Problem:
A positive integer has exactly eight positive factors. If two of the factors are 21 and 35, what is the positive integer?

For some integer $n$, a factor of $n$ is a non-zero integer that divides evenly into $n$. For example, 3 is a factor of 18 since $18 \div 3 = 6$, but 4 is not a factor of 18 since $18 \div 4 = 4.5$.

Solution:
Let $n$ represent the number we are looking for.

We know that four of the positive factors of $n$ are 1, 21, 35 and $n$. In our solution we will first find the remaining four positive factors and then determine $n$.

Since 21 is a factor of $n$ and $21 = 3 \times 7$, 3 and 7 must also be factors of $n$.

Since 35 is a factor of $n$ and $35 = 5 \times 7$, 5 must also be a factor of $n$.

Since 3 is a factor of $n$ and 5 is a factor of $n$, and since 3 and 5 have no common factors, $3 \times 5 = 15$ must also be a factor of $n$.

We have found all eight of the positive factors of the unknown number. The positive factors are 1, 3, 5, 7, 15, 21, 35 and $n$. We now need to determine $n$.

From the list of factors, we see that the prime factors of $n$ are 3, 5 and 7, and it follows that $n = 3 \times 5 \times 7 = 105$.

Therefore, the positive integer is 105.