

Problem

- a) All the license plates in the hamlet of Rickville have the form 6_8 QWL, where the blank is a single digit. How many cars can be licensed in Rickville with the pattern 6_8 QWL?
- b) All the license plates in Becville have the form 648 __WL, where the blank is a letter from the alphabet. How many cars can be licensed in Becville with the pattern 648 __WL?
- c) Suppose, instead, the license plates in Becville can have the form 6_ _ QW_ . Now how many cars can be licensed in Becville?



Extensions :

- 1. Below are pictures of typical license plates in Quebec and New Brunswick. If the digits can be any of 0,1,...,9 and the letters can be any of A,B,C,...Y,Z, how many cars can be licensed using either of these patterns of numbers and letters?



- 2. How would your answer to Extension 1. change for Quebec if the digit 0 is not permitted as the first of the three digits?
- 3. On the right is a picture of a typical Ontario license plate. How does the number of cars that could be licensed in Ontario compare to the number that could be licensed in Quebec or New Brunswick?



Hints

Hint 1 - c) What is the least number with the form $6_ _$, where the blanks are digits? What is the greatest such number?

Hint 2 - c) For each choice of the pair of digits following the 6, how many choices are there for the last letter in the license plate?

Suggestion: Once students realize there are 100 possibilities (600, 601, 602, ..., 699), point out that they could think of this as a product, (10 choices for the first digit) \times (10 choices for the second digit). Then ask what product is needed to find the total number of license plates.

Solution

- a) Since there are 10 possible digits that could fill the blank between the 6 and the 8, ten cars could be licensed in Rickville.
- b) Since there are 26 possible letters that could fill the blank before the W, 26 cars could be licensed in Becville.
- c) The two blanks after the 6 could be filled by any of the 100 pairs of digits 00, 01, 02, 03, ..., 97, 98, or 99. For each of these pairs, the last letter could be any one of the 26 letters A, B, ..., Z. So now $100 \times 26 = 2600$ cars could be licensed in Becville.

Extensions:

1. Both licenses have a triple of digits and a triple of letters. If all ten digits 0, 1, ..., 9 are permitted, and we know that a pair of digits has 100 possibilities, then a triple of digits could be formed by combining any pair of digits with any of the ten digits, so there would be $100 \times 10 = 1000$ possible triples of digits. We can similarly argue that a pair of letters would have 26 pairs of form A_, 26 of form B_, 26 of form C_, ..., 26 of form Z_. Thus there are $26 \times 26 = 676$ pairs of letters, each of which could be followed by any one of the 26 letters to give a triple of letters. Thus there are $676 \times 26 = 17\,576$ triples of letters. Since each of these could be used in combination with any one of the 1000 triples of digits, a total of $17\,576 \times 1000 = 17\,576\,000$ cars could be licensed in either Quebec or New Brunswick.
2. If the Quebec plate could not start with a 0, then there would only be 9 choices for the first digit, and hence only $100 \times 9 = 900$ possible triples of digits. Thus the total number of cars that could be licensed would be reduced to $17\,576 \times 900 = 15\,818\,400$ cars.
3. Since the fourth letter could be any of the 26 letters, for every plate from Quebec or New Brunswick, there would be 26 Ontario plates. Thus there are 26 times as many possibilities for Ontario plates as for Quebec or New Brunswick.