Game Board 1
Player 1 has a winning strategy for this game board. Player 1 should choose the square one over and one up from the square with the Splat as shown in the diagram below.

From here, Player 1 will mimic whatever Player 2 does in the following way:

- If Player 2 chooses a square from the remaining column, then Player 1 will choose the corresponding square in the remaining row.
- If Player 2 chooses a square from the remaining row, then Player 1 will choose the corresponding square in the remaining column.

For example, if Player 2 chooses the square marked as P2 (in Diagram 1 below), then Player 1 should chose the square marked as P1. The resulting game board is shown in Diagram 2 below.

If Player 1 keeps mimicking Player 2 in this way, eventually the square with the Splat will be the only square left after Player 1 makes a move. Then Player 2 must choose that square and lose the game.

Note: This solution is valid for any game board with an equal number of rows and columns (except for a 1 by 1 game board). To start, Player 1 should always choose the square one over and one up from the Splat. Then Player 1 mimics Player 2 as described above.

The strategy for Game Board 2 is on the next page.
Game Board 2

It turns out that Player 1 has a winning strategy for this game board but it is not easy to find or to describe. In one strategy, Player 1 can start with the following play.

From here, the strategy depends on what move Player 2 chooses to make. Player 1 will then have to react to this move and there are many different ways this can unfold. We will not present the strategy here but we encourage you to think about it and look into it further on your own. (Remember that this game is often called *Chomp*.)

It is actually possible to argue that Player 1 must have a winning strategy without actually finding one! An argument would involve explaining the following two facts:

1. Exactly one player (Player 1 or Player 2) must have a winning strategy for this game board.
2. It is not possible for Player 2 to have a winning strategy for this game board.

For point 1., it turns out that since there is no way for the game to end in a tie, we can argue that at least one player must have a winning strategy (although this argument is not as easy to explain carefully as you might think). By the definition of a winning strategy, it is not possible for both players to have a winning strategy for the same game board.

To argue point 2. we can use what is sometimes called a *strategy-stealing argument*. The idea here is as follows: If Player 2 had a winning strategy, then Player 1 could use that strategy before Player 2 has a chance to use it. The name “strategy stealing argument” is slightly misleading. A winning strategy must work regardless of the moves made by the other player and so cannot be “stolen” (at least not without a mistake!). That Player 1 can “steal” the winning strategy simply means that Player 1 must have had the winning strategy in the first place.

Let’s assume that Player 2 has a winning strategy. This means that no matter what first move Player 1 makes, Player 2 is guaranteed to win the game following this strategy.

In particular, this means that if Player 1 starts the game by selecting the top right hand square in the grid, then Player 2 must have a winning strategy from this point. This means Player 2 has a move in response to Player 1 removing the top right hand square that leaves Player 1 with a game board from which they cannot possibly win.

No matter what the game board looks like after Player 2 responds, Player 1 could have made their first move in a way that leaves this exact same game board! (Can you see why?) In this way, Player 1 can “steal” the strategy by making their first move to leave Player 2 with this game board from which they cannot win.

This argument may take a few times through to understand, but it achieves something quite remarkable. It explains that Player 1 has a winning strategy, but gives no hint whatsoever as to what the strategy is. In fact, it doesn’t even tell us what the first move should be!

*The above explanation that Player 2 cannot have a winning strategy involves a type of argument called a proof by contradiction.*