CEMC at Home
Grade 9/10 - Tuesday, April 7, 2020
Sum Code - Solution

Answers

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>12</td>
<td>5</td>
<td>4</td>
<td>16</td>
<td>22</td>
<td>19</td>
<td>20</td>
<td>10</td>
<td>2</td>
<td>7</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>N</td>
<td>O</td>
<td>P</td>
<td>Q</td>
<td>R</td>
<td>S</td>
<td>T</td>
<td>U</td>
<td>V</td>
<td>W</td>
<td>X</td>
<td>Y</td>
<td>Z</td>
</tr>
<tr>
<td>17</td>
<td>18</td>
<td>26</td>
<td>23</td>
<td>11</td>
<td>9</td>
<td>3</td>
<td>21</td>
<td>25</td>
<td>13</td>
<td>15</td>
<td>6</td>
<td>24</td>
</tr>
</tbody>
</table>

Explanation

Since we are told that H = 20, and there are two equations involving the letter H, a good place to start is with these equations.

\[ E = D \times D \quad V = C \times C \quad A = K + L \]
\[ B = E - D \quad Y \times Y = P + I \quad U = K \times T \]
\[ H = E + D \quad Y + M = P - Y \quad Z = O + W - K \]
\[ B = T \times D \quad P = V + 1 \quad O = W + C \]
\[ H = D \times C \quad R = F - R \quad X = T \times C \]
\[ J = C - T \quad S = R - J \quad Q = G - N + U \]

The equation \( H = D \times C \) tells us that D and C are a factor pair of 20. This means they could be 2 and 10 (in some order) or 4 and 5 (in some order). Note that they cannot be 1 and 20. (Why not?)

The equations \( E = D \times D \) and \( V = C \times C \) tell us more about this factor pair. If the factor pair is 2 and 10, then \( E \) and \( V \) are 4 and 100 (in some order). This is not possible since the numbers in this code only range from 1 to 26. Therefore, it must be the case that the factor pair \( D \) and \( C \) are 4 and 5 (in some order) which means that \( E \) and \( V \) are 16 and 25 (in some order).

Suppose \( D = 5 \) and \( C = 4 \). Then \( E = 25 \) and \( V = 16 \). Using the equation \( H = E + D \) we get that \( H = 25 + 5 = 30 \) which we know must be false. Since this is not the correct order of the factor pair, we know we must have \( D = 4 \) and \( C = 5 \). In this case, we get \( E = 16 \) and \( V = 25 \). We confirm with equation \( H = E + D \) that we get \( H = 20 \) as expected.

We now know for certain the values of \( D \), \( C \), \( E \) and \( V \). By substituting these values into all of the relevant equations above, we can also determine the values for \( B \), \( T \), \( J \), \( P \), and \( X \).

To proceed further, consider the equation \( Y \times Y = P + I \). This tells us that \( P + I \) is a perfect square. What does this tell us about possible values for \( I \) and \( Y \)? What does this information, combined with the equation \( Y + M = P - Y \), reveal about the value of \( M \)?

By substituting values we already know into equations, and combining equations that contain common letters, we can proceed to crack the rest of the code, as indicated in the answer key above.