



CEMC at Home

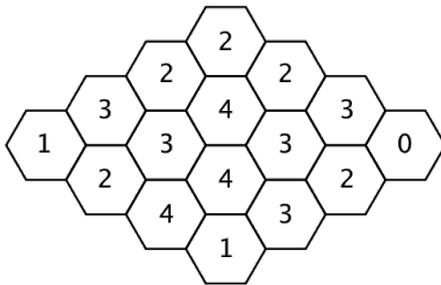
Grade 7/8 - Wednesday, March 25, 2020

Beehive - Solution

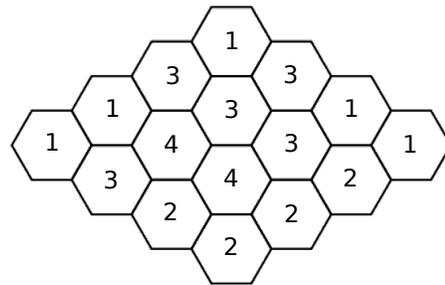
Question:

A bear studies how many hexagons in a honeycomb contain honey. For each hexagon, the bear records how many *other* hexagons touching this hexagon contain honey. The results of the bear's study are shown. How many hexagons contain honey?

Honeycomb 1

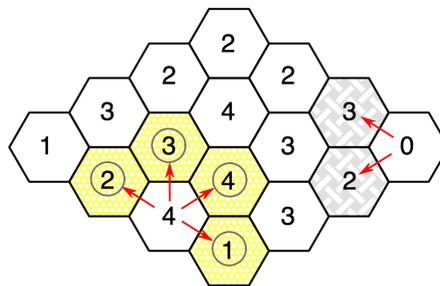


Honeycomb 2

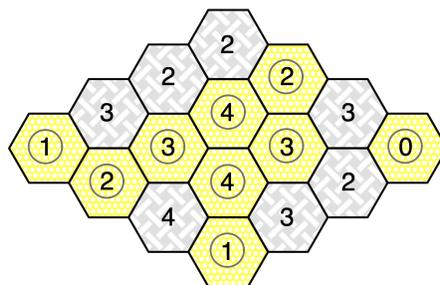


Solution Honeycomb 1:

One way to solve this is to start from a hexagon that contains a zero, because that tells us none of the hexagons touching it contain honey. Another way to solve this is to start from a hexagon that contains a number that is equal to the total number of hexagons touching it, because that tells us all of those touching hexagons contain honey. In this honeycomb we could do either strategy as shown below, where hexagons that contain honey are yellow with a circle around the number, and hexagons that do not contain honey are filled with a grey woven pattern.



After this first step, we can move through the honeycomb, determining which hexagons contain honey based on the number inside each hexagon and the conclusions we have already made about the hexagons touching it. The completed honeycomb should look like this.



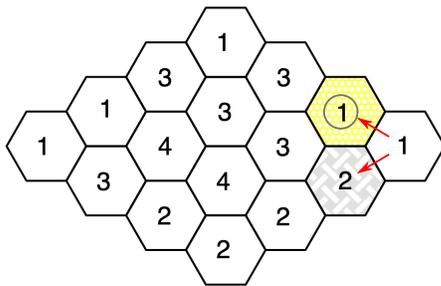
Therefore, 9 hexagons contain honey.



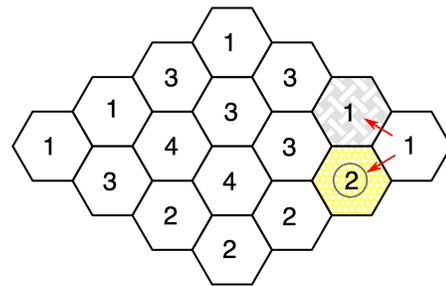
Solution Honeycomb 2:

This honeycomb is more challenging than the first honeycomb because there does not appear to be a good place to start. No hexagon contains a zero, and no hexagon contains a number equal to the total number of hexagons touching it. To solve this honeycomb one strategy is trial and error. Pick a hexagon to start with. Look at all the different options for the hexagons touching it.

For example, suppose we started with the rightmost hexagon in the honeycomb. This might be a good place to start because it is not touching many other hexagons. Since it contains the number one, that tells us that exactly one of the two hexagons touching it contains honey. So we have two options as shown below, where hexagons that contain honey are yellow with a circle around the number, and hexagons that do not contain honey are filled with a grey woven pattern.



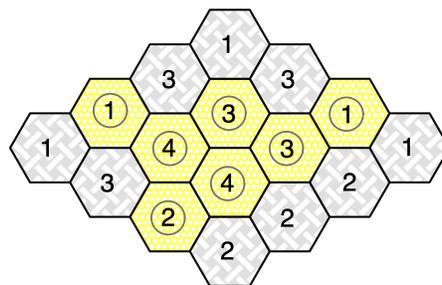
First possibility



Second possibility

We know that one of these pictures must be the correct one, but we cannot be sure which one is correct at this time. So how about we make a guess? For example, we can colour the honeycombs as shown in the second possibility above, and then see if we can move through the honeycomb colouring hexagons from there. If we find that something goes wrong, or we get stuck, then we can always go back to where we made our first choice and make a different choice.

In the end, there is only one way to colour the honeycomb that agrees with all of the numbers in the hexagons. The completed honeycomb should look like this.



Therefore, 7 hexagons contain honey.

Were you able to find this solution on your own? If not, then look at the picture above and check for yourself that this colouring works!