Part II: For the Teacher

Curriculum Areas

Problem 1 - Number Sense

Problem 2 - Number Sense and Pattern/Algebra

Problem 3 - Measurement and Geometry

Problem 4 - Geometry

Problem 5 - Probability

Problem 6 - Data Management

Hints and Suggestions:

Problem 1 c)

Hint 1 - Could the parents both have gone to the soccer game? Why or why not?

Hint 2 - Could all three children go to the soccer game? Why or why not?

Hint 3 - If both parents went to the basketball game, would there be enough money left for all three children to see a game?

Hint 4 - What is the least the parents' tickets could cost? What tickets could the children then buy?

Problem 2 a)

Suggestion: Supply students with a 1-100 hundreds chart.

Hint 1 - How many 3s are there from 1 to 10? How many 2s?

Hint 2 - For what numbers does 3 occur as a units digit? A tens digit?

Problem 2 b)

Hint 1 - Is the total number of 3s the same as the total number of 2s?

Problem 2 c)

Hint 1 - What number occurs only once as a tens digit?

Problem 3a)

Hint 1 - In how many different places could the extra area be added?

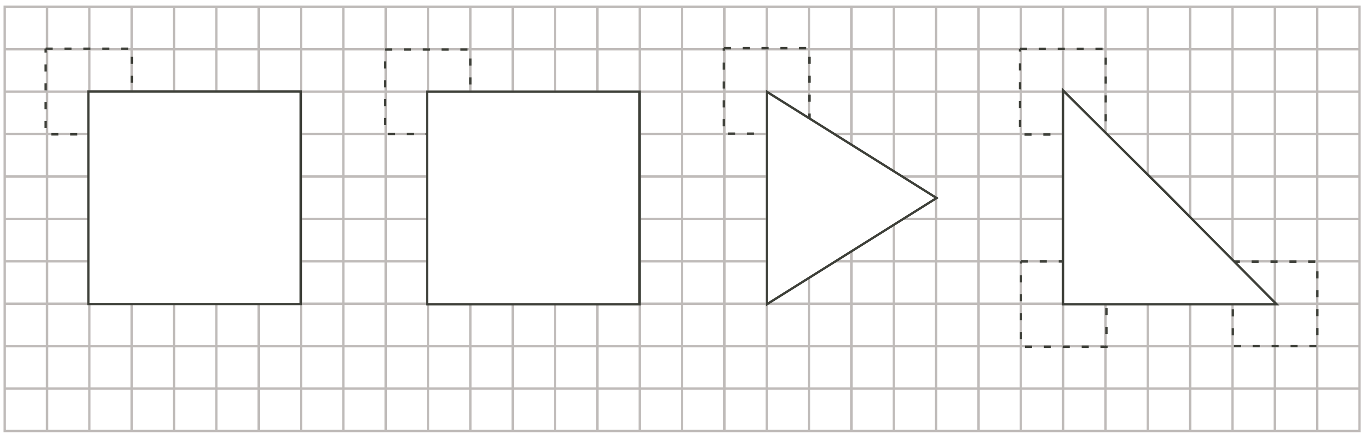
Hint 2 - Remember, she will re-use the old fencing.

Problem 3b)

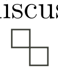
Hint 1 - What is the greatest amount of overlap possible? What is the least?

Problem 4

Suggestion: Students can easily create the various figures in this problem by cutting out the figures below, including the dotted tabs to allow them to attach the figures with pins at the vertices. Supply pins or tacks. (Alternatively, students can hold the figures with a pen or pencil point.) The figures could also be traced on bits of acetate using markers.



Problem 4c)

You may wish to discuss "What is a polygon?"(besides what you say when your parrot dies!). For example, the figure  is not (usually) considered to be a polygon.

Extension:

Hint 1 - What difference would it make if you used a different vertex of the triangle matched to the point A?

Problem 5a)

Hint 1 - You need to have one of each letter to win the lottery.

Problem 5c)

Suggestion: Review with students that the *likelihood* of an event equals

$$\frac{\text{the number of favourable outcomes}}{\text{the total number of possible outcomes}}$$

For example if you have 4 red marbles and 5 green marbles and 6 yellow marbles mixed together in a bag, and you reach your hand in without looking and grab one marble, the likelihood (probability) it is red is $\frac{4}{15}$, while the probability it is yellow or green is $\frac{11}{15}$.

Problem 5e)

Hint 1 - Does there have to be a winner?

Problem 6a)

Hint 1 - In listing the games to be played, be careful not to list the same game twice.

Problem 6b)

Hint 1 - Remember that when one team wins, another team loses.

Problem 6c) -e)

Hint 1 - Remember that there has to be a total of six wins and 6 losses.

Solutions

Problem 1

a) Since the tickets for the soccer game cost $(2 \times \$5.00) + (3 \times \$2.50) = \$17.50$, the change is $\$20.00 - \$17.50 = \$2.50$.

b) Since the cost ends with 50¢, they must have attended either soccer or basketball. We know from a) that soccer costs more than \$10.50, so they must have gone to the basketball game. Check: $(2 \times \$3) + (3 \times \$1.50) = \$6 + \$4.50 = \$10.50$

c)
1. If they all attend the hockey game, the cost is $(2 \times \$2.00) + (3 \times \$1.00) = \$7.00$. (This is the cheapest option)
2. If one parent and three children go to hockey, paying $(\$2.00 + 3 \times \$1.00)$, and the other parent attends basketball, paying $(1 \times \$3.00)$, the cost is \$8.00.
3. If the parents and one child go to hockey, paying $(2 \times \$2.00) + (1 \times \$1.00)$, and the other two children go to basketball, paying $(2 \times \$1.50)$, the cost is \$8.00.

Note that both parents to hockey, or one to hockey and one to basketball are the only possible combinations for the parents to leave enough money for all the children to see a game.

So there are three ways they can all get to see a game for a total cost not over \$8.00.

Note: Students may want to count different combinations of children/parents in 2. and 3. If so, there are two ways for 2. (Mom to hockey, Dad to basketball or vice versa), and three ways for 3. (child A to hockey and B, C to basketball, or B to hockey and A, C to basketball, or C to hockey and A,B to basketball.) This gives a total of six ways.

Problem 2

a) The digit 3 occurs 10 times as a tens digit (30 - 39), and 10 times as a units digit (3, 13, 23, ..., 93), giving a total of 20 times.

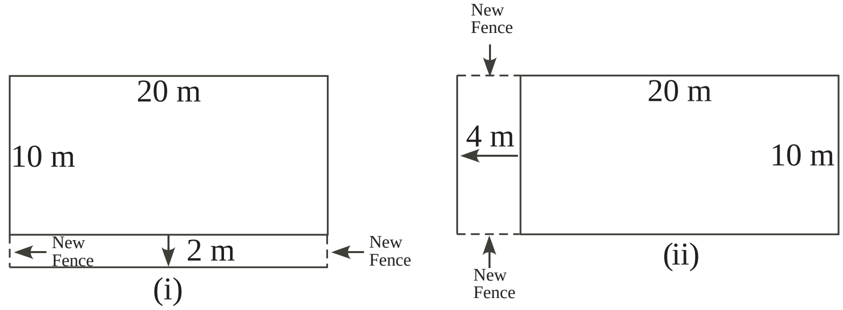
b) The answer is the same for the digits 2, 4, 5, ..., 9, for the same reasons.

c) The digits 0 and 1 give different answers. There are 11 occurrences of 0 (10, 20, ..., 90, 100), and 21 occurrences for 1 (20 as above, plus an extra in 100).

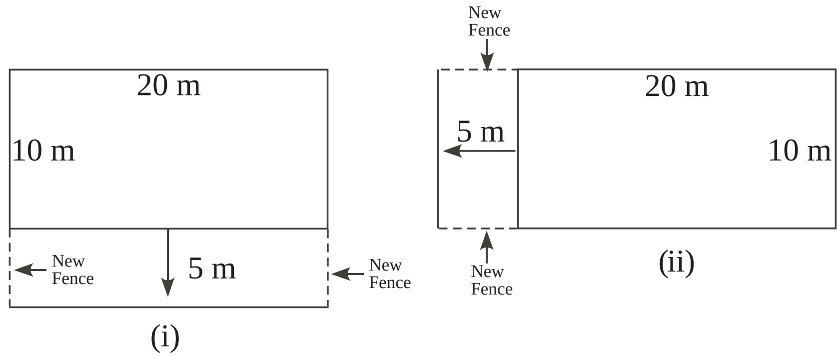
d) The total number of digits is thus $8 \times 20 + 11 + 21 = 192$.

Problem 3

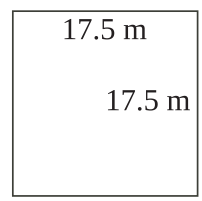
a) There are two ways Jan can move one side of the pen outwards to create the additional area of 40 square metres. They are both shown at the right. Clearly choice (i) uses only 4 metres of new fence, while (ii) uses 8 metres.



b) Jan could use the 10 metres of stored fencing in the same ways as in a), giving areas (i) $15 \times 20 = 300$ square metres, or (ii) $25 \times 10 = 250$ square metres.

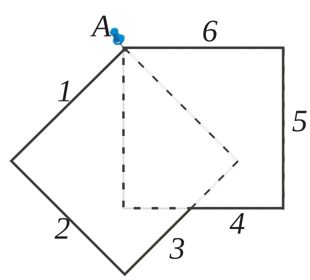


But there is a third alternative, which is to use all the fencing, $20 + 20 + 10 + 10 + 10 = 70$ metres to form a square with sides of 17.5 metres and area $17.5 \times 17.5 = 306.25$ square metres. This gives the largest possible area for the pen.

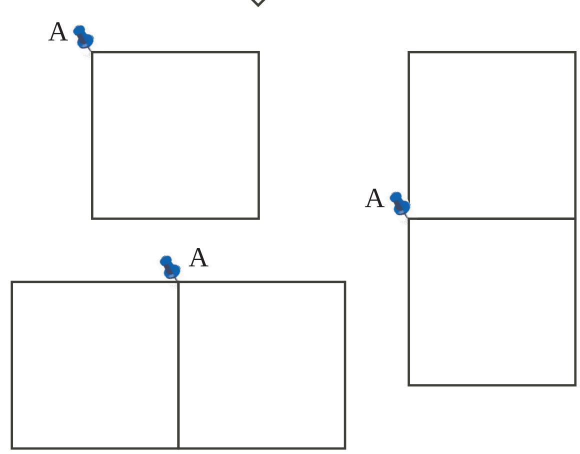


Problem 4

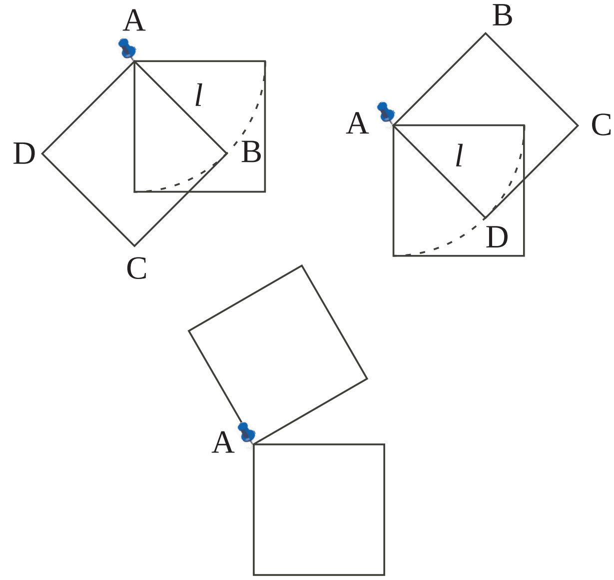
a) The polygon shown in the diagram has six sides.



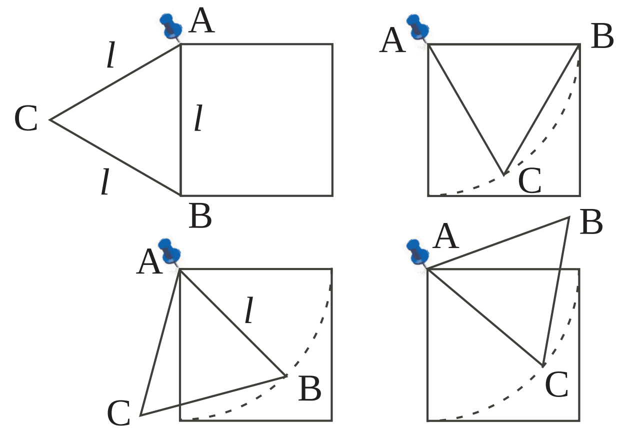
b) The least number of sides for a polygon formed by rotating the top square about the pin at A is four. This can have the shape of the original square (by simply overlapping the two squares completely), or a rectangular shape with one side double the other side, which can be formed in two ways, as shown. Thus there are just two such figures.



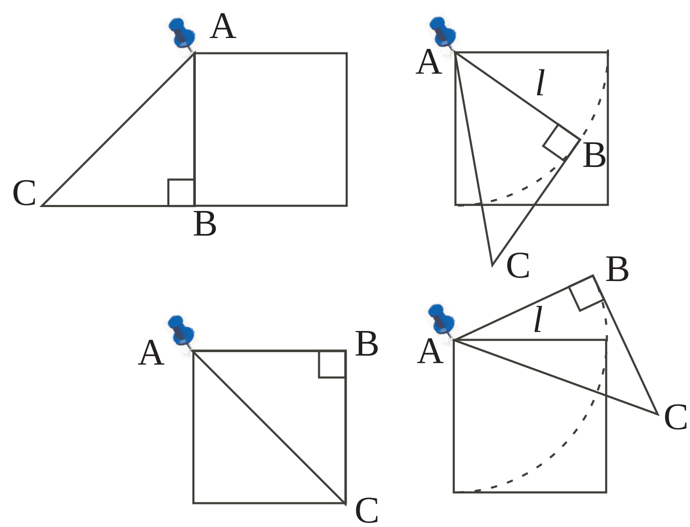
c) You could not form a polygon with more than 6 sides this way. Once you start rotating the top square, the vertex B travels on a circle of radius l, the side length of the square, giving a 6-sided polygon (top left). When the squares exactly overlap, the polygon has 4 sides. Then, as the top square is further rotated, vertex D travels on the same circle, and the polygon again has 6 sides (top right), until it reaches the rectangle shape with the top square above the bottom one. If we continue rotating about A, we have 8 sides (lower diagram), but not a polygon, since more than 2 sides meet at vertex A.



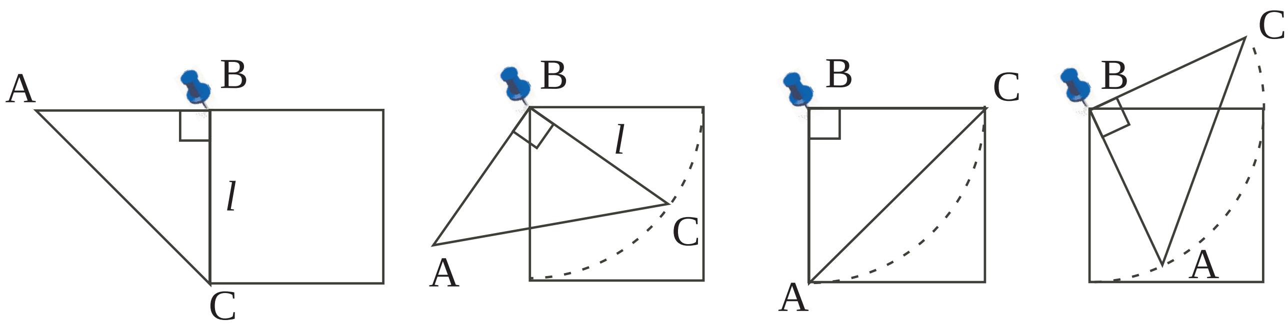
d) If the top figure is an equilateral triangle with side length the same as the square, we can reason as follows, starting with a side of the triangle coinciding with a side of the square (top left) to give a 5-sided polygon. As vertex B moves along a circle of radius l, we get a 6-sided figure (lower left), and then a 4-sided figure as AB coincides with the top of the square (top right), and 6-sided once again as the triangle moves above the square (lower right).



Extension:
Consider first the possibilities if the triangle is pinned to the square at one of the 45^o vertices. If the triangle is the right angle triangle which is half the square, then the polygon with the least number of sides has 4 sides (top left). A second way to get a polygon with 4 sides is when the triangle completely overlaps the square (lower left). With this triangle, we can form a polygon with 7 sides by any position with the side AB inside the square (top right). If AB is above the square, the polygon formed will have 6 sides as shown (lower right).



Suggestion: Have students explore what happens if the triangle is pinned at the other 45^o vertex C. (The possibilities are the same.)
Alternatively, if the triangle is pinned to the square at the right-angle vertex, only polygons with 4 sides or 6 sides are possible, as shown below.



Another, more interesting possibility is to explore the results if two congruent triangles are used (either the equilateral or isosceles triangle) instead of a triangle and a square. These can yield 8-sided polygons.

Problem 5

a) If the six boxes you bought happened to contain exactly the six letters Y, O, U, W, I, N, you would win. This is not very likely because of the large number of possible ways to select six boxes.

b) There could be at most 4 winners, since only 4 boxes have an N.

c) Since there are 15 boxes with a Y out of 60 boxes, the probability of randomly selecting a box with a Y is $\frac{15}{60}$, or $\frac{1}{4}$. Similarly, the probability of getting a box with and O is $\frac{13}{60}$, or an N is $\frac{4}{60}$, or $\frac{1}{15}$.

d) To *guarantee* having a winning set, you would have to buy enough boxes that your set includes at least one N. Since only 4 of the 60 boxes have an N, there are 56 with other letters. Thus to be sure of having at least one box with an N, you would have to buy 57 boxes.

e) At \$4 per box, if all the cookies are sold, the students will make $\$4 \times 60 = \$240$. They would make the *greatest* amount for the Red Cross if there are *no* lottery winners, in which case they need only pay the \$30 baking costs, leaving $\$240 - \$30 = \$210$. (On the other hand, if there are four winners, they make only $\$240 - \$30 - (4 \times \$50) = \$10$)!

Extension:
If you and three friends borrow \$240 and buy all 60 boxes, each of you wins \$50. Hence you will need to only pay $(\$240 - \$200) \div 4 = \$10$ each back to the person who lent you the money.

Problem 6

Current Standings are:

Team	Wins	Losses	Points
Majors	15	2	30
All-Stars	13	4	26
Champs	12	5	24
Generals	10	7	20

a) There are six games left to play, as follows: 1. Majors vs All-Stars, 2. Majors vs Champs, 3. Majors vs Generals, 4. All-Stars vs Champs, 5. All-Stars vs Generals, 6. Champs vs Generals.

b) Since the standing is determined solely by the number of wins, we can ignore the losses and points. Further, since we're asked for the highest the Majors can place, we first see whether they can stay on top.
This could happen for the following final standings in the Wins column:

Majors	15	15	15
All-Stars	14 (1 win)	15 (2 wins)	15 (2 wins)
Champs	14 (2 wins)	14 or 13 (2 or 1 win)	15 (3 wins)
Generals	13 (3 wins)	12 or 13 (2 or 3 wins)	11 (1 win)

Note that the last two columns are ties for first place.

c) Since there are only three more games for each team, the only team that could surpass the Majors is the All-Stars, if they win all three games, and the Majors lose all three of their games. In this case the final standings in the Wins column could be either of the following:

Majors	15
All-Stars	16 (3 wins)
Champs	13 or 12 (2 or 1 win)
Generals	12 or 13 (1 or 2 wins)

d) The Majors and All-Stars could tie for highest standing, or the Majors, All-Stars and Champs could have a three-way tie, as shown in the following final standings in the Wins column:

Majors	15	16 (1 win)	15
All-Stars	15 (2 wins)	16 (3 wins)	15 (2 wins)
Champs	14 or 13 (2 or 1 win)	14 or 13 or 12 (2, 1 or 0 wins)	15 (3 wins)
Generals	12 or 13 (2 or 3 wins)	10 or 11 or 12 (0, 1 or 2 wins)	11 (1 win)

e) The Generals could end up in a three-way tie for second place by winning all three of their games, as shown in the following final standings for the Wins column:

Majors	18 (3 wins)	17 (2 wins)
All-Stars	13	13
Champs	12	13 (1 win)
Generals	13 (3 wins)	13 (3 wins)

Comment: In each case, the final win column must total 56. Thus one way to explain possible outcomes is to add 0, 1, 2, or 3 to each team's original number of wins.