

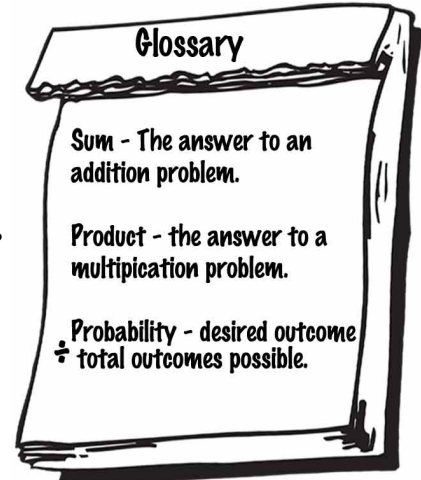
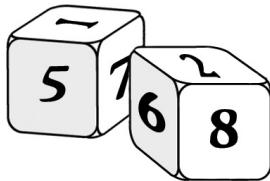
## Problem

Hamed has a pair of dice (number cubes). One has the odd numbers 1, 3, 5, 7, 9, 11 on its faces, and the other, the even numbers 2, 4, 6, 8, 10, 12.

- a) If he rolls the dice and adds the two numbers on the top faces:
- What is the probability that the sum is *odd*?
  - What is the probability that the sum is less than 15?
- b) Suppose instead that Hamed multiplies the two numbers:
- What is the probability that the product is *odd*?
  - What is the probability that the product is equal to 18?

Numbers on each die:

Die 1	1	3	5	7	9	11
Die 2	2	4	6	8	10	12



### *Extensions :*

- In a game, Hamed and Beth take turns rolling the two dice. Hamed gets a point every time the number on one die is exactly divisible by the number on the other die. Beth gets a point if the sum of the two numbers has a digit sum divisible by 4. (For example, if she rolls a 6 and an 11, with sum 17, then its digit sum is  $1+7=8$ , which is divisible by 4, so she get a point. But she gets no point if she rolls a 3 and an 8, since their sum is 11, with digit sum 2). Is this a fair game (i.e., do both Hamed and Beth have an equal chance to get a point)?

**Hints****Part a)**

**Hint 1** - If you add an odd number and an even number, will the sum be odd or even?

**Hint 2** - How many sums are possible in total from a roll of the two dice?

**Part b)**

**Hint 1** - If you multiply a number from one die by a number from the other, what sort of number will you get for the product?

**Hint 2** - How many products are possible in total from a roll of the two dice?

*Suggestion:* Listing the possibilities in tree form may be helpful for parts a)(ii) and b)(ii).

## Solution

Die 1 and Die 2 have numbers as shown:

Die 1	1	3	5	7	9	11
Die 2	2	4	6	8	10	12

a) Hamed rolls the dice and adds the two numbers:

- (i) Odd sums will occur in every case, since the sum of an even number and an odd number is always odd. Thus the probability is  $\frac{36}{36} = 1$  (i.e., it is a certain event that the sum will be odd).
- (ii) Here is a table showing the possible sums:

Die 2	Die 1	1	3	5	7	9	11
2		3	5	7	9	11	13
4		5	7	9	11	13	15
6		7	9	11	13	15	17
8		9	11	13	15	17	19
10		11	13	15	17	19	21
12		13	15	17	19	21	23

Thus the total possible number of sums less than 15 is  $1 + 2 + 3 + 4 + 5 + 6 = 21$ , out of a total of  $6 \times 6 = 36$  possible sums. Hence the probability of a sum less than 15 is  $\frac{21}{36} = \frac{7}{12}$ , a chance of 7 in 12.

Note to the Teacher: Students could also use trees to list the possible sums.

b) Hamed rolls the dice and multiplies the two numbers.

- (i) Since multiplying an even number by an odd number always gives an even number, there are no possible odd products. Thus the probability is  $\frac{0}{36} = 0$ .
- (ii) A product of 18 can be obtained in two ways,  $3 \times 6$ , or  $2 \times 9$ . Thus the probability is  $\frac{2}{36} = \frac{1}{18}$ .

### **Extension :**

**Hamed:** Odd numbers are only divisible by odd numbers. Thus no number on Die 2 can be evenly divided into by an odd number on Die 1. However, odd numbers can divide even numbers. Examining divisors from Die 1, possible divisions are:

- 1 into any of 2, 4, 6, 8, 10, or 12, giving 6 pairs;
- 3 into 6 or 12, giving 2 pairs;
- 5 into 10, giving 1 pair.

Thus there are 9 possible pairs in total, giving a probability of  $\frac{9}{36} = \frac{1}{4}$  of Hamed getting a point.

**Beth:** Examining the table of sums for part a) (ii) above, we see that the only sums with digit sums divisible by 4 are 13 ( $1 + 3 = 4$ ), and 17 ( $1 + 7 = 8$ ). Since 13 occurs 6 times, and 17 occurs 4 times, there are 10 possibilities. Thus the probability of Beth getting a point is  $\frac{10}{36} = \frac{5}{18}$ .

Conclusion: The game is not fair since Beth has a greater chance of getting a point.