Problem

a) If a boy named Biff Smith can solve a certain problem, he will win one million dollars in cash, tax free. Here is the problem he must solve: A set of five single digit numbers has the following properties:

- the *mode* of the numbers is 1;
- the *mean (average)* of the numbers is 4;
- the *median* of the numbers is 5.

What are the five numbers?

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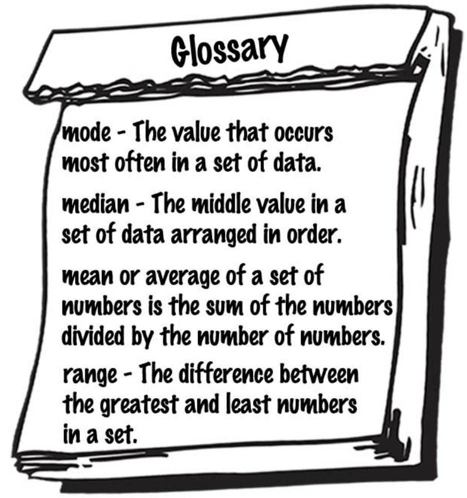

b) Biff can win an extra $100 000 if he can solve a second problem: Find a set of six whole numbers with a mean of 14.5, a mode of 1, and a range of 28. Is there more than one possible solution to this problem?

Hints

Hint 1 - a) If the numbers are taken in order, which number equals 5?

Hint 2 - a) How many occurrences of the number '1' must there be?

Hint 3 - a) What is the sum of the five numbers?

Hint 3 - b) What is the sum of the six numbers?

Solution

a) There are five numbers in the set. Since the mode is 1, the least two numbers must both be 1. Since the median is 5, the middle number must be 5. Since the mean is 4, the sum of the numbers must be $5 \times 4 = 20$. (By the difinition of mean, 4 = (sum of all five numbers) $\div$ 5, so the sum equals 5×4.) The sum of the first three numbers is $1 + 1 + 5 = 7$, so the last two must sum to 13. Hence the last two numbers must be 6 and 7, because we can't use 5 and 8, or the mode would not be 1, and we can't use 4 and 9 because 5 is the median, so the numbers would be out of order. Thus the set is {1,1,5,6,7}

b) Again, since the mode is 1, at least the first two numbers must be 1 (there could be more 1s in this case). Since the range is 28, the last number must be 29. Since the mean is 14.5, the six numbers must sum to $6 \times 14.5 = 87$, and thus the middle three numbers must sum to $87 - (1 + 1 + 29) = 56$. So Biff's solution can be any set of the form {1,1,M,N,P,29} , where M, N, and P are distinct whole numbers less than or equal to 29, and with a sum of 56. (In the final set of six numbers, 1 must be the number which appears most often.) Some examples are {1,1,1,10,45,29}, {1,1,15,18,23,29}, {1,1,16,19,21,29}. Thus there are many solutions to Biff's second problem.