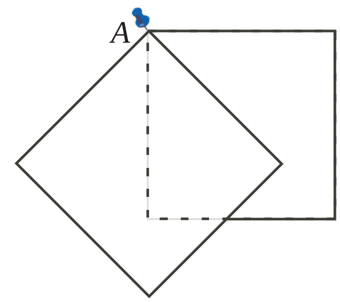
Problem

Picture two identical squares held together by a pin at A, so that the top square can be rotated about A while the bottom square remains fixed. Rotate the top figure partially as shown in the diagram. Trace the *outside* edges to form a polygon.

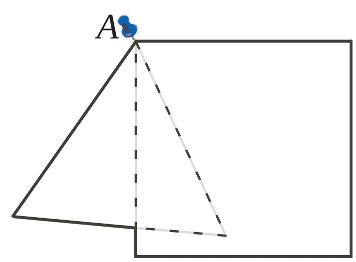


a) How many sides has this polygon?

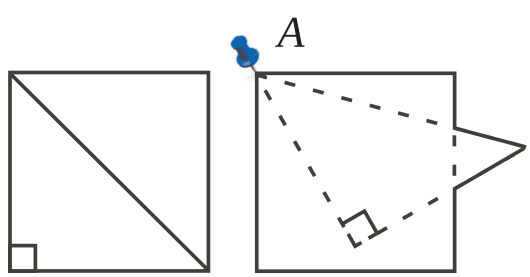
b) What is the least number of sides for a polygon formed by rotating the top square about the pin at A? How many different figures can you form with this (least) number of sides?

c) Could you form a polygon with more than 6 sides this way?

d) Repeat parts b) and c) if the top figure is an equilateral triangle with side length the same as the square on the bottom, with the pin A at one vertex of the triangle (and one corner of the square).

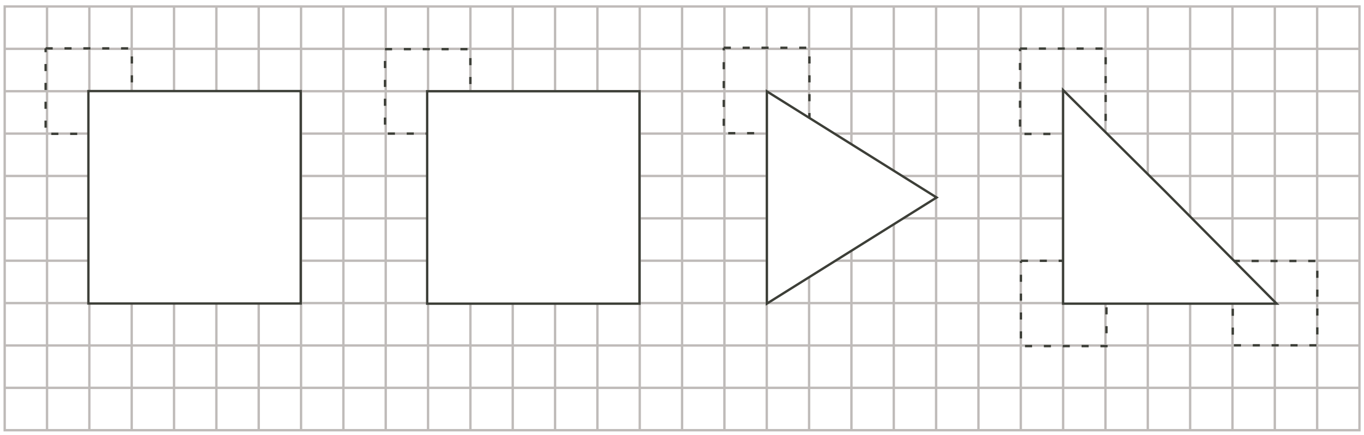


Extension:
Suppose instead that the triangle of part d) is the isosceles triangle that is one-half the square. Are your answers the same as in d)?



Hints

Suggestion: Students can easily create the various figures in this problem by cutting out the figures below, including the dotted tabs to allow them to attach the figures with pins at the vertices. Supply pins or tacks. (Alternatively, students can hold the figures with a pen or pencil point.) The figures could also be traced on bits of acetate using markers.



Part c)

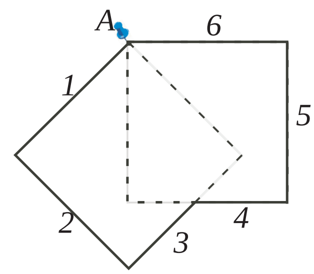
You may wish to discuss "What is a polygon?"(besides what you say when your parrot dies!). For example, the figure  is not (usually) considered to be a polygon.

Extension:

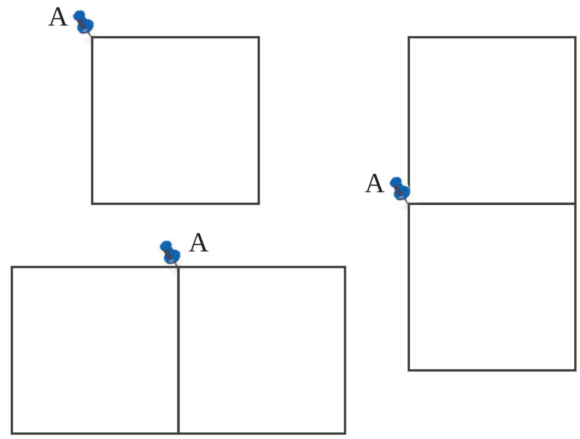
Hint 1 - What difference would it make if you used a different vertex of the triangle matched to the point A?

Solution

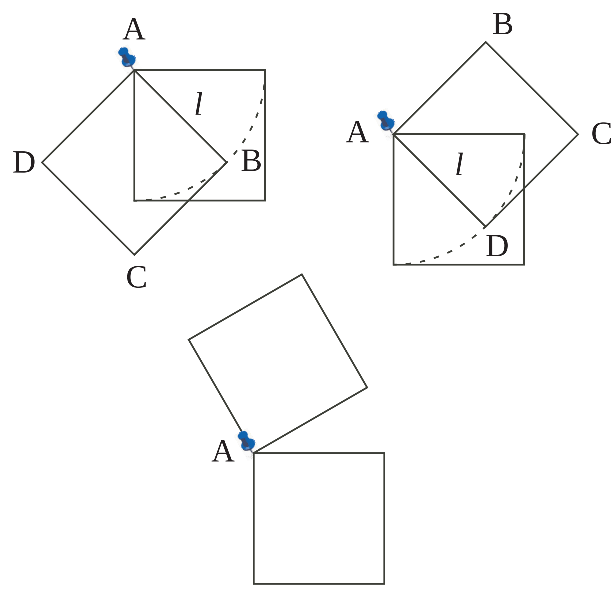
a) The polygon shown in the diagram has six sides.



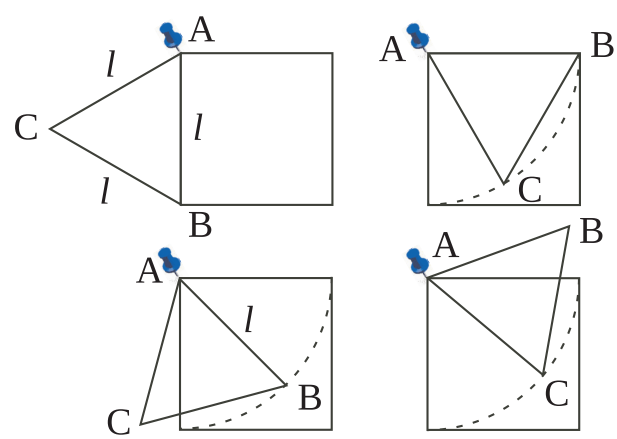
b) The least number of sides for a polygon formed by rotating the top square about the pin at A is four. This can have the shape of the original square (by simply overlapping the two squares completely), or a rectangular shape with one side double the other side, which can be formed in two ways, as shown. Thus there are just two such figures.



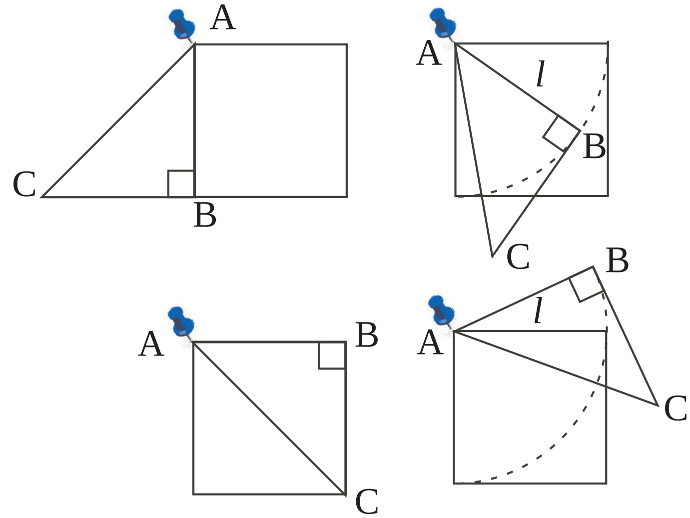
c) You could not form a polygon with more than 6 sides this way. Once you start rotating the top square, the vertex B travels on a circle of radius l, the side length of the square, giving a 6-sided polygon (top left). When the squares exactly overlap, the polygon has 4 sides. Then, as the top square is further rotated, vertex D travels on the same circle, and the polygon again has 6 sides (top right), until it reaches the rectangle shape with the top square above the bottom one. If we continue rotating about A, we have 8 sides (lower diagram), but not a polygon, since more than 2 sides meet at vertex A.



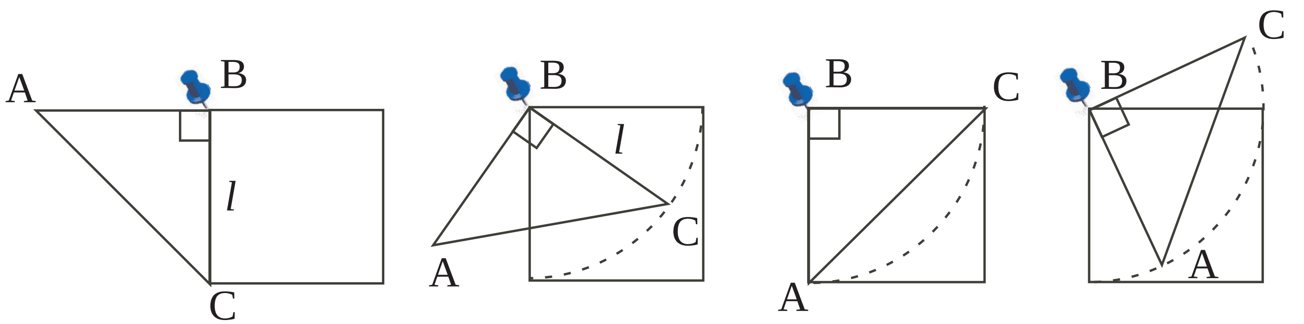
d) If the top figure is an equilateral triangle with side length the same as the square, we can reason as follows, starting with a side of the triangle coinciding with a side of the square (top left) to give a 5-sided polygon. As vertex B moves along a circle of radius l, we get a 6-sided figure (lower left), and then a 4-sided figure as AB coincides with the top of the square (top right), and 6-sided once again as the triangle moves above the square (lower right).



Extension:
Consider first the possibilities if the triangle is pinned to the square at one of the 45^o vertices. If the triangle is the right angle triangle which is half the square, then the polygon with the least number of sides has 4 sides (top left). A second way to get a polygon with 4 sides is when the triangle completely overlaps the square (lower left). With this triangle, we can form a polygon with 7 sides by any position with the side AB inside the square (top right). If AB is above the square, the polygon formed will have 6 sides as shown (lower right).



Suggestion: Have students explore what happens if the triangle is pinned at the other 45^o vertex C. (The possibilities are the same.)
Alternatively, if the triangle is pinned to the square at the right-angle vertex, only polygons with 4 sides or 6 sides are possible, as shown below.



Another, more interesting possibility is to explore the results if two congruent triangles are used (either the equilateral or isosceles triangle) instead of a triangle and a square. These can yield 8-sided polygons.