

Problem

The Grade 6 class at Happy Valley School is holding a cookie sale and lottery to raise money for the Red Cross. They have 60 small boxes, each containing 6 cookies and a small card printed with one of the six letters Y, O, U, W, I, N. The letters are distributed as follows: 15 boxes have cards with a Y, 13 with an O, 12 with a U, 10 with a W, 6 with an I, 4 with an N. To win the lottery, you have to buy boxes containing at least one of each of the six letters.



- Could you win if you bought just six boxes? Are you likely to win?
- How many winners could there be?
- Assuming the boxes look the same and are sold in random order, what is the probability that a buyer selecting one box will get one with a Y? an O? an N?
- How many boxes of cookies would you have to buy in order to *guarantee* that you have some set of six boxes containing Y, O, U, W, I, N (i.e., to guarantee that you've won the lottery)?
- If the students sell all the cookies for \$4 per box, how much will they make? If each lottery winner gets \$50, and baking the cookies cost \$30, what is the greatest amount they could make for the Red Cross?

Extension:

If you get together with three friends and borrow enough money from someone else to buy all 60 boxes of cookies to share among you, how much of your own money will each of you have to pay back to the person who lent you the money?



Hints**Part a)**

Hint 1 - You need to have one of each letter to win the lottery.

Part c)

Suggestion: Review with students that the *likelihood* of an event equals

$$\frac{\text{the number of favourable outcomes}}{\text{the total number of possible outcomes}}$$

For example if you have 4 red marbles and 5 green marbles and 6 yellow marbles mixed together in a bag, and you reach your hand in without looking and grab one marble, the likelihood (probability) it is red is $\frac{4}{15}$, while the probability it is yellow or green is $\frac{11}{15}$.

Part e)

Hint 1 - Does there have to be a winner?

Solution

- a) If the six boxes you bought happened to contain exactly the six letters Y, O, U, W, I, N, you would win. This is not very likely because of the large number of possible ways to select six boxes.
- b) There could be at most 4 winners, since only 4 boxes have an N.
- c) Since there are 15 boxes with a Y out of 60 boxes, the probability of randomly selecting a box with a Y is $\frac{15}{60}$, or $\frac{1}{4}$. Similarly, the probability of getting a box with an O is $\frac{13}{60}$, or an N is $\frac{4}{60}$, or $\frac{1}{15}$.
- d) To *guarantee* having a winning set, you would have to buy enough boxes that your set includes at least one N. Since only 4 of the 60 boxes have an N, there are 56 with other letters. Thus to be sure of having at least one box with an N, you would have to buy 57 boxes.
- e) At \$4 per box, if all the cookies are sold, the students will make $\$4 \times 60 = \240 . They would make the *greatest* amount for the Red Cross if there are *no* lottery winners, in which case they need only pay the \$30 baking costs, leaving $\$240 - \$30 = \$210$. (On the other hand, if there are four winners, they make only $\$240 - \$30 - (4 \times \$50) = \10 !)

Extension:

If you and three friends borrow \$240 and buy all 60 boxes, each of you wins \$50. Hence you will need to only pay $(\$240 - \$200) \div 4 = \$10$ each back to the person who lent you the money.