Part II: For the Teacher

Curriculum Areas

Problem 1 - Number Sense

Problem 2 - Measurement

Problem 3 - Number Sense and Geometry (Spatial Sense)

Problem 4 - Geometry

Problem 5 - Probability

Problem 6 - Geometry

Hints and Suggestions:

Problem 1

Suggestion: To enable a trial-and-adjustment approach, have students make number cards for the individual digits 1, 2, . . . , 9, and arrange them into sets of three with identical sums.

Hint 1 - How could you discover what should be the sum of the three numbers in each set?

Hint 2 - What numbers could NOT be together in the same set (e.g., Could 7 and 8 be together? Could 1 and 2 be together?)

Note to the Teacher: If Hint 1 is not enough, ask "What is the sum of all nine digits?" and then "How does this help to find the sum for each set?"

Suggestion: An alternative approach, once students have discovered that the sum of each set must be 15, is to suggest they simply make a list of all possible combinations of three digits having a sum of 15. Then they must look for three sets with no overlapping digits.

Problem 2

Hint 1 - Draw a careful diagram showing both the garden and the fence and their dimensions.

Suggestion: Supply students with graph paper to encourage accurate diagrams.

Problem 3

Hint 1 - Can you form a triangle with sides of lengths 2, 3, and 6? Why or why not?

Suggestion: Supply students with about 30 toothpicks each, with a single toothpick having unit length, and suggest they try to form the triangles.

Problem 4

Hint 1 - Which cube faces are NOT painted black?

Hint 2 - How many cube faces are there in total for 8 blocks?

Suggestion: Supply students with cube-a-links (or any set of cubic shapes) and have them use small pieces of masking tape or 'stickies' to denote the 'painted' faces.

Extension:

Hint 1 - How many cubes are there? How many faces in all?

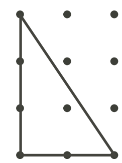
Hint 2 - Which cube faces are not painted? How many of them are there?

Problem 5 c)

Hint 1 - How many outcomes are possible in total?

Hint 2 - What are the ways a player can achieve the desired outcome?

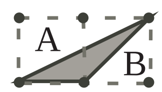
Problem 6 a)



Note to the teacher: We cannot use a base of more than one unit without enclosing a dot, except for the case when the base and height are both 2.

Hint 1 - What happens if you make the base of the right-angled triangle 2 units long, and the height 3 or more units?

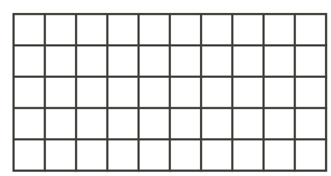
Problem 6 b)



Hint 1 - What is the area inside the dashed rectangle, but outside the first triangle?

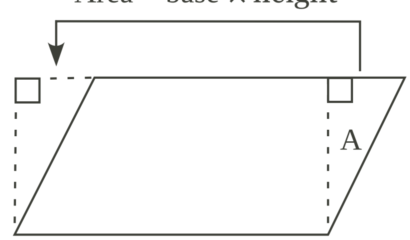
Suggestion: If this proves difficult, discuss the area outside the first triangle as "half of a 2×1 rectangle (area A) plus half of a 1×1 square (area B)". Thus the triangle has area $2 - 1 - \frac{1}{2} = \frac{1}{2}$. Alternatively, use the fact that the triangle has base 1 and height 1, and hence area $\frac{1}{2}(1 \times 1) = \frac{1}{2}$. If students are not familiar with the formula $\frac{1}{2}$ base $\times$ height, it can be motivated as follows:

1. For a rectangle, we know that the area equals the product of the number of units in the base length and the number of units in the height, which gives the total number of unit squares (or square units) in the rectangle.



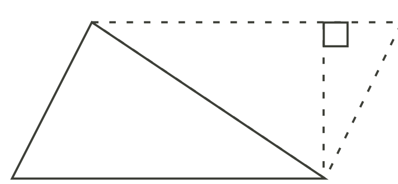
Area = base × height

2. For a parallelogram of the same base and height as the rectangle, if we cut off the triangle A on the right and add it to the left side as shown, we obtain a rectangle of exactly the same base and height. Hence the area of a parallelogram also equals its base length times its height.



Area = base × height

3. Since any triangle can be pictured as half of a parallelogram, we see that the area of a triangle is $\frac{1}{2}$ base $\times$ height.



Area = $\frac{1}{2}$ base × height

Note that a diagonal of a parallelogram divides it into two identical triangles, each with area half that of the parallelogram.

Solutions

Problem 1

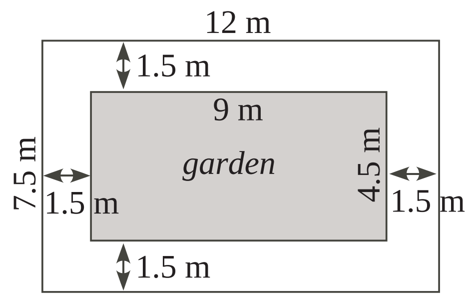
Since the sum of the digits 1, 2, ..., 9 is 45, and each of the three sets must have the same sum, each set must have sum 15. Thus the 7, 8 and 9 must be in separate sets. A list of the combinations of three digits with sum 15 (with no repeats in any set) give the possibilities $9+1+5$, $9+2+4$, $8+1+6$, $8+2+5$, $8+3+4$, $7+2+6$, $7+3+5$.
Selecting from this list, the only possible groups of three sets which use all nine digits just once are (i) {9, 1, 5}, {8 + 3 + 4}, {7 + 2 + 6}, or (ii) {9, 2, 4}, {8, 1, 6}, {7, 3, 5}.

Extensions:

1. Here are two solutions (there are others).
   (i) {9, 1, 2, 3}, {7, 8}, {6, 5, 4} or (ii) {8, 1, 2, 4}, {9, 6}, {7, 3, 5}
2. Answers will vary, depending on what set of numbers students start with. Have them check one another's work.

Problem 2

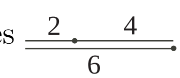
a) From the diagram, we see that if the garden is centred, the difference of 3 m in each dimension will be evenly split, giving a distance of 1.5 metres between the fence and each edge of the garden.

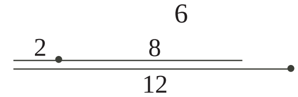


b) Since the new planting area will be $7.5 \times 12 = 90$ square metres, and the original garden was $4.5 \times 9 = 40.5$ square metres, Ben has added 49.5 square metres, i.e., he has more than doubled his planting area.

Problem 3

a) The factors of 24 are 2, 3, 4, 6, 8 and 12. Thus the five combinations which can form triangles are {2, 3, 4}, {3, 4, 6}, {3, 6, 8}, {4, 6, 8} and {6, 8, 12}.

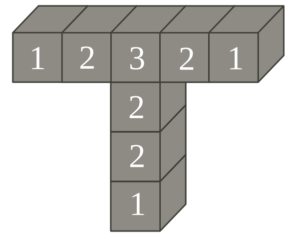
b) The key idea is that the sum of the lengths of any two sides must be greater than the third side.
For example, {2, 4, 6} can't form a triangle because $2 + 4 = 6$, so the sides 

do not 'contain' any area. Similarly for the set {2, 8, 12}, $2 + 8 = 10 < 12$ 

, so no triangle can be formed.
This is a famous mathematical theorem known as the 'Triangle Inequality': If a, b, c are the lengths of the sides of a triangle, then $a + b > c$.

Problem 4

Most students will simply count the painted faces one way or another, (e.g., One cube at a time, or all 'front' faces plus all 'top' faces plus all 'side' faces, etc.). Below is another approach which organizes the possible types of cubes depending on how many faces are painted.



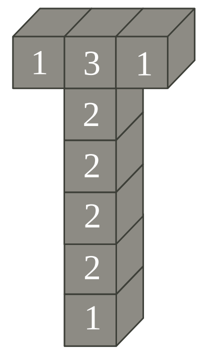
a) There are three basic types of blocks:

Type 1: Only 1 face not painted, so 5 faces are black;

Type 2: Only 2 faces not painted, so 4 faces are black;

Type 3: Three faces not painted, so 3 faces are black;

Clearly for the 'T' shown, we have 3 type 1 blocks, 4 type 2 blocks, and 1 type 3 block, so there are $(3 \times 5) + (4 \times 4) + (1 \times 3) = 34$ cube faces painted black.



b) If we reshape the 'T' as shown, we still have 3 blocks of type 1, 4 of type 2, and 1 of type 3, so the answers remains unchanged.

c) If we took apart the ''T, the different ways the individual cubes would be painted are as follows:

Type 1: All faces (5) except the one adjoining another block would be black.

Type 2: Four faces painted in a band which encircles the cube; left and right sides not painted.
Four faces painted in a horizontal band; top and bottom faces not painted.

Type 3: Front, top, and back painted; bottom, left and right sides not painted.

Extensions:

1. Labeling the block types as in part a), we have the following numbers of painted faces:

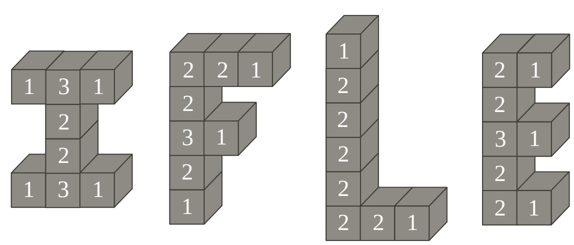
   I: $(4 \times 5) + (2 \times 4) + (2 \times 3) = 34$;

   F: $(3 \times 5) + (4 \times 4) + (1 \times 3) = 34$;

   L: $(2 \times 5) + (6 \times 4) = 34$;

   E: $(3 \times 5) + (4 \times 4) + (1 \times 3) = 34$.

   Thus all these letters have the same number of faces as the "T".

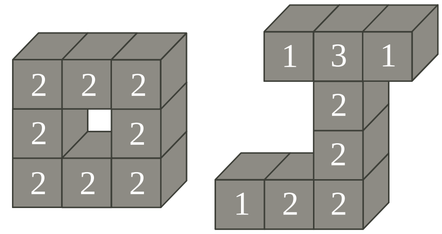


2. For each letter at right, the number of painted faces is:

   O: $(8 \times 4) = 32$;

   J: $(3 \times 5) + (4 \times 4) + (1 \times 3) = 34$.

   The "O" is different: it is the only letter which has no 'projections', i.e., it is an entirely closed loop.



Problem 5

a) Using the number spinner only, there is an equal chance of spinning any one of the four numbers, namely $\frac{1}{4}$.

b) Using the direction spinner only, the probability of spinning a 'left' is $\frac{1}{2}$, since 'left' occupies half the area of the circle.

c) Completing the tree enables students to see that:

   (i) there are 2 outcomes where a player gets a move of 2 to the left, and the total number of outcomes is 16; hence the probability is $\frac{2}{16}$ or $\frac{1}{8}$;

   (ii) there is only 1 outcome where a player gets a move of 1 to the right, and hence the probability is $\frac{1}{16}$.

Extensions:

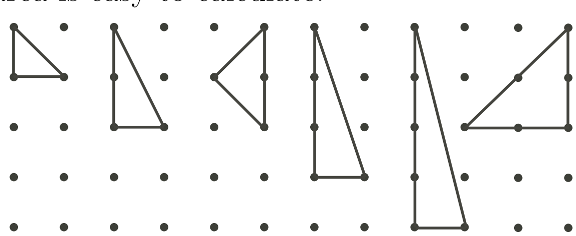
1. Another way to show this is as follows:
   Using their tree diagram, students can just list all the probabilities for all the possible events to see that none has a probability of $\frac{1}{12}$. For an event to have a probability of $\frac{1}{12}$, there would have to be a number n of outcomes such that $\frac{n}{16} = \frac{1}{12}$, i.e., $\frac{n}{16} \times 16 = \frac{1}{12} \times 16$, or $n = \frac{16}{12} = \frac{4}{3}$; but $\frac{4}{3}$ is not a whole number of outcomes, and hence no such outcome is possible. Using their tree diagram, students can just list all the probabilities for all the possible events to see that none has a probability of $\frac{1}{12}$.

2. One of the two 'left' parts of the tree is removed, since now 'left', 'right', and 'lose a turn' are equally likely. Thus the total number of outcomes is 12, and every outcome has equal probability, $\frac{1}{12}$, since no repeats are possible.

Problem 6

FIGURE IT OUT!

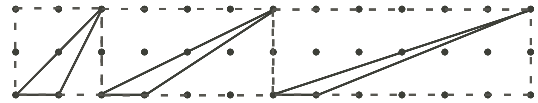
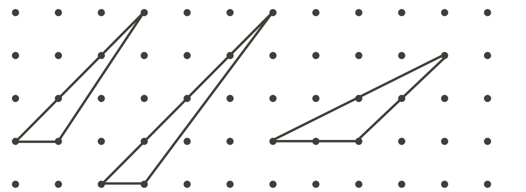
a) (i) Each right-angled triangle occupies half of a rectangle or square. Thus the area is easy to calculate.



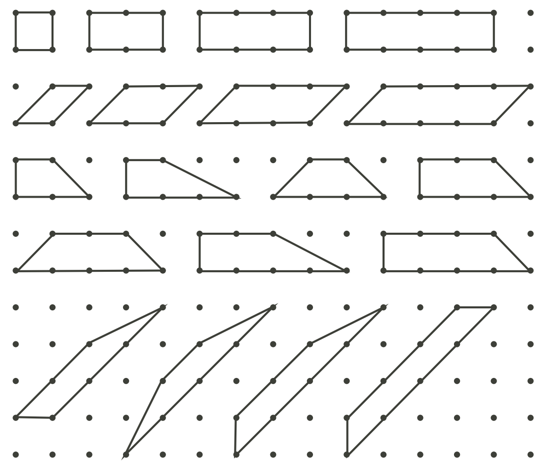
No. of Dots	Area (sq. units)
3	$\frac{1}{2}$
4	1
5	$1\frac{1}{2}$
6	2

   (ii) For each increase of 1 in the number of dots, $\frac{1}{2}$ square unit of area is added.

(iii) A triangle with 20 dots on the boundary has an area of $1 + \frac{1}{2}(20 - 4) = 9$ sq. units.

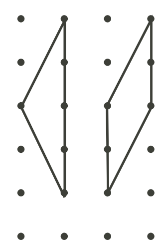



b) For non-right-angled triangles, we see that all those through 4 dots have base 1 unit and height 2 units and hence area 1 square unit, as in a). It is clear that a similar pattern to a) holds for all triangles of base 1 unit, e.g., 5 dots has height 3 and area $\frac{3}{2}$, 6 has height 4 and area 2, etc. A triangle with 6 dots could also have a height 2 and base 2 (hence area 2) as shown at the far right.

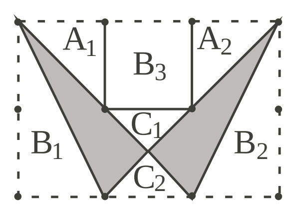


c) For rectangles, the areas are 1, 2, 3, 4, ... for 4, 6, 8, 10, ... dots, respectively, and the same holds for parallelograms. For trapezoids, those with an even number of dots have the same pattern as rectangles, while those with an odd number fall half-way between. Thus the pattern for the quadrilaterals appears to be: for 4, 5, 6, 7, 8, 9, 10, ... dots, the areas are 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, ... etc., the same as for triangles.

Extensions:



1. For eight polygons with area 2 square units, we can use the four triangles with 6 dots, the rectangle, parallelogram, and two trapezoids with 6 dots, giving 8 polygons already shown above with area 2. Two others are shown at right.



2. Note that areas A_1 and A_2 are both $\frac{1}{2}$ square unit: areas B_1, B_2 and B_3, are each 1 square unit, and area C_1 and C_2 are each $\frac{1}{4}$ square unit. Thus the unshaded area is $(2 \times \frac{1}{2}) + (3 \times 1) + (2 \times \frac{1}{4}) = 4\frac{1}{2}$ square units, leaving a W-shaped area of $6 - 4\frac{1}{2} = 1\frac{1}{2}$ square units.