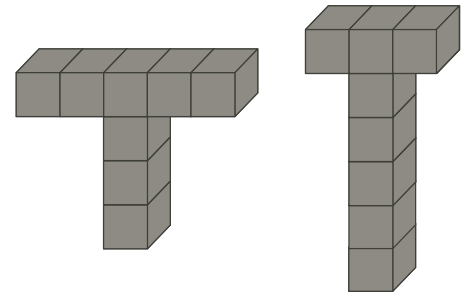


Problem

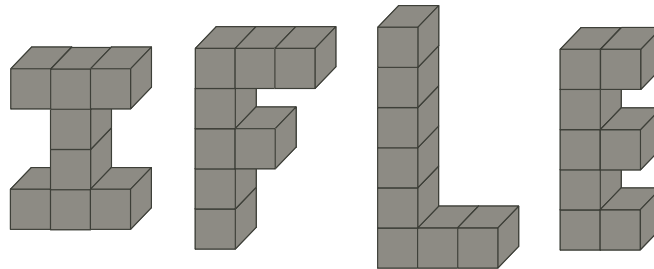
Eight cubes are connected in the shape of a capital “T”. Suppose you spray paint all the outside surfaces black.

- a) How many cube faces would be painted black?
- b) Does your answer to a) change if you make the “T” six blocks tall and the three blocks wide?
- c) If you took apart the “T”, describe the different ways the individual cubes would be painted in each of a) and b), WITHOUT turning the cubes.

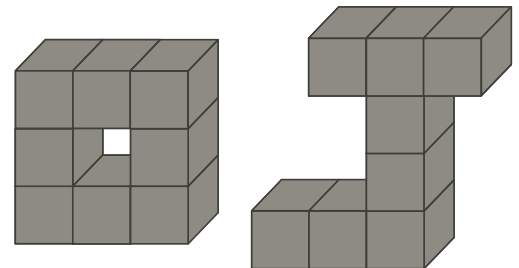


Extensions:

- 1. If you use 8 cubes to build the letters shown below, will there be more or fewer cube faces painted black than for the T?



- 2. Does your answer from 1 hold for the letters at the right?



Hints

Hint 1 - Which cube faces are NOT painted black?

Hint 2 - How many cube faces are there in total for 8 blocks?

Suggestion: Supply students with cube-a-links (or any set of cubic shapes) and have them use small pieces of masking tape or 'stickies' to denote the 'painted' faces.

Extension:

Hint 1 - How many cubes are there? How many faces in all?

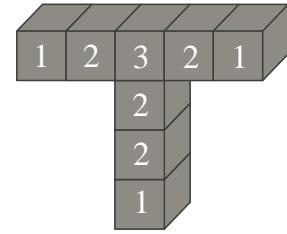
Hint 2 - Which cube faces are not painted? How many of them are there?

Solution

Most students will simply count the painted faces one way or another, (e.g., One cube at a time, or all ‘front’ faces plus all ‘top’ faces plus all ‘side’ faces, etc.). Below is another approach which organizes the possible types of cubes depending on how many faces are painted.

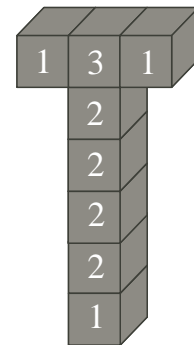
a) There are three basic types of blocks:

- Type 1: Only 1 face not painted, so 5 faces are black;
- Type 2: Only 2 faces not painted, so 4 faces are black;
- Type 3: Three faces not painted, so 3 faces are black;



Clearly for the ‘T’ shown, we have 3 type 1 blocks, 4 type 2 blocks, and 1 type 3 block, so there are $(3 \times 5) + (4 \times 4) + (1 \times 3) = 34$ cube faces painted black.

b) If we reshape the ‘T’ as shown, we still have 3 blocks of type 1, 4 of type 2, and 1 of type 3, so the answers remains unchanged.



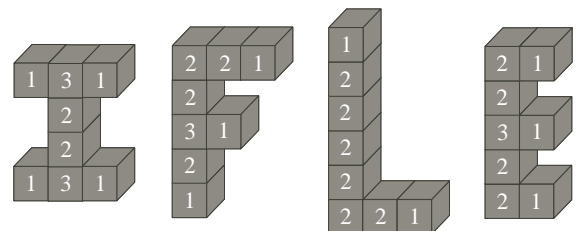
c) If we took apart the ‘T’, the different ways the individual cubes would be painted are as follows:

- Type 1: All faces (5) except the one adjoining another block would be black.
- Type 2: Four faces painted in a band which encircles the cube; left and right sides not painted.
Four faces painted in a horizontal band; top and bottom faces not painted.
- Type 3: Front, top, and back painted; bottom, left and right sides not painted.

Extensions:

1. Labeling the block types as in part a), we have the following numbers of painted faces:

- I: $(4 \times 5) + (2 \times 4) + (2 \times 3) = 34$;
- F: $(3 \times 5) + (4 \times 4) + (1 \times 3) = 34$;
- L: $(2 \times 5) + (6 \times 4) = 34$;
- E: $(3 \times 5) + (4 \times 4) + (1 \times 3) = 34$.



Thus all these letters have the same number of faces as the ‘T’.

2. For each letter at right, the number of painted faces is:

O: $(8 \times 4) = 32$;

J: $(3 \times 5) + (4 \times 4) + (1 \times 3) = 34$.

The “O” is different: it is the only letter which has no ‘projections’, i.e., it is an entirely closed loop.

