAE$, if we add in $\angle B$ and $\angle E$, we have all the angles in the three triangles. Thus $\angle C + \angle D + \angle A + \angle B + \angle E = 3 \times 180° = 540°$, i.e., the sum of the angles in the pentagon is 540°.
On the other hand, if we use the lower diagram, the pentagon has been divided into 5 identical triangles, each with angles summing to 180°, giving a total of $5 \times 180° = 900°$. But the angles at the centre of the pentagon form a complete revolution, i.e., a 'round' angle which measures 360°. Thus the five vertices must have angles that sum to $900° - 360° = 540°$, as before.



(ii) Since all five triangles are congruent and isosceles, each vertex angle equals 2θ, i.e., all five vertices have equal angles, each being $540° \div 5 = 108°$.

d)(i) For each of the regular polygons, the sum of the angles at the vertices and the size of each angle can be found by subdividing the polygon into triangles in one of the ways shown for the pentagon in part c). Three samples are shown below, and the values for each are given in the chart at right.



(ii) For each increase of 1 in the number of sides of the regular polygons, there is an increase of 180° in the sum of the angles at the vertices. Thus the totals for a 10-sided, 11-sided, and 12-sided polygon are 1440°, 1620°, and 1800°, respectively. The vertices are equal angles in all cases; answers are given in the chart.

No. of Sides	Sum of all angles at vertices	Size of angle at each vertex
3	180°	60°
4	360°	90°
5	540°	108°
6	720°	120°
7	900°	$128\frac{4}{7}^\circ$
8	1080°	135°
9	1260°	140°
10	1440°	144°
11	1620°	$147\frac{3}{11}^\circ$
12	1800°	150°

Extension:

1. For the irregular polygons, we can always use the first of the two methods from part c) to find the total number of degrees in the vertices, as shown below. However, since the sides are no longer of equal length, the vertices will not be equal angles. Thus only the first column of the table remains unchanged.