CEMI à la maison

9e et 10e année - Lundi 6 avril 2020

Jeunes pousses

Tu auras besoin de :

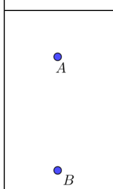
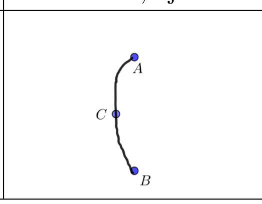
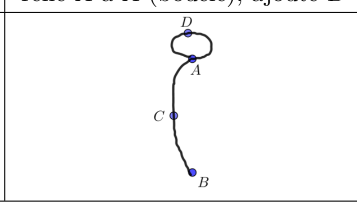
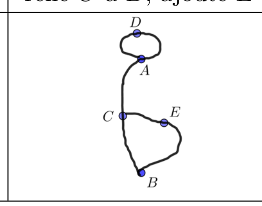
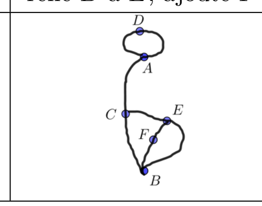
— Deux joueurs

— Une feuille de papier et un crayon

Comment jouer :

1. Commence avec deux ou trois points sur la page, raisonnablement espacés.
2. Les joueurs alternent les tours. Décidez quel joueur commence.
3. À ton tour, fais les actions suivantes (si possible, en fonction des restrictions indiquées en 4) :
   Trace une courbe reliant les deux points existants et ajoute un point à la courbe nouvellement tracée. *Note que cette courbe peut être tracée entre deux points différents ou sous la forme d'une boucle à partir d'un point vers lui-même.*
4. Voici les restrictions concernant les mouvements effectués en 3 :
   — Tu ne peux pas tracer une courbe si cela a pour résultat qu'un point ait plus de trois segments de courbe qui entrent ou sortent du point. En particulier, tu ne peux pas tracer une boucle sur un point qui a déjà plus d'un segment de courbe entrant ou sortant.
   — Tu ne peux pas tracer une courbe si elle doit croiser une courbe existante.
   — Le point ajouté ne peut pas être placé sur un point existant.
5. La dernière personne qui réussit à tracer une nouvelle courbe, en suivant les règles, gagne la partie !

Exemple d'un jeu complet commençant avec 2 points :

Début	Joueur 1 relie A à B, ajoute C	Joueur 2 relie A à A (boucle), ajoute D	Joueur 1 relie C à B, ajoute E	Joueur 2 relie B à E, ajoute F
				

Remarque qu'après ces quatre tours, le joueur 1 ne peut pas tracer de nouvelle courbe. Le joueur 1 ne peut pas tracer de courbe partant de A puisqu'il y a déjà trois segments de courbe qui entrent ou sortent de A (avec les deux de la boucle). C'est la même chose pour les points B, C, et E. Le joueur 1 ne peut pas relier D à F car la courbe croiserait une courbe existante et il ou elle ne peut pas dessiner de boucle sur D ou F car ils ont chacun déjà deux segments de courbe entrant ou sortant. Par conséquent, le joueur 2 gagne !

Jouez à ce jeu plusieurs fois, en commençant avec 2 points. Note le nombre total de tours qu'il faut pour gagner chaque partie. Y-a-t-il un certain nombre de tours après lequel la partie est garantie d'être terminée ?

Jouez à ce jeu plusieurs fois, en commençant avec 3 points. Y-a-t-il un certain nombre de tours après lequel la partie est garantie d'être terminée ? Comment compares-tu cette réponse à ta réponse pour le jeu commençant avec 2 points ?

Plus d'infos : Consulte la page internet du CEMI à la maison, mardi 14 avril, pour trouver une discussion sur Jeunes pousses.

CEMC at Home

Grade 9/10 - Monday, April 6, 2020
Sprouts - Solution

Sprouts starting with 2 dots

While playing games of Sprouts starting with 2 dots, you may have noticed that each game ended after at most 5 turns. Did you also notice that each game included at least 4 turns?

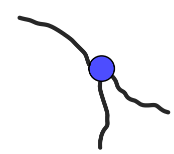
Sprouts starting with 3 dots

While playing games of Sprouts starting with 3 dots, you may have noticed that each game ended after at most 8 turns. Did you also notice that each game included at least 6 turns?

We encourage you to think about each of these observations and see if you can explain why this happened. We provide a discussion below to explain why any game starting with 3 dots must end after at most 8 turns.

A game of Sprouts starting with 3 dots must end after at most 8 turns

First, we note that the game is over as soon as no dots in the game can be part of a newly drawn curve. Remember that all dots in the game can have at most three curve segments attached to them. We will think of each dot as having three "slots" that can be filled. Curve segments can be attached to a dot M in a few different ways as shown below.



Curve drawn from M to another dot	Loop drawn at M	M added to a newly drawn curve

This curve takes up one slot on dot M	*This loop takes up two slots on dot M*	*This curve/loop takes up two slots on dot M*

The game starts with three dots and no curves. Since there are three dots in total, each having three slots, the game starts with nine available slots.

After one turn is complete, one of two things has happened:

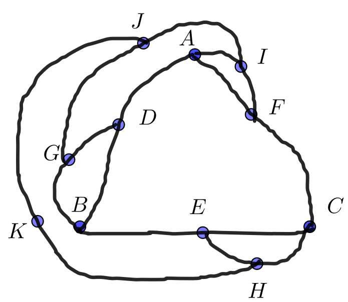
- a loop was added to one of the three dots, and a fourth dot was added to this loop, or
- a curve was drawn between two of the three dots, and a fourth dot was added to this curve.

In either case, after the first turn, there will be eight available slots remaining. Here we explain why:

Drawing a loop on a dot fills two of the three available slots on that particular dot, reducing us to $9 - 2 = 7$ slots available for the three original dots. However, a fourth dot is also added to this loop. Since two slots of this new dot are already taken, there is exactly one slot open on this new dot. This means there are $7 + 1 = 8$ slots available among the four dots now in the game. Notice that the situation is similar if a curve is drawn from one dot to another dot: we fill two slots (one on each dot at the ends of this curve), but gain one new slot from the fourth dot that is added.

In a similar way, we can argue that for *each turn that follows*, there is a net total loss of one slot per turn. After turn 2 there are 7 slots left, after turn 3 there are 6 slots left, and so on. If the game makes it to turn 8, then there can be only 1 slot left after turn 8 is complete. Since there must be at least 2 slots available in order for a new curve to be drawn, we can be sure that there is no legal move to make on turn 9. Therefore, we see that any game starting with 3 dots must end after at most 8 turns.

Did one of your games last exactly 8 turns? Note that our discussion above does not argue that a game can actually make it all the way to 8 turns, just that it is impossible for a game to make it to 9 turns. Below is an example of a game that lasted exactly 8 turns, which shows that 8 is the *maximum* number of turns attainable in a game starting with 3 dots.



Move	Endpoints	Added Point
1	A and B	D
2	B and C	E
3	A and C	F
4	B and D	G
5	E and C	H
6	A and F	I
7	G and I	J
8	H and J	K

Now try the following on your own:

- Explain why a game of Sprouts starting with 3 dots must last for at least 6 turns.
- Determine if a game of Sprouts starting with 3 dots can end in exactly 6 turns.

The game of Sprouts was invented by mathematicians John H. Conway (who died recently) and Michael S. Paterson at Cambridge University.

CEMI à la maison

9e et 10e année - Mardi 7 avril 2020
Code additionnel

Dans ce casse-tête, chacune des lettres de l'alphabet représente un nombre entier différent de 1 à 26. Ta tâche consiste à déterminer quel nombre est associé à chaque lettre. Pour commencer, on te donne : H = 20 et N = 17. Utilise les équations algébriques pour déchiffrer le code et trouver les autres correspondances.

A	B	C	D	E	F	G	H	I	J	K	L	M
							20					
N	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
17												

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 ~~17~~ 18 19 ~~20~~ 21 22 23 24 25 26

Équations algébriques

E = D × D

B = E − D

H = E + D

B = T × D

H = D × C

J = C − T

V = C × C

Y × Y = P + I

Y + M = P − Y

P = V + 1

R = F − R

S = R − J

A = K + L

U = K × T

Z = O + W − K

O = W + C

X = T × C

Q = G − N + U

Plus d'infos :

Consulte la page internet du CEMI à la maison, mardi 14 avril, pour la solution de Code additionnel.

CEMC at Home

Grade 9/10 - Tuesday, April 7, 2020
Sum Code - Solution

Answers

A	B	C	D	E	F	G	H	I	J	K	L	M
8	12	5	4	16	22	19	20	10	2	7	1	14
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
17	18	26	23	11	9	3	21	25	13	15	6	24

Explanation

Since we are told that H = 20, and there are two equations involving the letter H, a good place to start is with these equations.

$E = D \times D$	$V = C \times C$	$A = K + L$
$B = E - D$	$Y \times Y = P + I$	$U = K \times T$
$H = E + D$	$Y + M = P - Y$	$Z = O + W - K$
$B = T \times D$	$P = V + 1$	$O = W + C$
$H = D \times C$	$R = F - R$	$X = T \times C$
$J = C - T$	$S = R - J$	$Q = G - N + U$

The equation $H = D \times C$ tells us that D and C are a factor pair of 20. This means they could be 2 and 10 (in some order) or 4 and 5 (in some order). Note that they cannot be 1 and 20. (Why not?)

The equations $E = D \times D$ and $V = C \times C$ tell us more about this factor pair. If the factor pair is 2 and 10, then E and V are 4 and 100 (in some order). This is not possible since the numbers in this code only range from 1 to 26. Therefore, it must be the case that the factor pair D and C are 4 and 5 (in some order) which means that E and V are 16 and 25 (in some order).

Suppose D = 5 and C = 4. Then E = 25 and V = 16. Using the equation $H = E + D$ we get that $H = 25 + 5 = 30$ which we know must be false. Since this is not the correct order of the factor pair, we know we must have D = 4 and C = 5. In this case, we get E = 16 and V = 25. We confirm with equation $H = E + D$ that we get H = 20 as expected.

We now know for certain the values of D, C, E and V. By substituting these values into all of the relevant equations above, we can also determine the values for B, T, J, P, and X.

To proceed further, consider the equation $Y \times Y = P + I$. This tells us that $P + I$ is a perfect square. What does this tell us about possible values for I and Y? What does this information, combined with the equation $Y + M = P - Y$, reveal about the value of M?

By substituting values we already know into equations, and combining equations that contain common letters, we can proceed to crack the rest of the code, as indicated in the answer key above.

CEMI à la maison

9e et 10e année - mercredi 8 avril 2020
Acheter local

La semaine dernière, cinq personnes (Charlie, Manuel, Priya, Sal, et Tina) ont fait leur courses au marché de producteurs locaux. Chaque personne y est allée un jour différent (de lundi à vendredi), a acheté un article différent (carottes, bleuets, tomates, pommes ou patates) et a dépensé une somme d’argent différente (1,50 $, 2,00 $, 2,50 $, 3,50 $, ou 3,75 $).

Utilise les indices ci-dessous pour déterminer qui y est allé chaque jour, ce qu’ils ont acheté et pour combien.

1. Sal est allé au marché deux jours avant Priya.
2. Manuel a dépensé 3,75 $ au marché la veille du jour où quelqu’un a acheté des tomates.
3. Charlie a payé 2,50 $ pour des carottes le jour après que quelqu’un ait dépensé 3,50 $.
4. La personne qui y est allée mercredi a acheté des pommes.
5. Quelqu’un a acheté des patates pour 2,00 $ lundi.

Le tableau suivant pourra être utile pour organiser ta solution.

	Lundi	Mardi	Mercredi	Jeudi	Vendredi	Carottes	Bleuets	Tomates	Pommes	Patates	1,50 $	2,00 $	2,50 $	3,50 $	3,75 $
Charlie															
Manuel															
Priya															
Sal															
Tina															
1,50 $															
2,00 $															
2,50 $															
3,50 $															
3,75 $															
Carottes															
Bleuets															
Tomates															
Pommes															
Patates															

Plus d’infos :

Consulte la page internet du CEMI à la maison, mercredi 15 avril, pour une solution de Acheter local.

Ce type de casse-tête est connu sous le nom de casse-tête logique ou « puzzle logique » et nous en incluons parfois dans le Problème de la semaine. En voici un qui s’appelle Un carnaval d’hiver.

CEMC at Home
Grade 9/10 - Wednesday, April 8, 2020
Buying Local - Solution

Answer

- Charlie bought carrots for \$2.50 on Friday
- Manuel bought apples for \$3.75 on Wednesday
- Priya bought tomatoes for \$3.50 on Thursday
- Sal bought blueberries for \$1.50 on Tuesday
- Tina bought potatoes for \$2.00 on Monday

Explanation

There are many different ways to arrive at the answers above. You may have used the chart provided with the problem to keep track of matches that were confirmed or deemed impossible while examining and combining the different clues. Below we present an explanation in words only. It may be helpful to follow along by filling out the chart given with the problem as you read.

The apples were purchased on Wednesday (clue 4). The potatoes were purchased for \$2.00 on Monday (clue 5). The carrots were not purchased on Tuesday because then the potatoes would cost \$3.50 instead of \$2.00 (clue 3). So the carrots were purchased for \$2.50 on either Thursday or Friday. The tomatoes were not purchased on Tuesday because then the potatoes would cost \$3.75 instead of \$2.00 (clues 2 and 5). The tomatoes were not purchased on Friday because if they were, then the carrots were purchased on Thursday, and the carrots would cost \$3.75 instead of \$2.50 (clues 2 and 3).

So the potatoes were purchased for \$2.00 on Monday, the blueberries were purchased on Tuesday, the apples were purchased on Wednesday, the tomatoes were purchased on Thursday, and the carrots were purchased for \$2.50 on Friday.

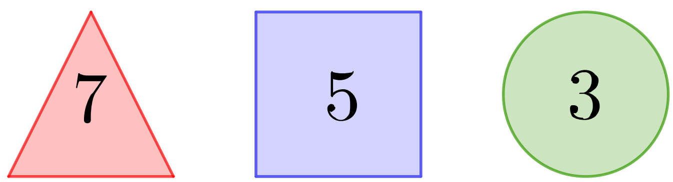
Manuel spent \$3.75 on Wednesday purchasing apples (clue 2). Charlie spent \$2.50 on Friday purchasing carrots, and someone spent \$3.50 on Thursday purchasing tomatoes (clue 3). Since the potatoes cost \$2.00 (clue 5) someone spent \$1.50 on Tuesday purchasing blueberries. Priya, Sal, and Tina went to the market on Monday, Tuesday, and Thursday in some order. Since Sal went two days before Priya (clue 1), then it must be the case that Sal went on Tuesday, Priya went on Thursday, and Tina went on Monday.

Le CEMI à la maison présente Le problème de la semaine

9e et 10e année - jeudi 9 avril 2020
Des jetons jetés

Trois sacs contiennent des jetons. Le sac vert contient 22 jetons verts circulaires, chacun numéroté avec un nombre entier entre 1 et 22. Le sac rouge contient 15 jetons rouges triangulaires, chacun numéroté avec un nombre entier entre 1 et 15. Le sac bleu contient 10 jetons bleus carrés, chacun numéroté avec un nombre entier entre 1 et 10.

Chaque jeton dans un des sacs a la même probabilité d'être pigé qu'un autre jeton dans le même sac. Il y a un total de $22 \times 15 \times 10 = 3300$ combinaisons différentes de jetons créées en pigeant un jeton de chaque sac. Note que piger le jeton rouge avec le 7, le jeton bleu avec le 5 et le jeton vert avec le 3 est une combinaison différente que piger le jeton rouge avec le 5, le jeton bleu avec le 7 et le jeton vert avec le 3. L'ordre n'est pas important.



Tu piges un jeton de chaque sac. Quelle est la probabilité que deux jetons ou plus soient numérotés avec un 5 ?

Plus d'infos :

Consultez la page du CEMI à la maison, jeudi 16 avril, pour trouver la solution à ce problème. Vous pouvez également vous inscrire au Problème de la semaine en cliquant sur le lien ci-dessous et recevoir la solution ainsi qu'un nouveau problème, par courriel, jeudi 16 avril.

Cette ressource du CEMI à la maison correspond au Problème de la semaine pour les 9e et 10e années. Le Problème de la semaine est une ressource gratuite. Chaque semaine, des problèmes provenant de divers domaines mathématiques sont publiés en ligne et envoyés par courriel aux enseignants afin qu'ils les utilisent avec leurs étudiants. Les problèmes sont disponibles pour les étudiants de la 3e jusqu'à la 12e année. Les solutions aux problèmes sont envoyées une semaine après, en même temps que le nouveau Problème de la semaine.

Pour plus d'informations et vous inscrire au Problème de la semaine, rendez-vous sur : https://www.cemc.uwaterloo.ca/resources/potw-f.php

7 5 3

Problem of the Week
Problem D and Solution
Tokens Taken

Problem

Three bags each contain tokens. The green bag contains 22 round green tokens, each with a different integer from 1 to 22. The red bag contains 15 triangular red tokens, each with a different integer from 1 to 15. The blue bag contains 10 square blue tokens, each with a different integer from 1 to 10.

Any token in a specific bag has the same chance of being selected as any other token from that same bag. There is a total of $22 \times 15 \times 10 = 3300$ different combinations of tokens created by selecting one token from each bag. Note that selecting the 7 red token, the 5 blue token and 3 green token is different than selecting the 5 red token, 7 blue token and the 3 green token. The order of selection does not matter.

You select one token from each bag. What is the probability that two or more of the selected tokens have the number 5 on them?

Solution

Solution 1

There are 22 different numbers which can be chosen from the green bag, 15 different numbers which can be chosen from the red bag, and 10 different numbers which can be chosen from the blue bag. So there are a total of $22 \times 15 \times 10 = 3300$ different combinations of numbers which can be produced by selecting one token from each bag.

To count the number of possibilities for a 5 to appear on at least two of the tokens, we will consider cases.

1. Each of the selected tokens has a 5 on it.

   This can only occur in 1 way.

2. A 5 appears on the green token and on the red token but not on the blue token.

   There are 9 choices for the blue token excluding the 5. A 5 can appear on the green token and on the red token but not on the blue token in 9 ways.

3. A 5 appears on the green token and on the blue token but not on the red token.

   There are 14 choices for the red token excluding the 5. A 5 can appear on the green token and on the blue token but not on the red token in 14 ways.

4. A 5 appears on the red token and on the blue token but not on the green token.

   There are 21 choices for the green token excluding the 5. A 5 can appear on the red token and on the blue token but not on the green token in 21 ways.

Summing the results from each of the cases, the total number of ways for a 5 to appear on at least two of the tokens is $1 + 9 + 14 + 21 = 45$. The probability of 5 appearing on at least two of the tokens is $\frac{45}{3300} = \frac{3}{220}$.

Solution 2

This solution uses a known result from probability theory. If the probability of event A occurring is a, the probability of event B occurring is b, the probability of event C occurring is c, and the results are not dependent on each other, then the probability of all three events happening is $a \times b \times c$.

The probability of a specific number being selected from the green bag is $\frac{1}{22}$ and the probability of any specific number not being selected from the green bag is $\frac{21}{22}$.

The probability of a specific number being selected from the red bag is $\frac{1}{15}$ and the probability of any specific number not being selected from the red bag is $\frac{14}{15}$.

The probability of a specific number being selected from the blue bag is $\frac{1}{10}$ and the probability of any specific number not being selected from the blue bag is $\frac{9}{10}$.

In the following we will use $P(p, q, r)$ to mean the probability of p being selected from the green bag, q being selected from the red bag, and r being selected from the blue bag. So, $P(5, 5, \text{ not } 5)$ means that we want the probability of a 5 being selected from the green bag, a 5 being selected from the red bag, and anything but a 5 being selected from the blue bag.

$$\begin{aligned}
& \text{Probability of 5 being selected from at least two of the bags} \\
= \; & \text{Probability of 5 from each bag} + \text{Probability of 5 from exactly 2 bags} \\
= \; & \text{P}(5,\ 5,\ 5) + \text{P}(5,\ 5,\ \text{not } 5) + \text{P}(5,\ \text{not } 5,\ 5) + \text{P}(\text{not } 5,\ 5,\ 5) \\
= \; & \frac{1}{22} \times \frac{1}{15} \times \frac{1}{10} + \frac{1}{22} \times \frac{1}{15} \times \frac{9}{10} + \frac{1}{22} \times \frac{14}{15} \times \frac{1}{10} + \frac{21}{22} \times \frac{1}{15} \times \frac{1}{10} \\
= \; & \frac{1}{3300} + \frac{9}{3300} + \frac{14}{3300} + \frac{21}{3300} \\
= \; & \frac{45}{3300} \\
= \; & \frac{3}{220}
\end{aligned}$$

The probability of 5 appearing on at least two of the tokens is $\dfrac{3}{220}$.