Part II: For the Teacher

Curriculum Areas

Problem 1 - Number Sense

Problem 2 - Measurement

Problem 3 - Pattern/Algebra

Problem 4 - Probability

Problem 5 - Geometry and Spatial Sense

Problem 6 - Measurement and Data Management

Hints and Suggestions:

Problem 1 a)

Hint 1 - What numbers could go in circles B and C?

Hint 2 - What numbers could go in circles A and D?

Problem 1 c)

Hint 1 - What number MUST go in circle B?

Extension:

Hint 1 - Start with the one of the two known possibilities for circle B. What does that tell you about A and C?

Hint 2 - Pick a value for circle M. Does it work?

Problem 2 a)

Hint 1 - What is the total cost of the bag of oranges?

Problem 2 b)

Hint 1 - What is the cost of buying 2 bags per week over the month?

Problem 3 (i)

Hint 1 - If your Mom has 12 loonies to share equally between you and your sister, how many will each of you get?

Hint 2 - What is the total number of loonies in the two bags in equation (B)?

Hint 3 - What is the total number of loonies in the three bags in equation (C)?

Extension:

Hint 1 - Is 3 + 8 a different choice than 8 + 3 for $\Box$ and $\Diamond$ for any part of the Extension?

Problem 4

Suggestion: Before students begin this problem, you may wish to review the idea that the probability of a desired outcome or event is the ratio of the number of ways the desired event can occur to the total number of outcomes possible.

Problem 4 c)

Hint 1 - What are the possible total values of the different sets of remaining coins?

Hint 2 - How many of these total values exceed 30 cents?

Extension:

Hint 1 - How does the fact that there are now two pennies affect the probabilities?

Problem 5

Suggestion: Have students outline several pool tables on graph paper for each question to avoid excessive erasing.

Problem 5 a)

Hint 1 - How can the squares on the grid paper help you to draw a 45° angle?

Problem 5 b)

Hint 1 - If you shot the ball from corner C at 45° from the table edges, where would it go?

Extension:

Hint 1 - If a ball lands in pocket C, where was its final rebound? What about if it lands in pocket A? pocket B? pocket D?

Suggestion: Use differenly coloured pencils to experiment with drawing different paths for parts b) and c), and the Extension.

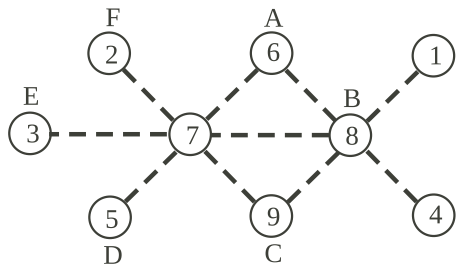
Problem 6

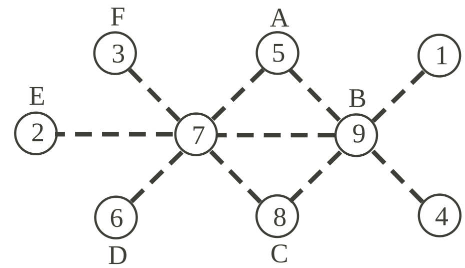
Suggestion: Make sure the drip rate is slow enough for accurate counting (say, no more than one droplet per second), but not too slow.

Solutions

Problem 1

a),b) There are only two possible numbers for circle B, namely, 8 or 9. Once B is chosen, A, C, and E are fixed, which in turn determines D and F. The two possible solutions are:

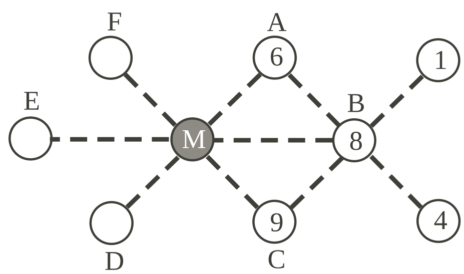




c) When the centre-left number is 5 instead of 7, we have two possible scenarios for the horizontal line of three circles. If B = 8, then 8 + 5 + E = 18 would require E = 5, which is already used. If B = 9, then 9 + 5 + E = 18 would require E = 4, which is already used. Thus no solutions are possible when 7 is replaced with 5.

Extension:

1. To show why 7 is the only possible choice, just try the possibilities one-by-one. There are still only two possible values for B, namely, 8 or 9. Here is the argument for B = 8, with initial configuration:



The four numbers available for the remaining four spaces are 2, 3, 5, and 7. So all we need to do is try each of these four for one of the spaces. Since we have already seen that M = 7 has two solutions, and M = 5 has none, we need only test M = 2 and M = 3. If we choose M = 2, then B + M + E = 18 gives E = 18 - 8 - 2 = 8, which is already used. If M = 3, then A + M + D = 18 gives D = 18 - 6 - 3 = 9, which is already used. Thus, if B = 8, the only possible choice of M is 7.

A similar argument shows that B = 9 also yields no solutions other than M = 7.

Problem 2

2 a) The total cost of the bag is \$0.99 x 3 = \$2.97. Since $\frac{1}{8}$ of the total cost is for the peel, the peel costs $\frac{1}{8} \times \$2.97 = \$0.37125 \approx \$0.37$. Thus the fruit itself costs $\approx$ \$2.60.

2 b) The cost to buy 2 bags per week over the month is

$$2 \times \$0.99 + 2 \times \$0.97 + 2 \times \$1.02 + 2 \times \$0.95 = \$7.86\,.$$

The cost to buy 8 bags the first week is $8 \times \$0.99 = \$7.92$. Thus it is less expensive to buy the bags weekly.

2 c) If tansportation costs, and the extra time required for four trips are considered in the decision, then the 6 cents difference is not enough to warrant buying the bags weekly.

Problem 3

(i) Each bage in equation (A) contains 6 loonies.
Since $\boxed{\text{B}} + \boxed{\text{B}}$ = 7 - 3, or 4 loonies, each bag in (B) contains 2 loonies.
Since $\boxed{\text{C}} + \boxed{\text{C}} + \boxed{\text{C}}$ = 19 - 4, or 15 loonies, each bag in (C) contains 5 loonies.

(ii) Since $\boxed{\text{A}} + \boxed{\text{A}}$ = 12 loonies, $\boxed{\text{B}} + \boxed{\text{B}}$ = 4 loonies, and $\boxed{\text{C}} + \boxed{\text{C}} + \boxed{\text{C}}$ = 15 loonies, we see that the total number of loonies in all seven bags is the sum of the left sides of these three equations, which must equal 12 + 4 + 15, or 31 loonies, the sum of the right sides.

(iii) This story matches equation (B), where the bag $\boxed{\text{B}}$ represents the amount each twin has saved, and their desired total is $7.00.

(iv) Have several students read their stories, and discuss with the other students how well the stories match the given equations.

Extension:

1.a) If $\square + \lozenge = 11$, the possible values of $\square$ and $\lozenge$ are, in pairs,
$(\square, \lozenge) = (1, 10), (2, 9), (3, 8), (4, 7), (5, 6), (6, 5), (7, 4), (8.3), (9, 2), (10, 1)$.

b) If $\square - \lozenge = 3$ as well, then the only possibility is $(\square, \lozenge) = (7, 4)$.

c) If $\square \times \lozenge = 24$, then possible pairs are $(\square, \lozenge) = (3, 8)$ or $(\square, \lozenge) = (8, 3)$.

d) If $\square \times \lozenge = 20$, there are NO possible solutions.

Problem 4

a) Since only one of the four coins in Xiaomei's pocket is a dime, the probability that she has pulled out a dime is one in four, or $\frac{1}{4}$. (This assumes that she does not look at or feel the coins, but just reaches into her pocket and pulls out the first coin she touches.)

b) The completed table is shown at right.

Coin in hand	Remaining coins
5¢	1¢, 10¢, 25¢
1¢	5¢, 10¢, 25¢
10¢	1¢, 5¢, 25¢
25¢	1¢, 5¢, 10¢

c) Since the total value of the remaining coins is greater than 30 cents for 3 of the 4 outcomes, the probability is $\frac{3}{4}$. Thus only 1 outcome has a total value less than 30 cents, giving a probability of $1 - \frac{3}{4} = \frac{1}{4}$.

Extension:

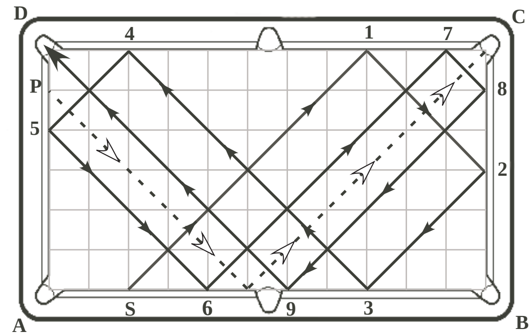
(i) Since there are now 5 possible outcomes (there are 2 ways to choose a penny), the probability of choosing a dime is now $\frac{1}{5}$. Similarly, there are now 4 ways she can end up with the value of the remaining coins greater than 30 cents, so that probability is now $\frac{4}{5}$, and the probability the total is less than 30 cents is now $\frac{1}{5}$.

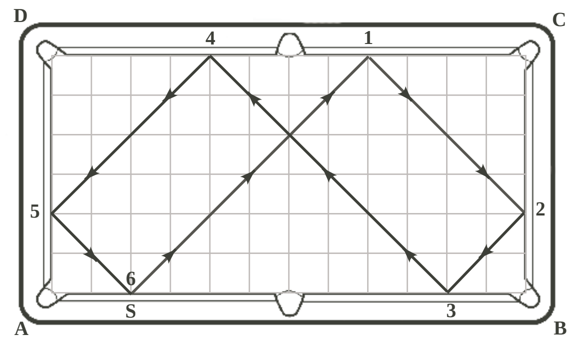
(ii) The remaining value is greater than 40 cents only if she pulls out a penny. Thus the probability is $\frac{2}{5}$, since there are 2 ways this desired outcome can occur, given that there are two different pennies.

Problem 5

a) The ball lands in pocket D after 9 rebounds, as shown by the solid path on the diagram.

b) Working backwards from pocket C, we see that we would have to shoot the ball from position P, one unit below pocket D, in order for it to land in C with only one rebound.





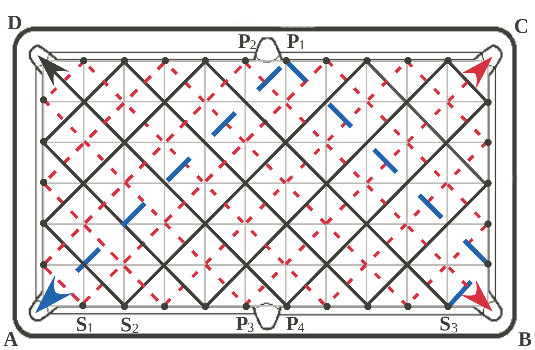
c) On this 12 by 6 table, a ball shot from S never lands in a pocket; the sixth rebound occurs at the starting point S, and the ball repeats the same path again and again.

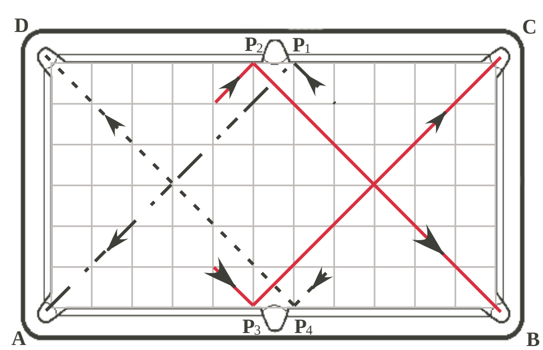
Extension:

1. The diagram at left below shows all the paths taken by a ball shot in either direction from each of the points S_1, S_2, S_3 . Note that, as suggested in the HINT, these paths include all possible paths taken by a ball shot from ANY point in the set. That is, eventually, a path from one of these three points hits any other chosen point in the set, after which the two paths would coincide.

   The diagram at right below reveals that a ball which lands in pocket A had its final rebound at the point P_1 , moving from right to left; if it lands in pocket B, C, or D, it had its final rebound at P_2 (from left to right), P_3 (from left to right), or P_4 (from right to left), respectively.

   Examining the paths in the left diagram, we see that balls leaving S_1 or S_3 eventually hit either P_2 or P_3 , while balls leaving S_2 eventually hit either P_1 or P_4 . Since this covers all paths from all points in the set, we see that there is NO point in the set from which a ball can be shot at a 45° angle and NOT land in one of the pockets.





Problem 6

Solutions here will depend on the data collected by each team. If concerned about potential mess or water play, the teacher can do parts a) and b) with the class and only one dripping tap, and then have students complete parts c), d), e), and the Extension.