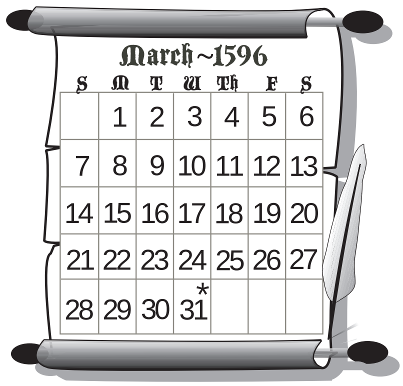
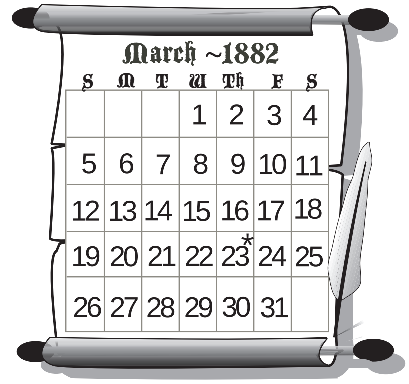
Problem

Below are three calendars, one for the month of March in the year 1596, one for May, 1718, and one for March, 1882. Use these calendars to search for patterns by answering the questions below.



* birthdate of Rene Descartes

* birthdate of Maria Gaetana Agnesi



* birthdate of Emmy Noether

a) Pick a number in the third week of one of the calendars.

   (i) How is it related to the number directly above?

   (ii) How is it related to the number directly below?

   (iii) Would this be true of any calendar?

   Explain your answers.

b) Pick a number in the middle of a week in one of the calendars.

   (i) How is it related to the number on the right?

   (ii) How is it related to the number on the left?

   (iii) Is this true of any calendar?

   Explain your answers.

c) Pick a number somewhere near the middle of one of the calendars; call that number n. There are eight other numbers adjacent to n, forming a square with a total of nine numbers.
   A sample is shown at right.(Use the same square for (i), (ii), (iii), (iv).)

6	7	8
13	14	15
20	21	22

   (i) From the square of nine numbers, find two sets of three numbers such that the sum of the three numbers is the same for each set, and the average of the three numbers is the number n.

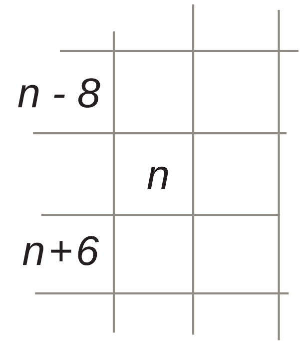
   (ii) Find two more such sets, different from those you found in (i).

   (iii) Would the same choices of such sets work for any 'middle' number n you choose? Discuss this with a friend who picked a different number. Explain your answer.

   (iv) Are there two sets of three numbers in the square which have the same sum, but for which the average is NOT equal to n ? If so, list them.

Extensions:

1. (i) The diagram at right shows a partially completed square of nine numbers from a calendar, like the one you chose above, but in a more general form, with the middle number labelled n. By observing how n is related to the other eight numbers, fill in the rest of the square with numbers written in terms of n.
   HINT: Use your results from a) and b) above.

   (ii) Using your completed square, explain why sets of three numbers picked in the way you chose in c)(i),(ii) above, will always have an average of n, while those from c(iv) will not.

2. Write a brief paragraph on each of the mathematicians Rene Descartes, Marie Agnesi and, Emmy Noether. Research your facts on the web or in reference books.

Hints

Part c)

Hint 1 - Could the three numbers in each of two rows have the same sum? Why/or why not?

Hint 2 - Could the three numbers in each of two columns have the same sum? Why/or why not?

Hint 3 - Is there a row with the same sum as one of the columns?

Hint 4 - How do the sums of the diagonals compare?

Hint 5 - Is it necessary to stick to a row, column, or diagonal?

Solution

Using the middle number $n = 14$ in the third week of the second calendar gives the excerpt shown at right, which can be used to illustrate the solutions for a), b), c) as follows. In each case, the appropriate numbers or calculations for this excerpt are given in brackets in the solution.

6	7	8
13	14	15
20	21	22

a) The middle number (14) is 7 greater than the number directly above it, and 7 less than the number directly below it, due to the difference of one week (7 days) between each pair of dates. This would be true of any calendar laid out horizontally in weeks of 7 days.

b) The middle number (14) is one less than the number on its right, and one greater than the number on its left. This would also be true of any calendar laid out horizontally in weeks of 7 days.

c) The key idea in parts (i), (ii), and (iii) is that for a set of three numbers to have an average of n (14), their sum must equal $3n$ (42). Many such sets exist. From the information in parts a) and b), obvious choices which include the middle number n (14) are the middle column (7 + 14 + 21 = 42), and the middle row (13 + 14 + 15 = 42), or the diagonals (6 + 14 + 22 = 42, or 8 + 14 + 20 = 42). In each case, the first number is less than n by the same amount as the last number is greater than n.

Other possibilities (NOT containing the middle number n) include, for example, upper right + middle left + middle bottom (8 + 13 + 21 = 42), or upper left + middle right + middle bottom (6 + 15 + 21 = 42), or upper middle + middle right + bottom left (7 + 15 + 20 = 42). In these sets, the key idea is to pick three numbers which are each from a different row and a different column than the others, which guarantees that their sum will be $3n$.

For part (iv), where we want two sets with the same sum, but NOT with sum $3n$, we can pick, for example, the upper left + bottom middle + bottom right (6 + 21 + 22 = 49) and the upper right + bottom left + bottom middle (8 + 20 + 21 = 49). (Other possible pairs exist.)

Extension:

1. Observing the relationship between n and the surrounding numbers, we see that the square can be written in the form shown at right. Thus, for example, the averages of the diagonals are $\frac{1}{3}(n-8+n+n+8) = \frac{3n}{n} = n$, or $\frac{1}{3}(n+6+n+n-6) = \frac{3n}{n} = n$. (Proofs for any of the sets found in parts a), b), and c)(i), (ii) above are similar.) On the other hand, for the sets we chose in part c)(iv) above, the general sums give an average of $\frac{1}{3}(n-8+n+7+n+8) = \frac{3n+7}{n} \neq n$, and $\frac{1}{3}(n-6+n+6+n+7) = \frac{3n+7}{n} \neq n$. (Other pairs may have different sums.)

$n - 8$	$n - 7$	$n - 6$
$n - 1$	n	$n + 1$
$n + 6$	$n + 7$	$n + 8$